THE ENIGMA OF THE LEANING TOWER OF PISA

The Sixth Spencer J. Buchanan Lecture

by

John B. Burland

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INTRODUCTION

In 1989 the civic tower of Pavia collapsed without warning, killing four people. The Italian Minister of Public Buildings and Works appointed a commission to advise on the stability of the Pisa Tower. The commission recommended closure of the Tower to the general public and this was instituted at the beginning of 1990. There was an immediate outcry by the Mayor and citizens of Pisa who foresaw the damage that the closure would inflict on the economy of Pisa, heavily dependent on tourism as it is. In March 1990 the Prime Minister of Italy set up a new Commission, under the chairmanship of Professor Michele Jamiolkowski, to develop and implement measures for stabilising the Tower. It was the sixteenth commission this century and its membership covered a number of disciplines including structural and geotechnical engineering, architecture, architectural history, archaeology and restoration.

It is instructive to imagine a tower, founded on jelly and slowly inclining to the point at which it is about to fall over. Any support would also have to rest on the jelly. Worse still, the masonry composing the tower is so fragile that it could explode at any time. This is a reasonable description of the state of the Leaning Tower of Pisa, and helps to explain why stabilising it represents the ultimate civil engineering challenge.

DETAILS OF THE TOWER AND GROUND PROFILE

Fig. 1 shows a cross-section through the Tower. It is nearly 60m high and the foundations are 19.6m in diameter. The weight of the Tower is 14,500t. At present the foundations are inclined due south at about 5.5 degrees to the horizontal. The average inclination of the axis of the Tower is somewhat less due to its slight curvature as will be discussed later. The seventh cornice overhangs the first cornice by about 4.5m.

Construction is in the form of a hollow cylinder. The inner and outer surfaces are faced with marble and the annulus between these facings is filled with rubble and mortar within which extensive voids have been found. The spiral staircase winds up within the annulus.

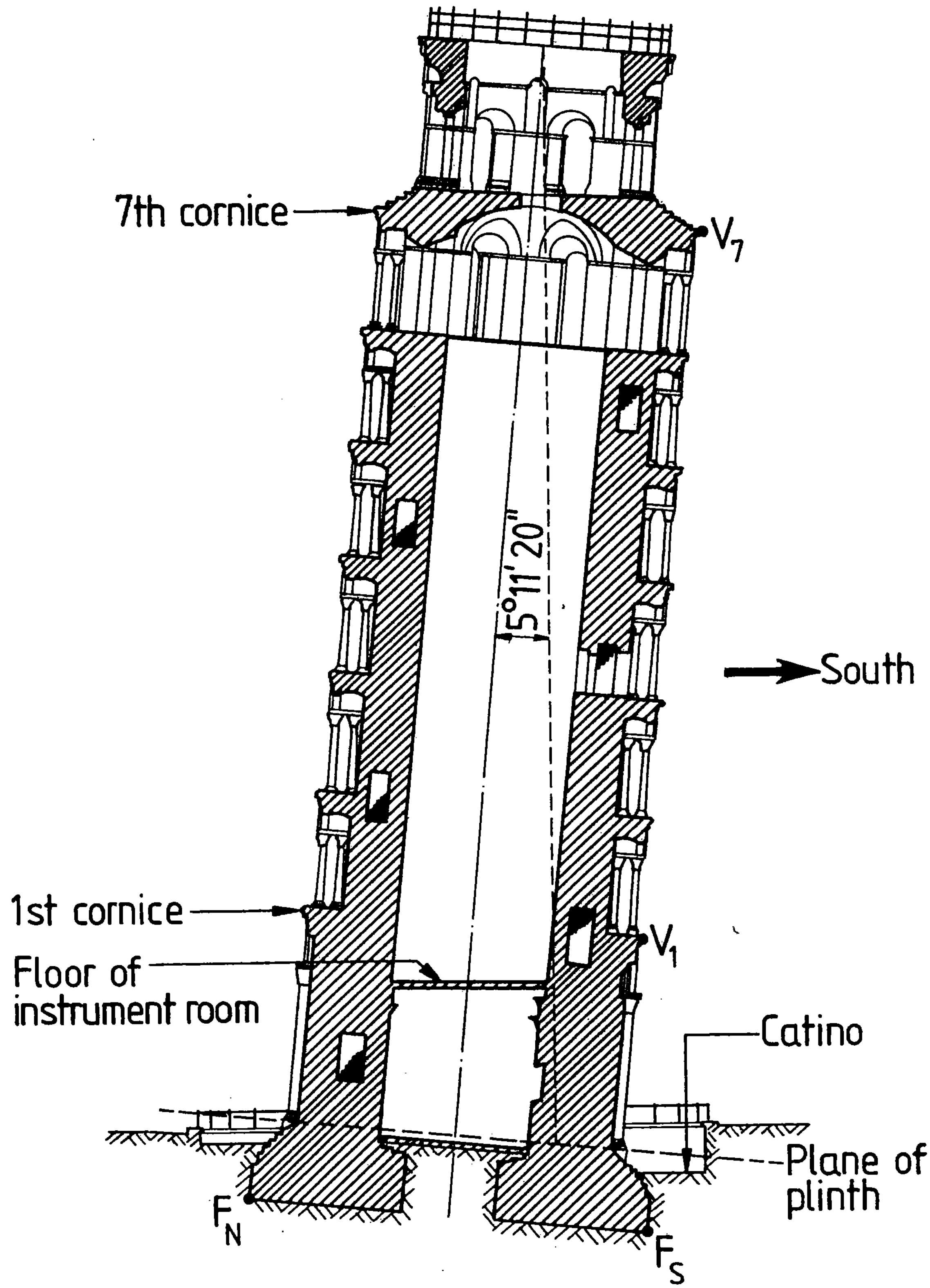

Fig. 1 Cross-section through Tower

Fig. 2 shows the ground profile underlying the Tower. It consists of three distinct layers. Layer A is about 10m thick and primarily consists of estuarine deposits laid down under tidal conditions. As a consequence the soil types consist of rather variable sandy and clayey silts. At the bottom of Layer A is a 2m thick medium dense fine sand layer (the upper sand). Based on sample descriptions and cone tests, the material to the south of the Tower appears to be more clayey than to the north and the sand layer is locally much thinner. Therefore, to the south Layer A could be expected to be slightly more compressible than to the north.

Layer B consists of soft sensitive normally consolidated marine clay which extends to a depth of about 40m. The upper clay, known as the Pancone Clay, is very sensitive to disturbance which causes it to lose strength. The lower clay is separated from the Pancone Clay by a sand layer (the intermediate sand) overlain by a layer if stiffer clay (the intermediate clay). The Pancone Clay is laterally very uniform in the vicinity of the Tower. Layer C is a dense sand which extends to considerable depth (the lower sand).

The water table in Horizon A is between 1m and 2m below ground surface. Pumping from the lower sand has resulted in downward seepage from Layer A with a vertical pore pressure distribution through Layer B which is slightly below hydrostatic.

The many borings beneath and around the Tower show that the surface of the Pancone Clay is dished beneath the Tower from which it can be deduced that the average settlement is between 2.5m and 3.0m.

HISTORY OF CONSTRUCTION

The Tower is a campanile for the Cathedral, construction of which began in the latter half of the 11th Century. Work on the Tower began on 9th August 1173 (by the modern calendar). By about 1178 construction had progressed to about one quarter of the way up the fourth storey when work stopped. The reason for the stoppage is not known but had it continued much further the foundations would have experienced a bearing capacity failure within the Pancone Clay. The work recommenced in about 1272, after a pause of nearly 100 years, by which time the strength of the clay had increased due to consolidation under the weight of the Tower. By about 1278 construction had reached the 7th cornice when work again stopped - possibly due to military action. Once again there can be no doubt that, had work continued, the Tower would have fallen over. In about 1360 work on the bell chamber was commenced and was completed in about 1370 - nearly 200 years after commencement of the work.

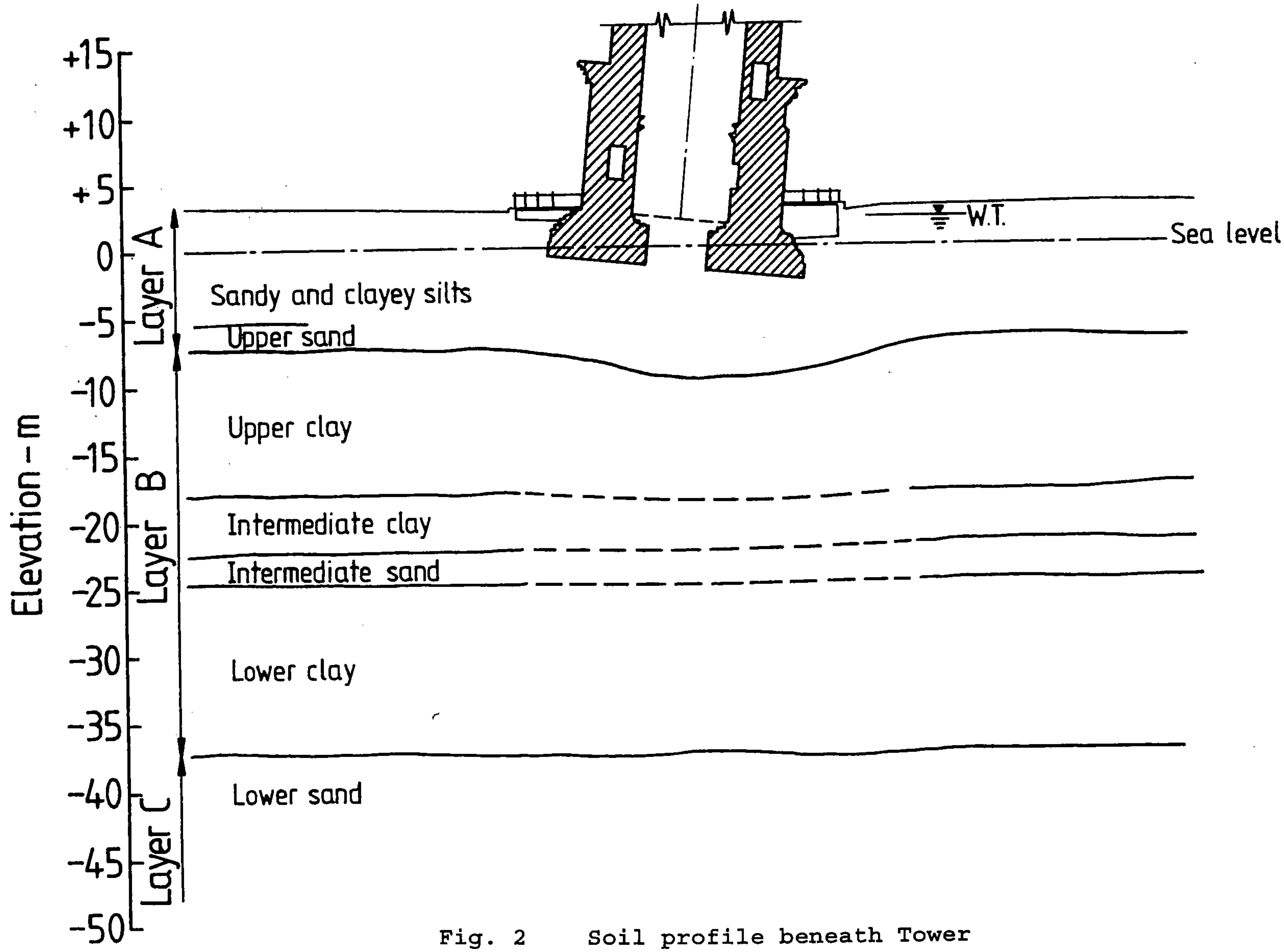

Fig. 2 Soil profile beneath Tower

It is known that the Tower must have been tilting to the south when work on the bell chamber was commenced as it is noticeably more vertical than the remainder of the Tower. Indeed on the north side there are four steps from the seventh cornice up to the floor of the bell chamber while on the south side there are six steps. Another important historical detail is that in 1838 the architect Alessandro Della Gherardesca excavated a walk-way around the foundations. This is known as the catino and its purpose was to expose the column plinths and foundation steps for all to see as was originally intended. This activity resulted in an inrush of water on the south side, since here the excavation is below the water table, and there is evidence to suggest that the inclination of the Tower increased by as much as half a degree. As is described later, Gherardesca left us an unpleasant surprise.

HISTORY OF TILTING

One of the first actions of the Commission was to undertake the development of a computer model of the Tower and the underlying ground that could be used to assess the effectiveness of various possible remedial measures. Calibration of such a model is essential and the only means of doing this is to attempt to simulate the history if tilting of the Tower during and subsequent to its construction. Hence it became apparent very early on that we needed to learn as much as possible about the history of the tilt of the Tower. In the absence of any documentary evidence all the clues to the history of tilt lie in the adjustments made to the masonry layers during construction and in the shape of the axis of the Tower.

Over the years a number of measurements of the dimensions of the Tower have been made and many of them are conflicting. The Polvani Commission measured the thickness of each of the masonry layers and its variation around the Tower [1]. This information has proved extremely valuable in unravelling the history of tilt.

Fig. 3 shows the shape of the axis of the Tower deduced from the measured relative inclinations of the masonry layers assuming that construction proceeded perpendicular to each masonry layer. This shape compares favourably with other independent measurements at a few locations up the Tower. It can be seen that the axis is curved. For years the Tower has been unkindly referred to as having a banana shape. I prefer to call it a question mark (?) so as to reflect the enigma of the Tower.

Some important observations can be made from the measurements on the masonry layers. For most of the storeys, construction took place using parallel sided blocks of masonry. With one or two notable exceptions adjustments only took place close to each floor using tapered blocks. The most important exception can be seen in Fig. 3 where there is an obvious kink one quarter the

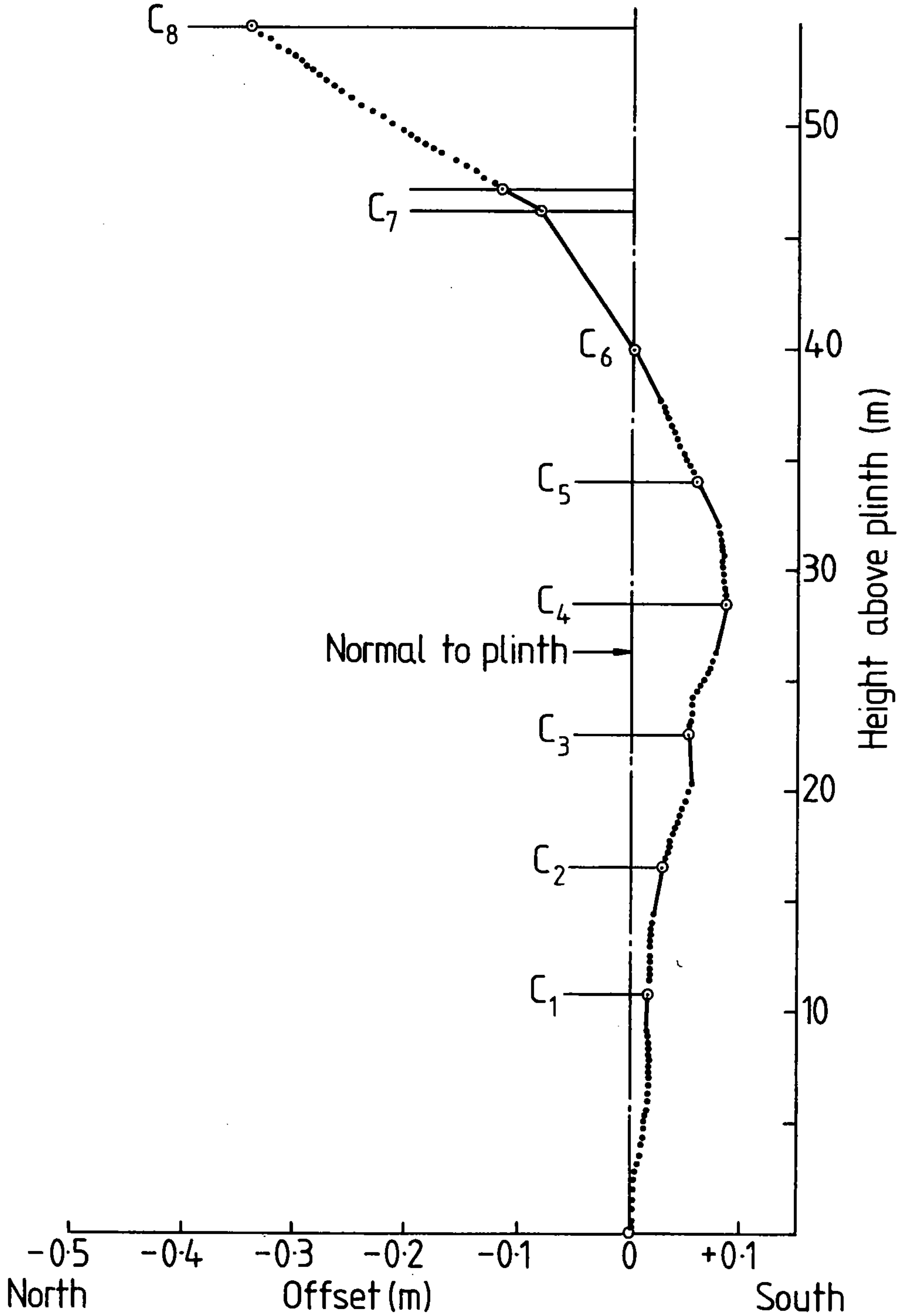

Fig. 3 Shape of axis of Tower deduced from relative inclinations of masonry layers

way up the fourth storey. It will be recalled that the construction remained at this level for about 100 years. Evidently the Tower was tilting significantly when work recommenced and the masons made adjustments to correct it.

We see that the history of the tilting of the Tower is tantalisingly frozen into the masonry layers. If only we knew the rules that the masons followed in adjusting for the tilt we would be able to unravel the history. We have to put ourselves in the place of a mason or architect in the 12th or 13th Century and ask ourselves:

> 'What is the most practical thing to do when you arrive at a given floor and find that the tower is out-of-plumb?'

A widely accepted hypothesis is that the masons would always try to keep the masonry layers horizontal and the Polvani Commission adopted this. Although this seems reasonable for a low aspect ratio building like a cathedral, it does not make sense for a tower since it would tend to perpetuate the overall out-of-plumb. After a few trials, a child building a tower of bricks on a carpet will soon learn to compensate for any tilt by attempting to place successive bricks over the centre of the base of the tower i.e. by bringing the centre of the tower back vertically over the centre of the foundations (or possibly even further, away from the direction of tilt). Therefore an alternative hypothesis is one in which the masons aimed to bring the centre line of the tower back, vertically over the centre of the foundations at the end of each storey. The architectural historians on the Commission were satisfied that the masons would have had the technology to make such an adjustment, particularly as the stones for each storey were carved and assembled on the ground prior to hoisting into position.

Fig. 4 shows the re-constructed history of inclination of the foundations of the Tower using the alternative hypothesis [2]. In this figure the weight of the Tower is plotted against the deduced inclination. It can be seen that initially the Tower inclined slightly to the north amounting to about 0.2 degrees in 1272. As construction proceeded the Tower began to move towards the south at an increasing rate. In 1278, when construction had reached the seventh cornice, the tilt was about 0.6 degrees. During the 90 year pause, the tilt increased to about 1.6 degrees. After the completion of the bell chamber in about 1370 the inclination of the Tower increased dramatically. The point dated 1817 is based on measurements made by two British architects Cressy and Taylor using a plumb line. A further measurement was made by the Frenchman Ruhault de Fleury in 1859 which showed that the excavation of the catino by Gherardesca in 1838 caused a significant increase of inclination. The history of tilting depicted in Fig. 4 has been used to calibrate numerical and physical models of the Tower and underlying ground.

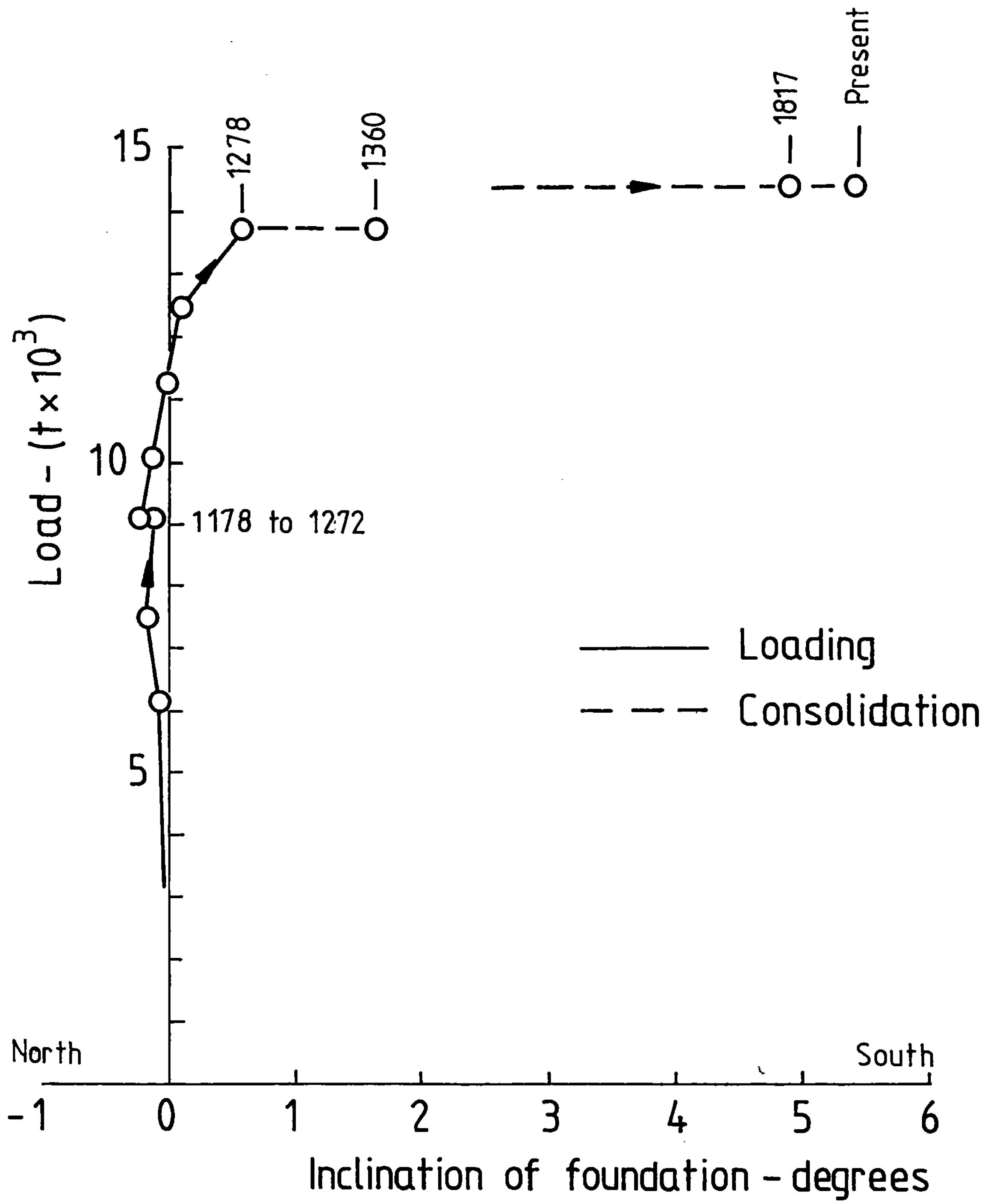

Fig. 4 Deduced history of inclination of Tower during and subsequent to construction

COMPUTER MODELLING OF THE MOVEMENTS OF THE TOWER

The analysis was carried out using a suite of finite element geotechnical computer programs developed at Imperial College and known as ICFEP [3]. The constitutive model is based on Critical State concepts [4] and is non-linear elastic work-hardening plastic. Fully coupled consolidation is incorporated so that time effects due to the drainage of pore water out of or into the soil skeleton are included.

It must be emphasised that the prime objective of the analysis was to develop an understanding of the mechanisms controlling the behaviour of the Tower [5]. Accordingly a plane strain approach was used for much of the work and only later was three-dimensional analysis used to explore certain detailed features.

The layers of the finite element mesh matched the soil sub-layering that had been established from numerous extensive soil exploration studies. Fig. 5 shows the mesh in the immediate vicinity of the foundation. In Layer B (see Fig. 2) the soil is assumed to be laterally homogeneous. However a tapered layer of slightly more compressible material was incorporated into the mesh for Layer A1 as shown by the shaded elements in Fig. 5. This slightly more compressible region represents the more clayey material found beneath the south side of the foundation as discussed in Section 2. In applied mechanics terms the insertion of this slightly more compressible tapered layer may be considered to act as an 'imperfection'. The overturning moment generated by the lateral movement of the centre of gravity of the Tower was incorporated into the model as a function of the inclination of the foundation as shown in Fig 5.

The analysis was carried out in a series of time increments in which the loads were applied to the foundation to simulate the construction history of the Tower. The excavation of the catino in 1838 was also simulated in the analysis. Calibration of the model was carried out by adjusting the relationship between the overturning moment generated by the centre of gravity and the inclination of the foundation. A number of runs were carried out with successive adjustments being made until good agreement was obtained between the actual and the predicted present day value of the inclination.

Fig. 6 shows a graph of the predicted changes in inclination of the Tower against time, compared with the deduced historical values. It is important to appreciate that the only point that has been pre-determined in the analysis is the present-day value. The model does not simulate the initial small rotation of the Tower to the north. However, from about 1272 onwards there is remarkable agreement between the model and the historical inclinations. Note that it is only when the bell chamber

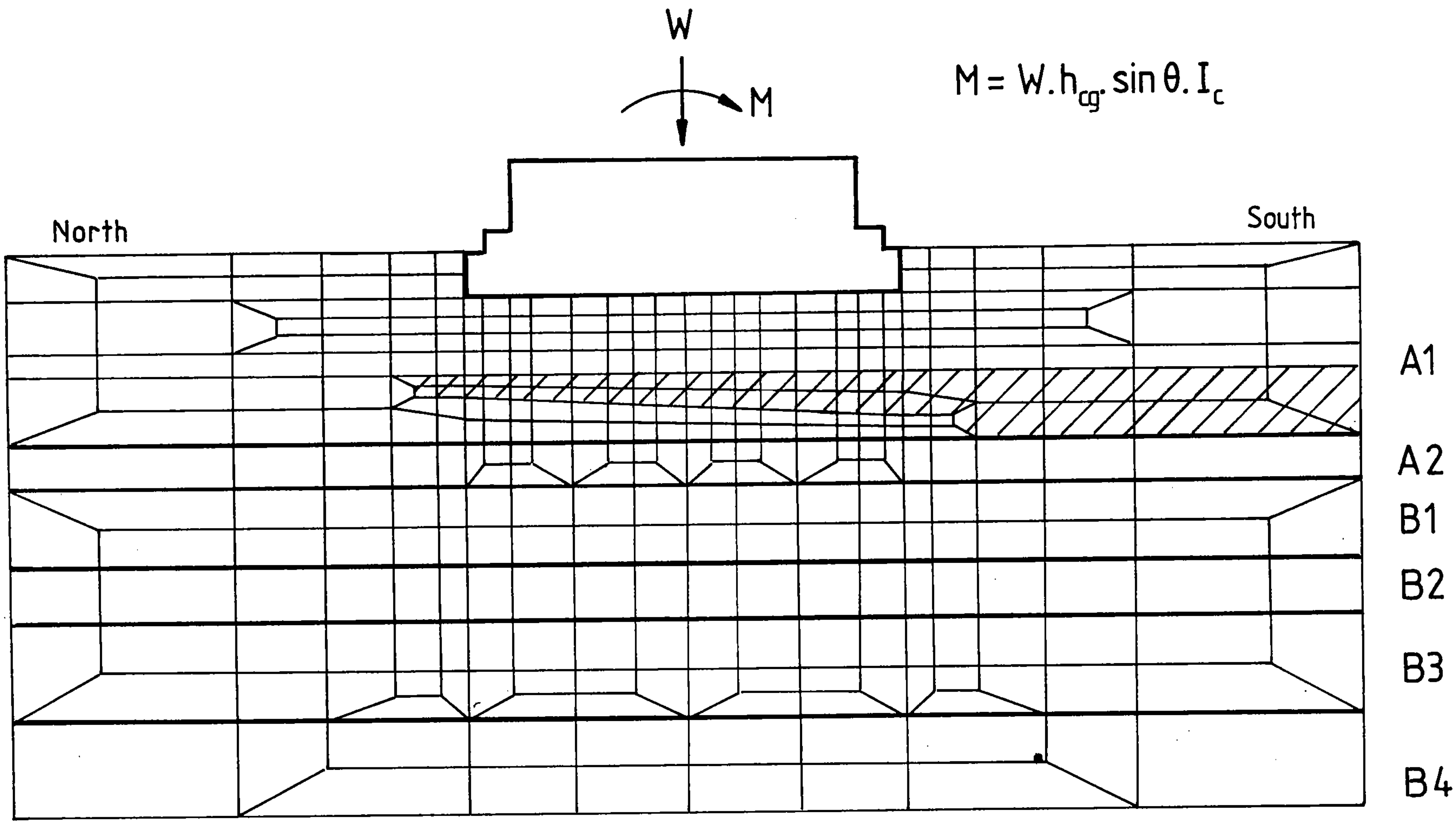

Fig. 5 Finite element mesh in the vicinity of the tower foundation

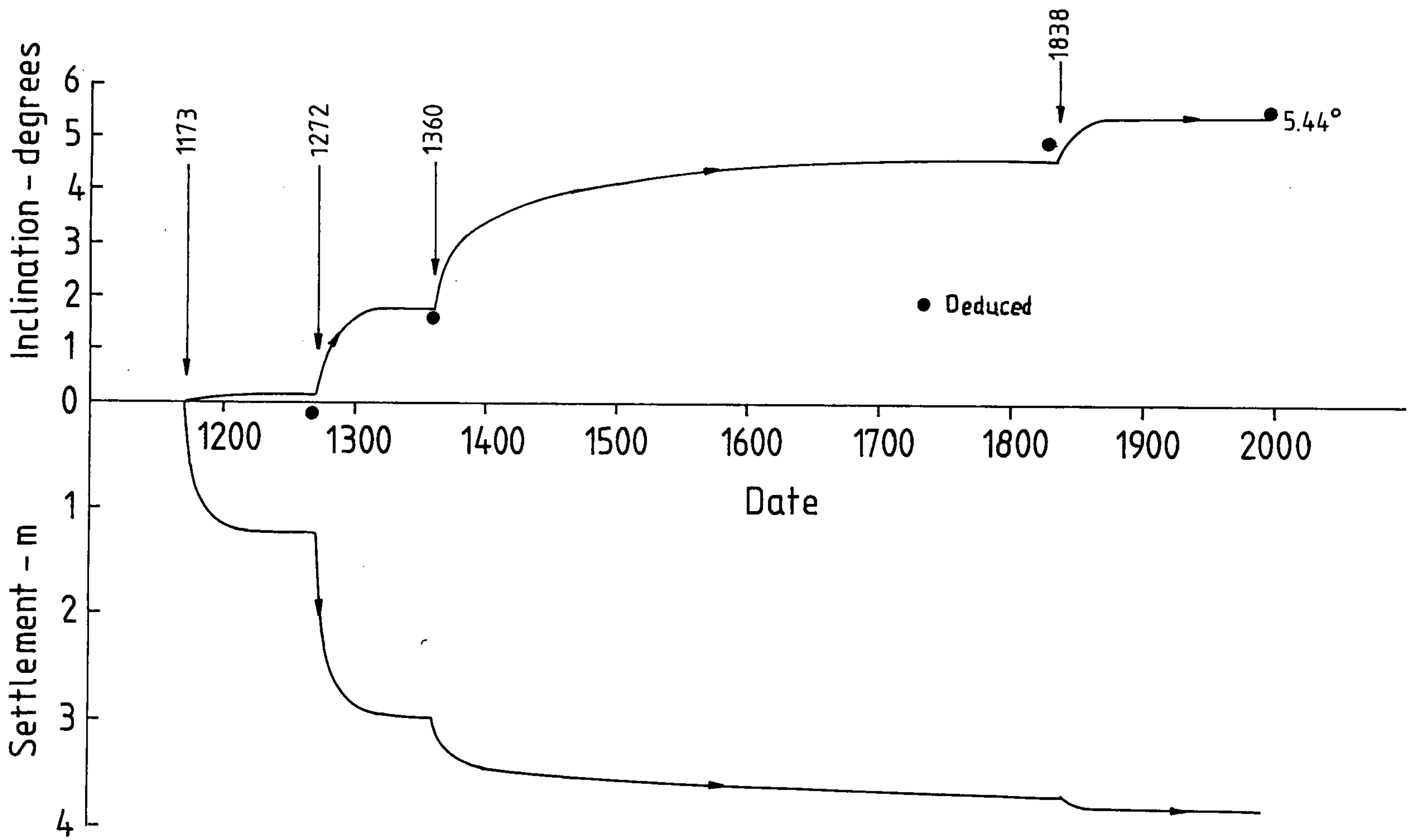

Fig. 6 Relationship between time, inclination and settlement for the plane strain Finite Element simulation of the history of the Pisa Tower

was added in 1360 that the inclination increases dramatically. Also of considerable interest is the excavation of the catino in 1838 which results in a predicted rotation of about 0.75 degrees. It should be noted that the final imposed inclination of the model tower is 5.44 degrees which is slightly less than the present-day value of 5.5 degrees. It was found that any further increase in the final inclination of the model tower resulted in instability - a clear indication that the Tower is very close to falling over.

Burland and Potts [5] concluded from a careful study of the computer model that the impending instability of the Tower foundation is not due to a shear failure of the ground but can be attributed to the high compressibility of the Pancone Clay. This phenomenon was called 'leaning instability' by the late Edmund Hambly [6] who used it to explain the lean of the Pisa Tower. No matter how carefully the structure is built, once it reaches a critical height the smallest perturbation will induce leaning instability. As pointed out by Hambly: '... leaning instability is not due to lack of strength of the ground but is due to insufficient stiffness, i.e. too much settlement under load'. Children building brick towers on a soft carpet will be familiar with this phenomenon!

In summary, the Finite Element model gives remarkable agreement with the deduced historical behaviour of the Tower. It is important to emphasise that the predicted history of foundation inclinations and overturning moments were self generated and were not imposed externally in a pre-determined way. The only quantity that was used to calibrate the model was the present-day inclination. The analysis has demonstrated that the lean of the Tower results from the phenomenon of leaning instability due to the high compressibility of the Pancone Clay. The role of the layer of slightly increased compressibility beneath the south side of the foundations is to act as an 'imperfection'. Its principal effect is to determine the direction of lean rather than its magnitude. The main limitation of the model is that it is a plane strain one rather than fully three-dimensional. Also, the constitutive model does not deal with creep so that no attempt has been made to model the small time-dependent rotations that have been taking place this century and which are described in the next Section. Nevertheless the model provides important insights into the basic mechanisms of behaviour and has proved valuable in assessing the effectiveness of various proposed stabilisation measures. Its role in evaluating the effectiveness of the temporary counterweight solution is described later.

OBSERVED BEHAVIOUR OF THE TOWER THIS CENTURY

Change of inclination

For most of this century the inclination of the Tower has been increasing. The study of these movements has been important in developing an understanding of the behaviour of the Tower and has profoundly influenced the decisions taken by the Commission. It is important to appreciate that the magnitudes of the movements are about three orders of magnitude less than the movements that occurred during construction. Thus changes in inclination are measured in arc seconds rather than degrees (one arc second equals 1/3600th of a degree).

Fig. 7 is a plan view of the *Piazza dei Miracoli* showing the location of the Baptistry, Cathedral and Tower. Since 1911 the inclination of the Tower has been measured regularly by means of a theodolite. The instrument is located at the station marked E and the angles between station D and the first cornice (V1 in Fig. 1) and between station D and the seventh cornice (V7) are measured. The difference between these two angles is used to calculate the vertical offset between the seventh and first cornice and hence the inclination of the Tower.

In 1928 four levelling stations were placed around the plinth level of the Tower and were referred to a bench mark on the Baptistry. Readings were taken in 1928 and 1929 but not again until 1965. In 1965, fifteen levelling points were installed around the Tower at plinth level and about seventy surveying monuments were located around the Piazza.

In 1934 a plumb line was installed in the Tower, suspended from the sixth floor and observed in an instrument room whose location is shown in Fig. 1. The instrument was designed by the engineers Girometti and Bonechi and is known as the GB pendulum. Also in 1934 a 4.5m long spirit level was installed within the instrument room. The instrument rests on brackets embedded in the masonry and can be used to measure both the north-south and the east-west inclination of the Tower. The instrument was designed by the officials of the Genio Civile di Pisa and is known as the GC level.

Fig. 8 shows the change of inclination with time since 1911. From 1934 to 1969 the GC level was read regularly once or twice a year except during the second world war. For some reason readings with the GC level ceased in 1969 but fortunately precision levelling on the fifteen points around the Tower began in 1965 and continued regularly until 1985. In 1990 Professor Carlo Viggiani and I read the GC level again and found that the inclination agreed to within a few seconds of arc with that derived from the precision levelling around the plinth.

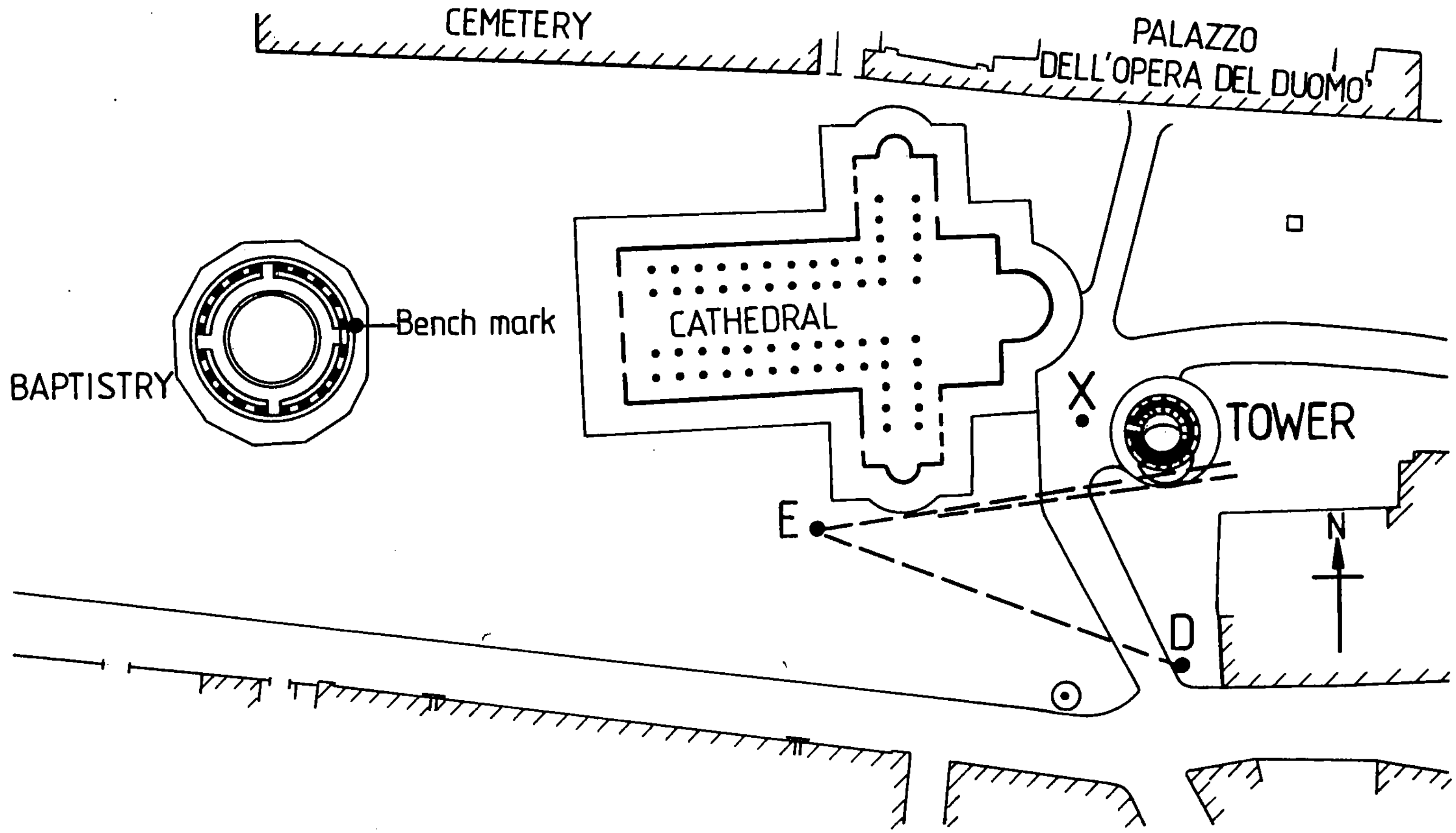

Fig. 7 Plan of the Piazza dei Miracoli

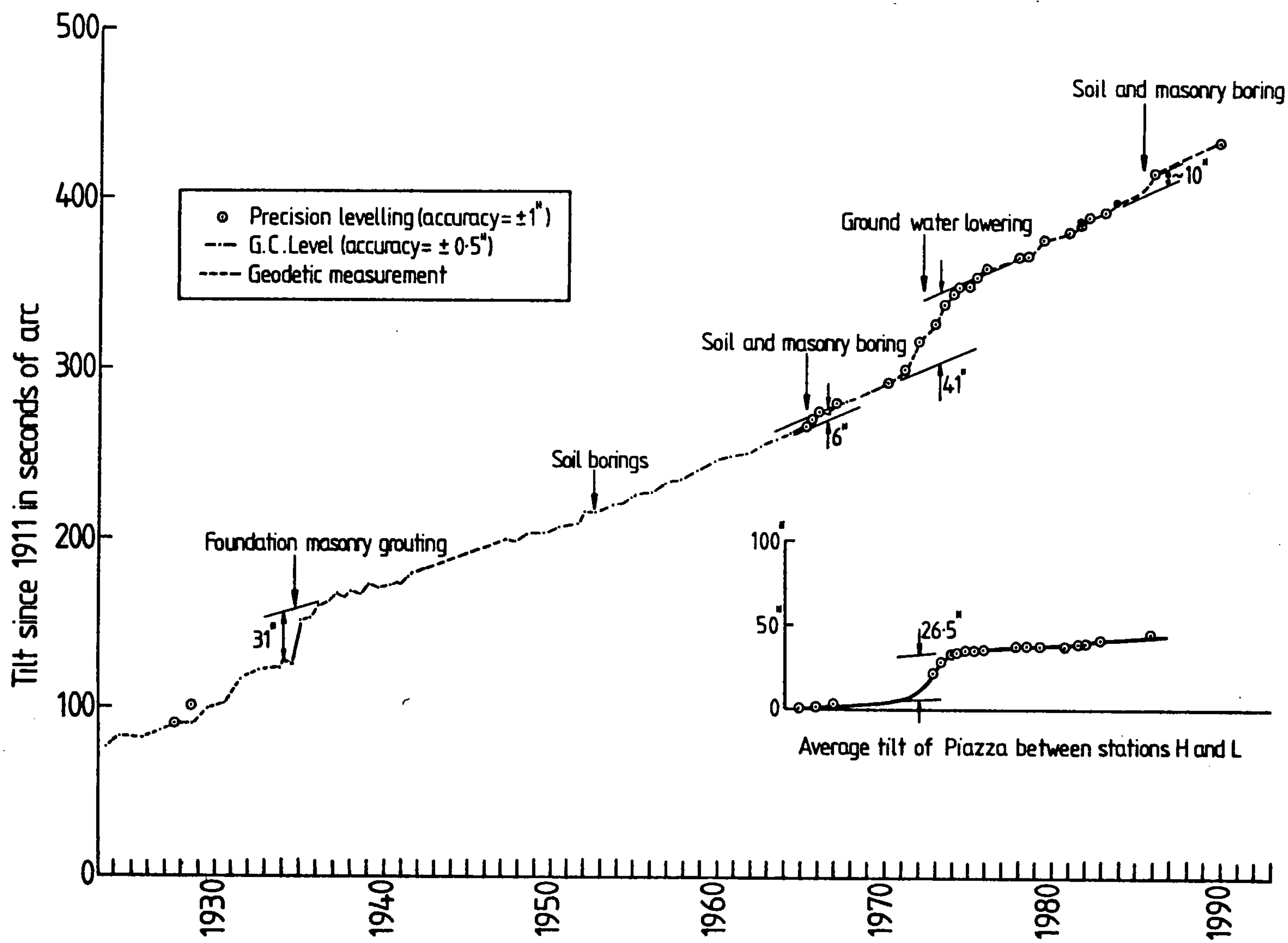

Fig. 8 Change in inclination of the foundations since 1911

It can be seen from Fig. 8 that the inclination-time relationship for the Tower is not a smooth curve but contains some significant 'events'. In 1934, Girometti drilled 361 holes into the foundation masonry and injected about 80t of grout with a view to strengthening the masonry. This activity caused a sudden increase in tilt of 31 arc seconds. Like Gherardesca, Girometti also left us an unpleasant surprise as is described later. In 1966 some soil and masonry drilling took place and caused a small but distinct increase of tilt of about 6 arc seconds. Again, in 1985 an increase in tilt of 10 arc seconds resulted from masonry boring through the foundations. In the late 1960's and early 1970's pumping from the lower sands caused subsidence and tilting towards the south-west of the *Piazza*. This induced a tilt of the Tower of about 41 arc seconds. When pumping was reduced the tilting of the Tower reduced to its previous rate. It is clear from these events that the inclination of the Tower is very sensitive to even the smallest ground disturbance. Hence any remedial measures should involve a minimum of such disturbance. The rate of inclination of the Tower in 1990 was about 6 seconds of arc per annum or about 1.5mm at the top of the Tower.

The motion of the Tower foundation

Previously, studies have concentrated on the changes of inclination of the tower. Little attention has been devoted to the complete motion of the foundations relative to the surrounding ground. The theodolite and precision levelling measurements help to clarify this. It will be recalled from Fig. 7 that angles were measured relative to the line ED. Hence it is possible to deduce the horizontal displacements of the tower relative to point D. Fig. 9 shows a plot of the horizontal displacement of point V_1 on the first cornice relative to point D since 1911. Also shown, for comparison, is the relative vertical displacements between the north and south sides of the foundation (points F_N and F_S). It can be seen that up to 1934 the horizontal movement of V_1 was very small. Between 1935 and 1938, following the work of Girometti, point V_1 moved southwards by about 5mm. No further horizontal movement took place until about 1973 when a further southward movement of about 3mm to 4mm took place as a result of the ground water lowering. A further small horizontal movement appears to have taken place in about 1985 as a result of masonry drilling at that time. These observations reveal the surprising fact that during steady state creep-rotation point V_1 on the first cornice does not move horizontally. Horizontal movements to the south only take place when disturbance to the underlying ground takes place.

Study of the precision levelling results shows that between 1928 and 1965 the centre of the foundations at plinth level rose by 0.3mm relative to the Baptistry - a negligible amount. Between 1965 and 1986 the relative vertical

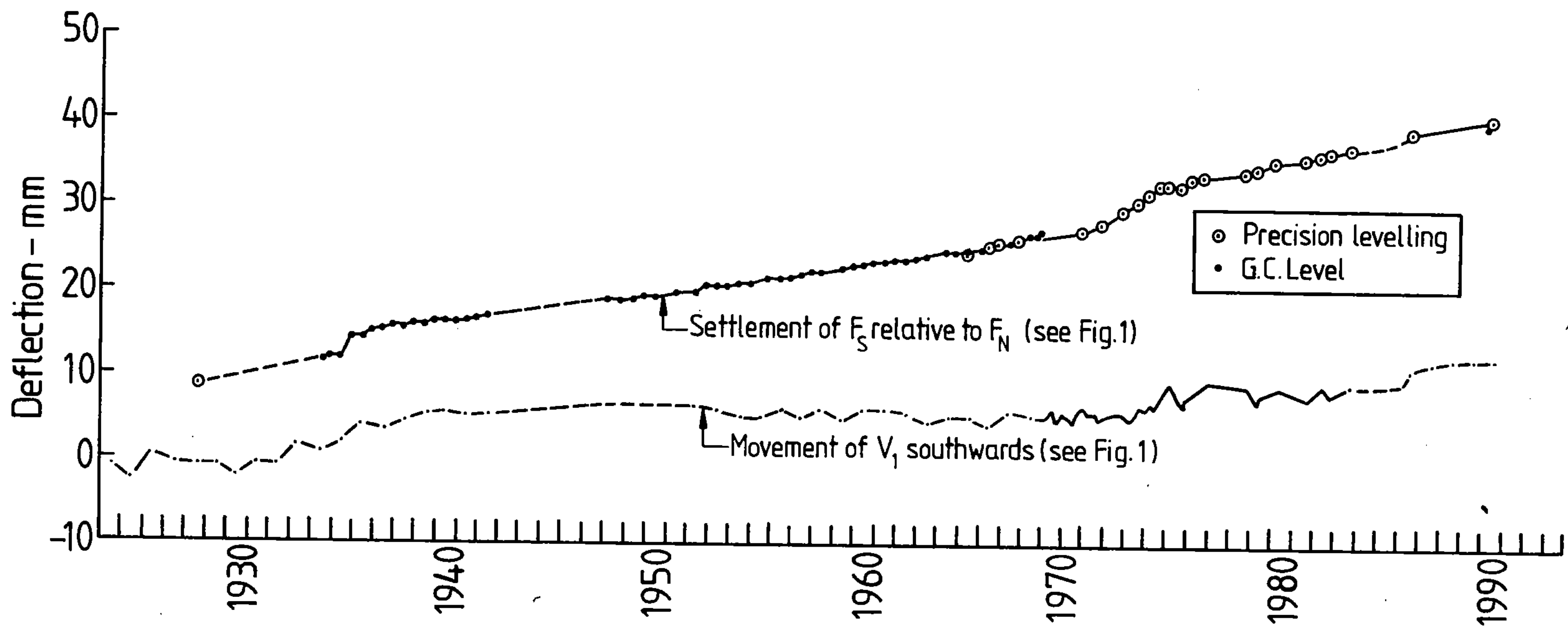

Fig. 9 Horizontal displacement of V1 (see Fig. 1) on the first cornice since 1911

displacement between the centre of the plinth and a point a few metres away from the tower was again negligible. Thus, not only does point V_1 not move horizontally during steady-state creep, but also negligible average settlement of the foundations has taken place relative to the surrounding ground.

The observations described above can be used to define the rigid-body motion of the tower during steady-state creep-rotation as shown in Fig. 10. It can be seen that the Tower must be rotating about a point approximately located level with point V_1 and vertically above the centre of the foundation. The direction of motion of points F_N and F_S are shown by vectors and it is clear that the foundations are moving northwards with F_N rising and F_S sinking.

Conclusions from the observed motion of the Tower foundations

The discovery that the motion of the Tower is as shown in Fig. 10 has turned out to be a most important finding in a number of respects. Previously it had been believed that the foundations were undergoing creep settlements with the south side settling more rapidly than the north. However, the observation that the north side had been steadily rising led to the suggestion that the application of load to the foundation masonry on the north side could be beneficial in reducing the overturning moment [7].

The form of foundation motion depicted in Fig. 10 leads to the very important conclusion that the seat of the continuing long-term tilting of the Tower lies in Horizon A and not within the underlying Pancone Clay as had been widely assumed in the past. It can therefore be concluded that this stratum must have undergone a considerable period of ageing since last experiencing significant deformation. Thus, in developing the computer model, it is reasonable to assume that the clay has an increased resistance to yield subsequent to the excavation of the catino in 1838. This conclusion has proved of great importance in the successful analysis of the effects of applying the lead counterweight [5].

The continuing foundation movements tend to be seasonal. Between February and August each year little change in the north-south inclination takes place. In late August or early September the Tower starts to move southward and this continues through till December or January amounting to an average of about 6 arc seconds. In the light of the observed motion of the Tower foundations, the most likely cause of these seasonal movements is thought to be the sharp rises in ground-water level that have been measured in Horizon A resulting from seasonal heavy rainstorms in the period September to December each year. Thus, continuing rotation of the foundations might be substantially reduced by controlling the water table in Horizon A in the vicinity of the Tower.

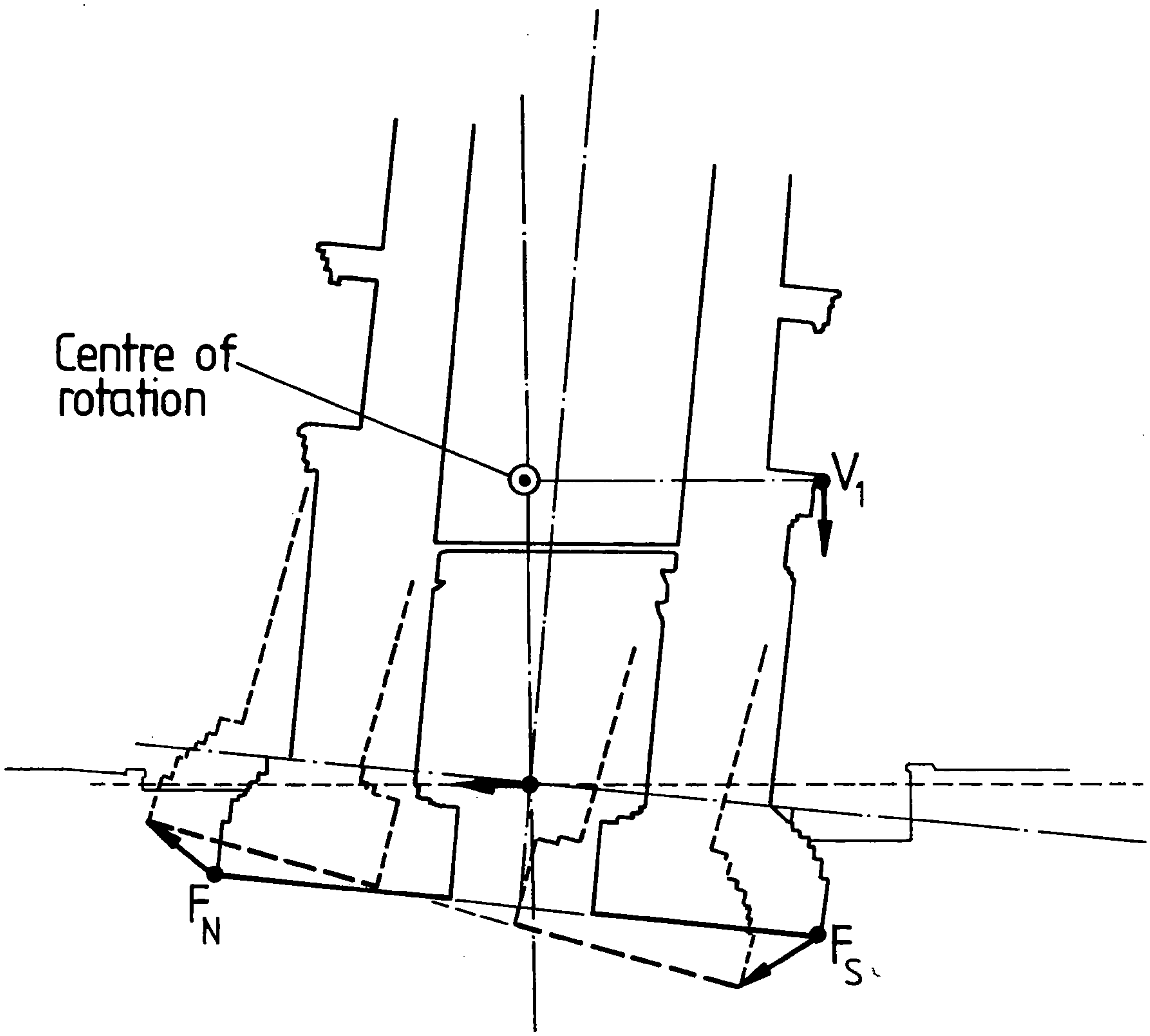

Fig. 10 Motion of the Tower during steady state creep-rotation

TEMPORARY STABILISATION OF THE TOWER

There are two distinct problems that threaten the stability of the Tower. The most immediate one is the strength of the masonry. It can be seen from the cross section in Fig. 1 that at first floor level there is a change in cross section of the walls. This gives rise to stress concentrations at the south side. In addition to this, the spiral staircase can be seen to pass through the middle of this change in cross section giving rise to a significant magnification in the stresses. The marble cladding in this location shows signs of cracking. It is almost impossible to assess accurately the margin of safety against failure of the masonry, but the consequences of failure would be catastrophic. The second problem is the stability of the foundations against overturning.

The approach of the Commission to stabilisation of the Tower has been a two stage one. The first stage has been to secure an increase in the margin of safety against both modes of failure as quickly as possible by means of temporary measures. Having achieved this, the second stage is to develop permanent solutions recognising that these would require time to carry out the necessary investigations and trials. Significant progress has been made with the first stage. It is a prerequisite of restoration work that temporary works should be non-destructive, reversible and capable of being applied incrementally in a controlled manner.

Temporary stabilisation of the masonry

The masonry problem has been tackled by binding lightly pre-stressed plastic covered steel tendons around the Tower at the first cornice and at intervals up the second storey as shown in Fig. 11. The work was carried out in the summer of 1992 and was effective in closing some of the cracks and in reducing the risk of a buckling failure of the marble cladding. The visual impact has proved to be negligible.

Temporary stabilisation of the foundations

As mentioned previously, the observation that the northern side of the foundation had been steadily rising for most of this century led to the suggestion that application of load to the foundation masonry on the north side could be beneficial in reducing the overturning moment. Clearly such a solution would not have been considered if it had not been recognised that leaning instability rather than bearing capacity failure was controlling the behaviour of the Tower or if the north side of the foundation had been settling.

Before implementing such a solution it was obviously essential that a detailed analysis should be carried out. The purpose of such an analysis was two-fold: (i) to ensure that the proposal was safe and did not lead to any undesirable effects and (ii) to provide a best estimate of

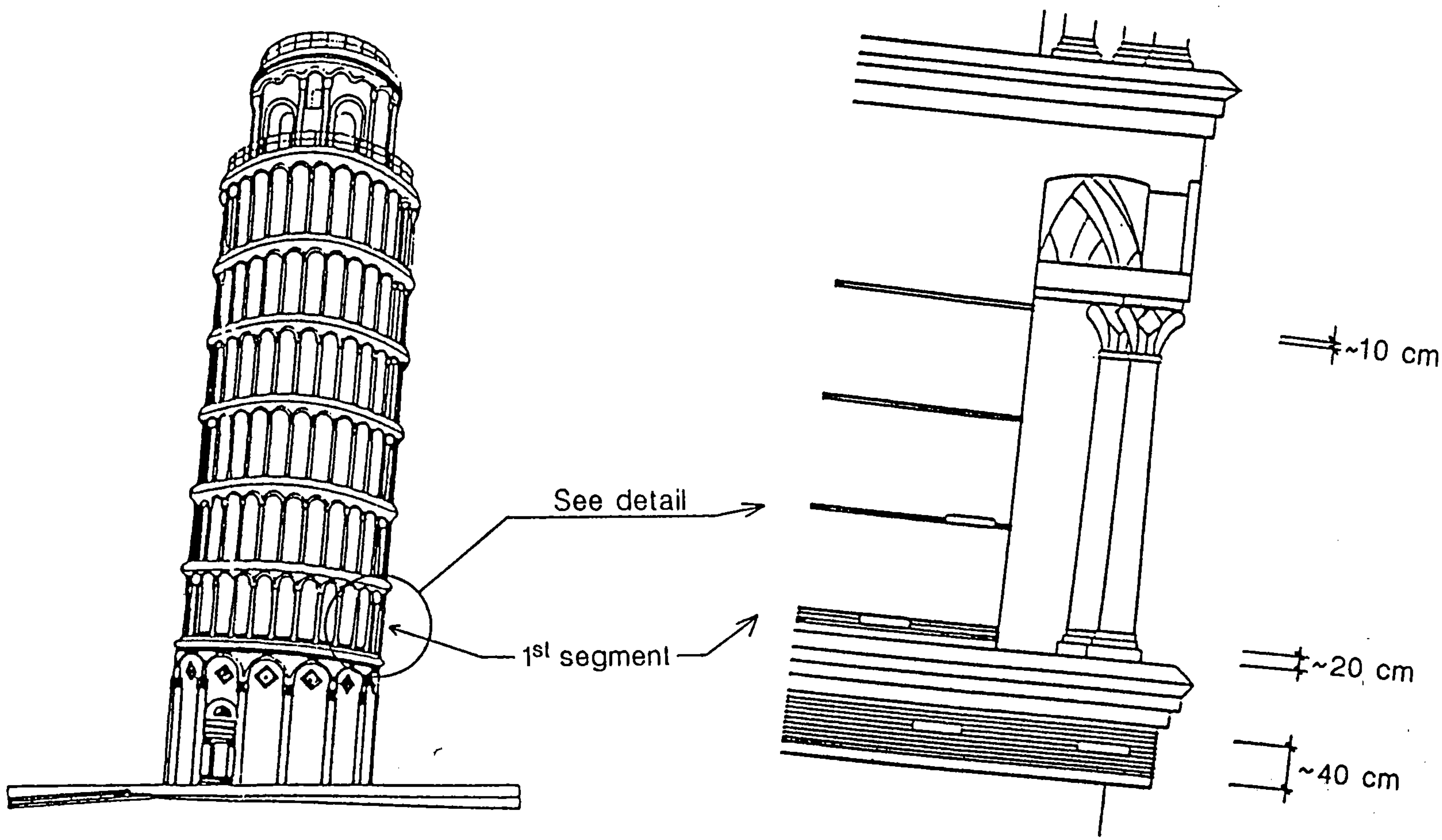

Fig. 11 Temporary stabilisation of the masonry with light circumferential pre-stressing

the response against which to judge the observed response of the Tower as the load was being applied. A detailed description of the analysis is given by Burland and Potts [5] who found that a satisfactory result was only forthcoming if the effects of ageing of the underlying Pancone Clay was incorporated in the computer model. The justification for such ageing lay in the observed motion of the foundations depicted in Fig. 10 as described in the previous section. The computer analysis indicated that it was safe to apply up to a maximum of 1400t load to the north side of the foundation masonry. Above that load there was a risk that the underlying Pancone Clay would begin to yield resulting in a southward rotation of the Tower and excessive settlement of the foundations.

Accordingly a design was developed by Professors Leonhardt and Macchi for the application of a north counterweight and the details of construction are shown in Fig. 12. It consists of a temporary prestressed concrete ring cast around the base of the tower at plinth level. This ring acts as a base for supporting specially cast lead ingots which were placed one at a time at suitable time intervals. The movements experienced by the tower are measured with a highly redundant monitoring system consisting of the following: (i) precision inclinometers and levellometers installed on the wall of the ground floor room, (ii) high precision levelling of 8 survey stations mounted on the wall of the above room and (iii) external high precision levelling of 15 bench marks located around the tower plinth and 24 bench marks located along north-south and east-west lines centred on the tower. All the levels are related to a deep datum installed in the Piazza dei Miracoli by the Commission.

Observed Response

Burland et al [7] describe the response of the Tower to the application of the counterweight. Construction of the concrete ring commenced on 3rd May 1993 and the first lead ingot was placed on 14th July 1993 (see Fig. 13). The load was applied in four phases with a pause between each phase to give time to observe the response of the tower. Fig. 14 shows the application of one the lead ingots. The final phase was split in two either side of the Christmas break. The last ingot was placed on 20th January 1994.

Fig. 15 shows the change of inclination of the tower towards the north during the application of the lead ingots as measured by the internal high precision levelling and the inclinometer placed in the north-south plane. The agreement between the two independent monitoring systems is excellent. (Note that Fig. 15 does not include the inclination induced by the weight of the concrete ring which amounted to about 4 arc seconds). It can be seen that the amount of creep between the phases of load is small. However, subsequent to completion of loading, time dependent northward inclination continued. On 20th

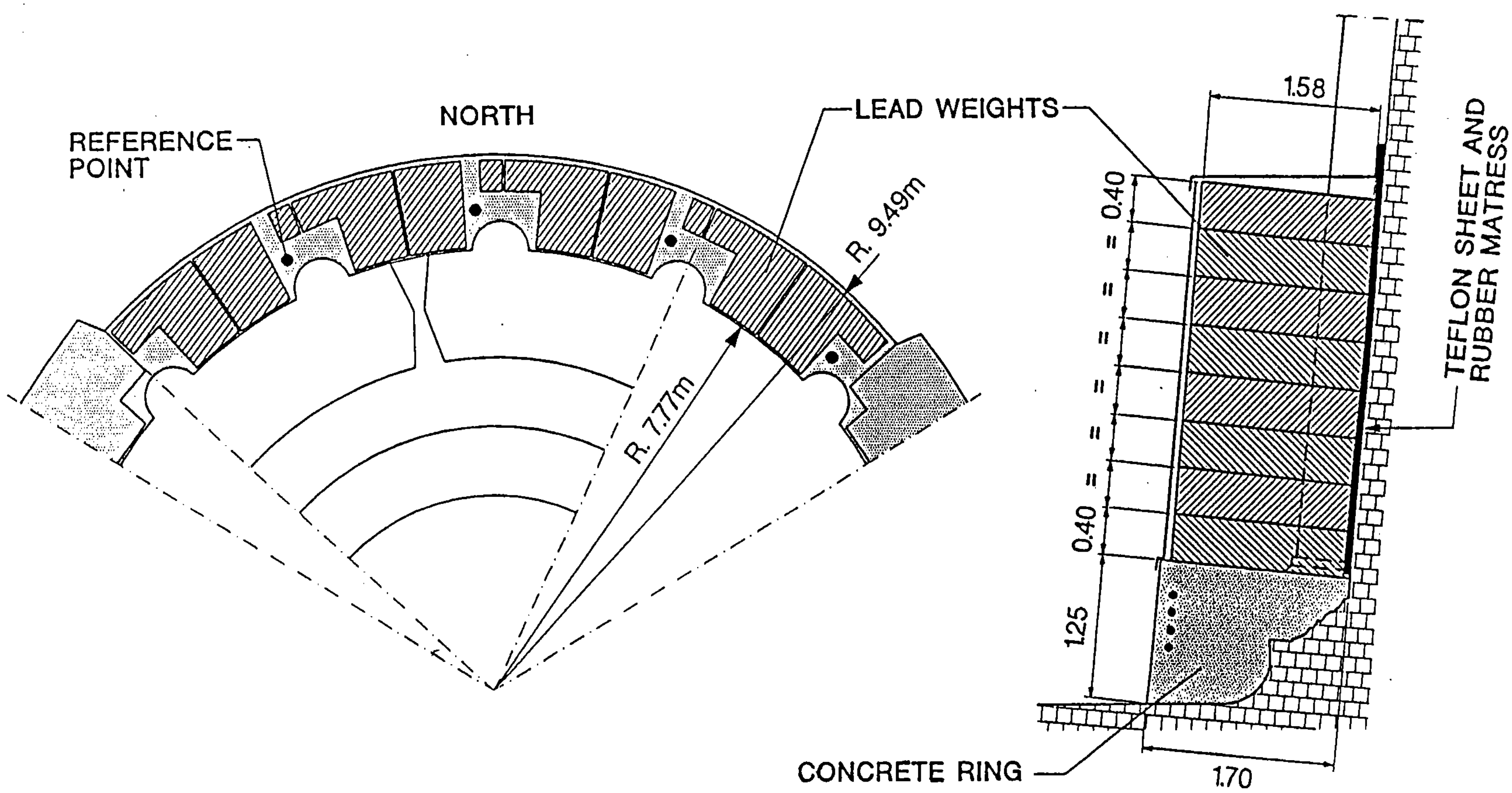

Fig. 12 Details of the north counterweight

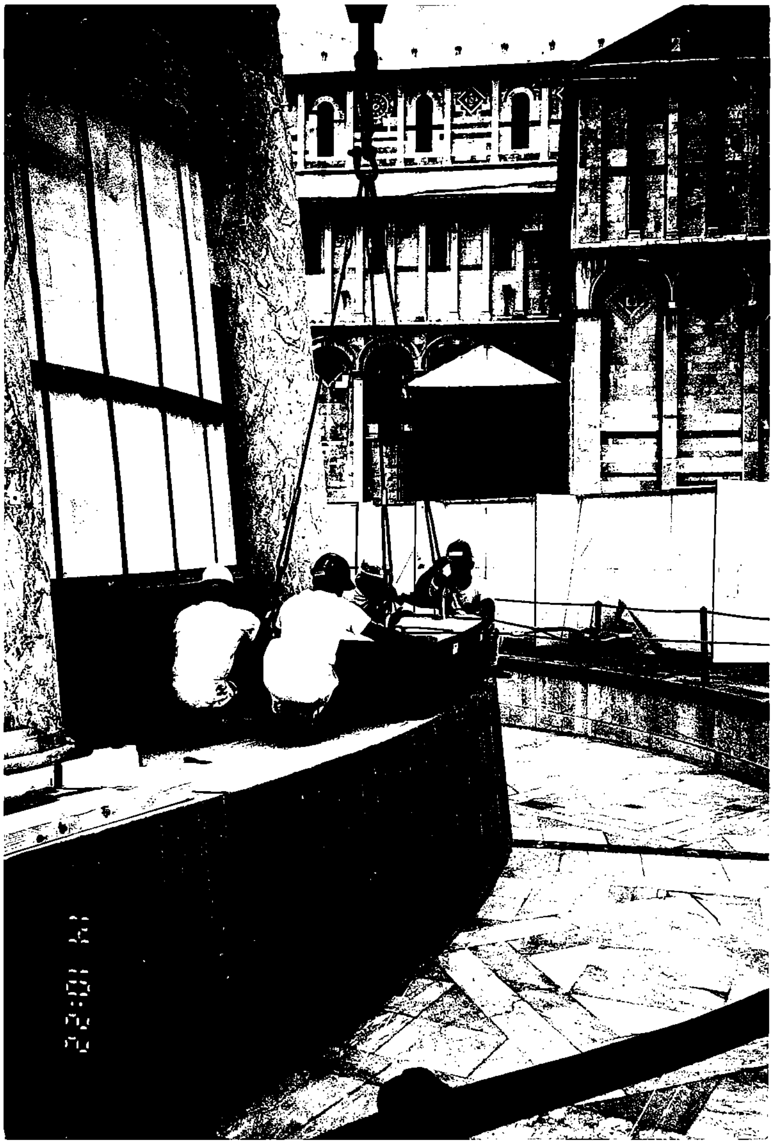

Fig. 13   Placing the first lead ingot on 14th July 1993

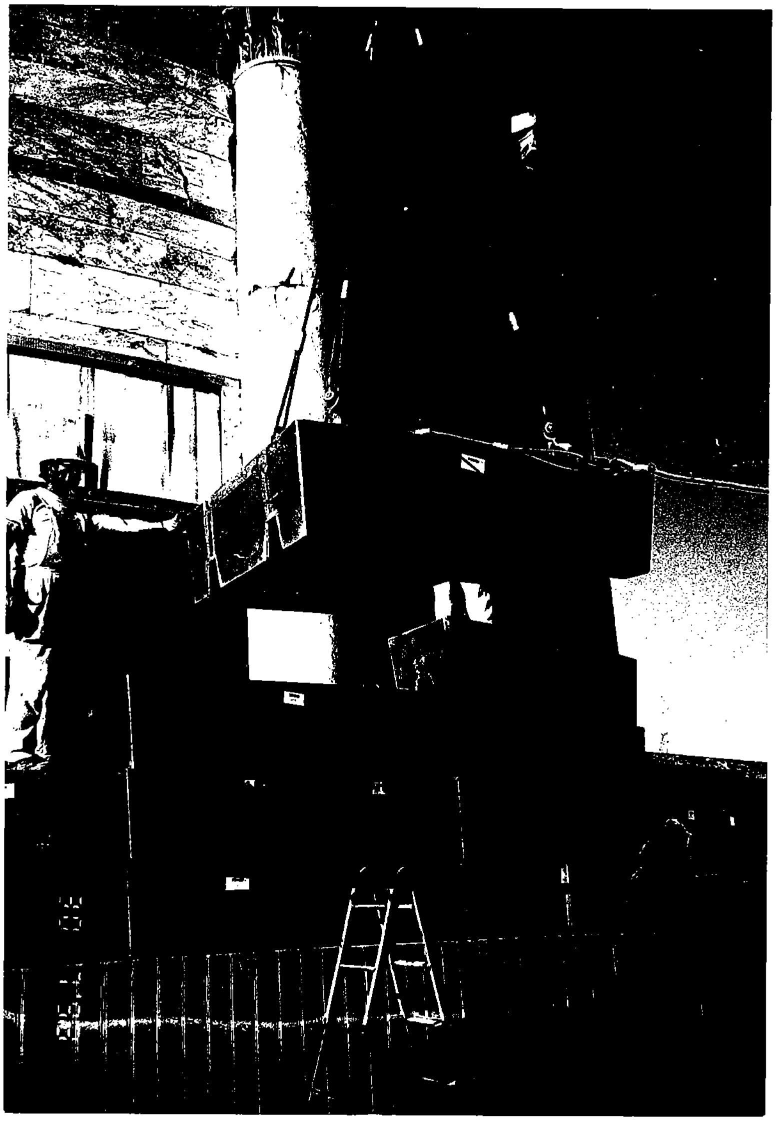

Fig. 14 Construction of the north counter-weight

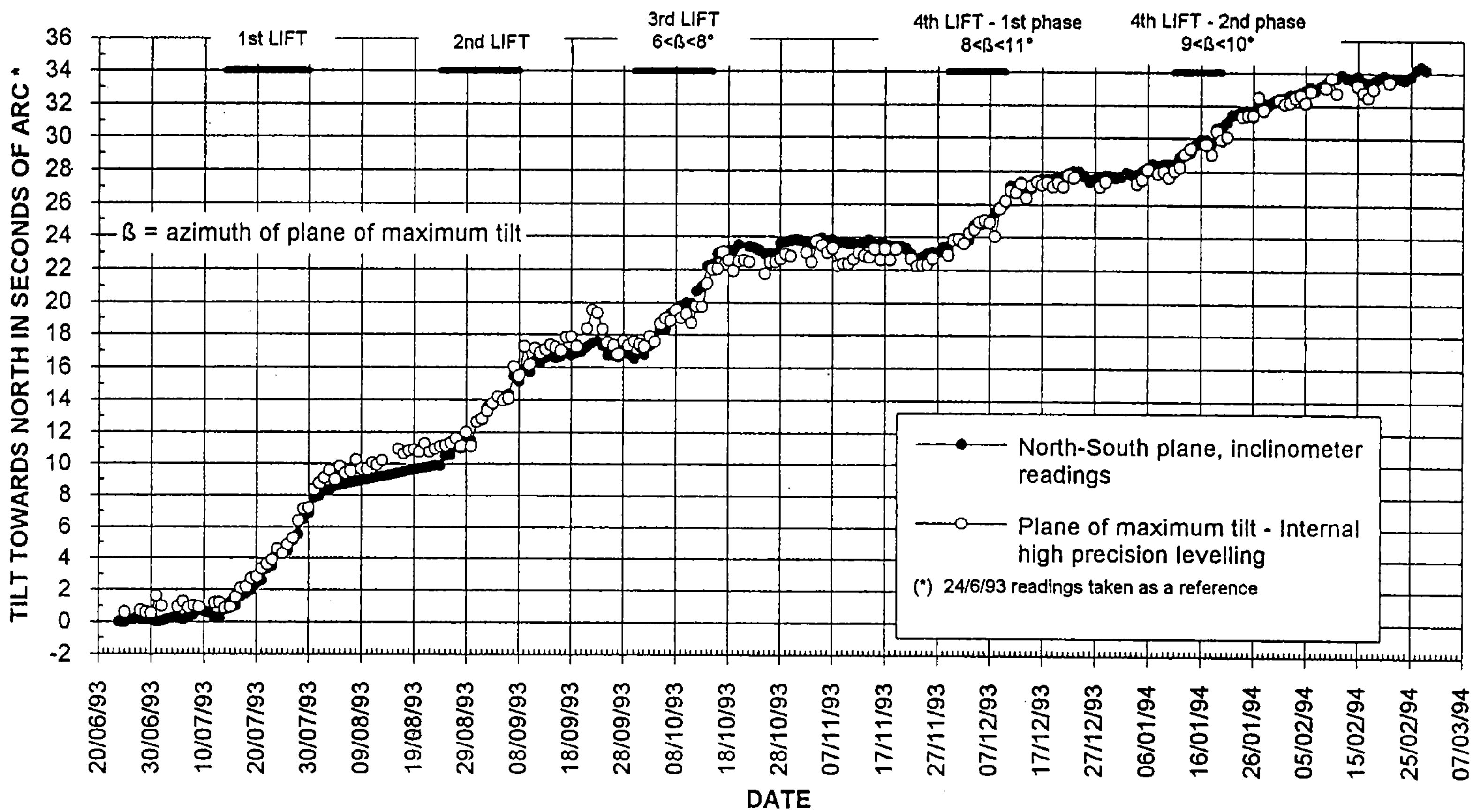

Fig. 15 Observed change of inclination of the tower during application of the counterweight