Dynamic optimal real-time algorithm for signals (DORAS): Case of isolated roadway intersections

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\textbf{ABSTRACT}

This paper studies intersection signal control in which traffic arrivals from all approaches along with the queues are assumed known. The control policy minimizes the overall intersection delay by deciding the green intervals for signal phases dynamically as driven by real-time traffic but subject to a set of constraints such as min/max green time for each phase. This paper models intersection vehicle delay by assuming continuous vehicle arrival and departure, and presents the optimal condition for green signal switch. Prior to this work, there does not appear to have been a continuous model on optimal control applied to the general intersection. Two numerical algorithms are proposed: optimum based (DORAS) and queue-based heuristic (DORAS-Q) respectively. Numerical tests are conducted via discrete simulation using an actual intersection data covering peak, mid-day and midnight hours, respectively. Comparison is conducted between actuated, DORAS, DORAS-Q and OPAC III. The tests show that the latter three methods all perform significantly better than the actuated.

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1. Problem statement

This paper studies the isolated intersection signal control problem, in which there is a finite set of signal phases denoted by $\Phi$ for allocation of traffic right of way (ROW) in order to avoid vehicular traffic conflict at the intersection and to improve intersection performance. Future vehicle arrivals as well as the current queueing of approaches are assumed known on a continuous basis. The green signal switches between phases. Each phase grants the ROW to a fixed set of approaches, and traffic from approaches in the same phase concurrently move through the intersection. For example, a phase may grant green indications to both through and left-turning traffic from a direction. And at any given time, only one phase of signal is given the right of way via green indications and the signal indications are red for all other phases. An all-red period is typically required between any two phases for intersection traffic clearance and safety, but is not necessarily so such as in the situation of lagging phase. For technical convenience, the all-red period during signal switches is also considered a phase in this paper, which grants the ROW to an empty set of approaches. All the lost effective green time due to signal switch is considered to be incorporated in the all-red interval. In the analytical study, the green time for each phase there only contains the effective green time as discussed in Newell (1989). The discharge process is assumed known under the

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1 WTRI hosted the first author a short time during this research.

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queueing and traffic arrival condition with a given signal indication. There is a set of rules such as minimum/maximum green durations for each phase or a pre-defined sequence of phases. A special case, the set of rules is empty, which implies maximum potential for intersection efficiency. This set of rules is pre-defined in order to consider practical implementation. A control policy is one that decides on the sequence and durations of the phases. The control objective is to find a policy that minimizes the average vehicle delay at the intersection.

To facilitate reading, a few terminologies are briefly explained here according to literature. An approach may be considered as one or more lanes with similar traffic movement crossing the intersection from a direction, that is, lanes for one of the movements: left-turn, right-turn, and through. To make the problem tractable and to simplify the analytical presentation, we assume that each approach has its own dedicated lanes. Approaches are served with ROW through phases. In the set of finite phases, no two phases may be granted the ROW at the same time, implying conflict ROW between any two phases in the set $\Phi$.

The intersection under this study is general in terms of geometry, traffic arrivals and signal phasing. For convenience of presentation, we only consider the case where the sequence of phases is determined without losing much practical applicability. In addition, as a special case, only protected movement is allowed, which greatly simplifies calculation of departures. Of course, if permitted movement is allowed, an estimation method for estimating the total discharged vehicles would be necessary, which can also be accommodated by the proposed method and is beyond discussion in this paper. At any time in the control process, a decision of signal change solely depends on the current queueing and future arrivals and has nothing to do with the past traffic processes.

Roadway intersection traffic signal control plays an important role in urban life. Millions of urban travelers experience delay at signalized intersections on a daily basis. Signal control is one of the few fundamental challenges in traffic operations. The impact from an improved traffic control is almost immediately visible and tangible. The emerging data rich environment in transportation due to the wide use of GPS, cellular phones, sensors, vehicle to vehicle communication, and vehicle infrastructure integration brings about great opportunities for real-time, information driven intersection traffic control, which has not been possible before. This paper attempts to answer such a question: given information about intersection queueing and traffic arrivals, what is the optimal policy for signal control at an isolated intersection? This question has been intermittently approached in a long history of literature and much remains to be further explored. Note that our real-time policy is intended to be globally optimal. In contrast, the traditional control are usually parametric, which means setting up best parameters that are restricted to specific control schemes such as setting up the critical gap for the actuated control. Fig. 1b illustrates ignorance of most approach traffic in an actuated control compared with a full stream data environment of Fig. 1a as in an optimum based study.

2. Brief literature for signal control

For decades, a tremendous amount of research have been devoted to signal control, a summary of which is seen in Gartner (1983). Briefly, signal control has experienced from pre-timed to vehicle actuated and to improved versions often called adaptive control. The pre-timed control functions with pre-determined signal cycle length and split, and does not respond to real-time traffic, although literature in this area such as Webster (1958) also has significant impact on the setup of maximum green time of each phase for the actuated control. The actuated control uses vehicle detectors for close arrivals in order to determine extension of green time to allow the closely spaced arrivals to pass the intersection without a stop. As in Fig. 1b, critical gap is the control parameter to determine closeness of two consecutive arrivals, disregarding traffic
situations at other approaches. The actuated signal is a typical example of parametric control. Parametric control is operated through improved setup for parameters, often restricted to specific schemes disregarding the global optimality of the employed scheme. Adaptive signal control builds on the actuated by incorporating more information for the current approach or some limited incremental information for other approaches as well. Real time optimal control aims at optimal adaptation of signals to traffic in view of the global and dynamic traffic situation.

2.1. Vehicle actuated signal control

Vehicle actuated control has two types: semi actuated and fully actuated. The former has loop detectors implemented only on the minor approach and does not switch signal from the major approach unless vehicle arrivals are detected on the minor approach. The latter implements loop detectors on all approaches, which is superior to the former. In the latter, each phase controls the green duration within a range of minimum and maximum limits. Within this range, the signal switch is determined by vehicle arrivals at the advance loop detector. If a vehicle is detected arriving, the green interval is extended by a minimal amount just enough to allow the detected vehicle to pass through the intersection. This minimum extended amount is called critical gap in literature. The green indication is terminated either when no vehicle is detected to be arriving within the time of a gap or when the green time maxes out.

Literature about actuated vehicle control is rich largely because for decades this method has been considered among the most adaptive and most practically feasible. Newell (1989) conducts a review to this literature. Many analytically rigorous studies are conducted starting from the 1960s (Cowan, 1978; Dunne, 1967) to the most recent years (Akcelik, 1994; Lin, 1982, as examples), mainly on intersection performance or parameter setup under actuated control.

A few studies specially examine the optimal size of the critical gap in the actuated signal control such as Newell (1969) and Wang et al. (2010). In practice, the critical gap is typically set up between 2 and 3 s. However, no matter how ‘optimal’ the critical gap is, it is a parameter restricted to this special control scheme. It does not aim for the global optimality of the intersection performance. As in Fig. 1, the preset critical gap treats the traffic situation in Fig. 1a (e.g. heavy traffic) the same way as for Fig. 1b. Our proposed optimal control policy readily explains the suboptimal nature of actuated control in Section 5.3.

2.2. Adaptive signal control

For its name sake, it is about adapting signal by at least partially considering the near future traffic data to incrementally optimize the local signal timing, mainly about cycle length and split, not necessarily for the global optimum. Miller (1965) represents one of the early adaptive signal controls. A summary of adaptive signal control is respectively seen in NCHRP Synthesis Report 403 Stevanovic (2010) and Zhao and Tian (2012), in which the few famous adaptive systems are summarized including OPAC (Gartner, 1983; Gartner et al., 1991) and RHODES (Head et al., 1992). Notably, Dunne and Potts (1964; 1967) propose a control based on queue lengths of approaches. A queue clearance policy is analyzed, in which signal switches when the queue is cleared for the current green phase and when the signal has not reached it green maximum. Zijverden and Kwakernaak (1969) propose a rolling horizon, short term optimization method, which is perfected along that line in Gartner (1982,1983).

Worthy of special mention, there is a line of literature that attempts to deal with optimal signal control. These studies hold the greatest promise to improve the adaptive signal control to its maximum potential. Specially, Gartner (1982,1983) proposes a dynamic programming model, later termed OPAC I in Gartner (2002), that defines a control policy to be a series of signal switch points. Optimization of these switch points is equivalent to optimizing the intersection control. This definition of policy is also adopted by Sen and Head (1997). The control progresses by sequentially determining signal switch points according to traffic situation, which is consistent to our proposed methodology here. Gartner’s (1983) model is intended for implementation in a real time, dynamic traffic situation. The optimal policy as defined in OPAC I presents a critically important benchmark to other approximation algorithms. However, the optimal decision at any moment depends on future optimal decisions, which is hard to implement when the optimization horizon is long and the state space as partially represented by the queue lengths at the intersection is large. Therefore OPAC III and OPAC IV are proposed (Gartner, 1982, 1983, 2002) as simplified optimization. OPAC II sets a stage of fixed length for optimization and moves the control forward stage by stage, where each stage roughly has the length of a signal cycle, while OPAC IV moves the control forward on a rolling time horizon using the virtual fixed-cycle concept. In an alternative way, some earlier literature tries to examine the structure of the optimal policy with restrictive assumptions on traffic and intersection geometry.

In particular, Grafton and Newell (1967) examines the optimal policy by assuming deterministic, uniform, and continuous traffic arrivals and departures at an intersection between two one-way streets, which reveals that the queue clearance policy proposed in Dunne and Potts (1967) may not be optimal. The authors find that the queue clearance policy is suboptimal when the saturation flow rates between approaches have a significant difference. Grafton and Newell’s findings may be easily explained by our general result in this paper. Sen and Head (1997) apply a discrete dynamic programming (DP) model to the intersection control, and only exhaustive search is implied to find the optimal solution. Here, our result translates delay minimization directly into intersection efficiency maximization in order to decide on the optimal switch points. Our derived optimal condition for signal switch allows to propose new algorithms. Our proposed algorithms named DORAS and DORSA-Q show impressive performance in our signal simulation studies.
3. Theoretical framework of intersection control

We present the models and analytical processes in this section by first introducing the notations.

Notations

\( (n, \tau) \) intersection state variable, where \( n = [n_i] \) with \( n_i \) representing queued vehicles for approach \( i \), \( \tau \) represents the time before the end of the control horizon.

\( \theta \) control policy that determines signal switch from one approach to another during the entire control horizon. With slight notational abuse, we use \( \theta(t) \) for the specific signal control at time \( t \).

\( \Phi \) set of phases. \( N \) is the total number of phases in \( \Phi \). The phases are indexed from 0 to \( N - 1 \) sequentially according to the order in a signal cycle with 0 being the current one with green signal. The index is therefore on a rotational basis.

\( \phi_0 \) set of approaches in phase \( k \) \( \in \Phi \). \( k = 0 \) means current approaches with ROW.

\( D^i(t_1, t_2) \) total arrivals of vehicles from approach \( i \) during time interval \( (t_1, t_2) \) under policy \( \theta \). \( D_i(\Delta t) \) means total arrivals in the past period \( \Delta t \) with slight notational abuse.

\( \Omega \) set of constraints for signal timing, which includes minimum/maximum green times, sequence of phases, etc.

\( \hat{q}_i \) currently observed queue length for approach \( i \) at the decision point.

\( \Delta q_i \) expected additional vehicles to be discharged beyond \( \hat{q}_i \) for approach \( i \) during a normally run-phasing in the current cycle forward.

\( q_i \) the total number of vehicles discharged for approach \( i \) during the current signal cycle that starts from the current phase \( \theta \). \( q_i = \hat{q}_i + \Delta q_i \).

\( s_i \) saturation flow rate for approach \( i \).

\( L \) green time loss due to signal switch, referred to as all-red interval in this paper.

\( s_0 \) current intersection efficiency under the current green phase.

\( e_i \) next (or switch-to) phase intersection efficiency equivalent.

\( \lambda^i(t) \) continuous vehicle arrival rate from approach \( i \) at time \( t \) under policy \( \theta \).

\( d^i(t) \) vehicles discharge rate for approach \( i \) at time \( t \). \( d_i(t) = \lambda_i(t) \) when approach \( i \) has the ROW and when no queue exists for approach \( i \). \( d_i(t) = 0 \) when approach \( i \) does not have the ROW. \( d_i(t) = s_i(t) \) when approach \( i \) has the ROW and when a queue exists being cleared. In summary, the controller assigns situational values to \( d_i \).

\( t^\tau \) critical point of time at which the signal switches from one phase to the next.

\( w^i(n, \tau) \) total intersection vehicle waiting time from state \((n, \tau)\) till the end of the control horizon under policy \( \theta \).

\( t^\tau \) limit to time \( t = \lim_{n \to 0} t - \Delta \).

\( t^- \) limit to time \( t = \lim_{n \to \infty} t - \Delta \).

\( \pi_i(t_1, t_{k+1}) \) green duration from time \( t_1 \) to \( t_{k+1} \) for phase \( i \). In phase \( i \), the pre-determined set of approaches get the right of way.

\( \alpha_i/\beta_i \) minimum/maximum green time limit for phase \( i \).

We first clarify about the optimal policy. The control policy consists of a series of signal phases that last till the end of the control horizon, which sequentially takes the ROW (except for the all-red interval \( \pi_R \)). We denote a policy \( \theta \) as a set \( \{\pi_1(t_1, t_2), \pi_R(t_2, t_2 + L), \pi_2(t_2 + L, t_3), \pi_3(t_3 + L, t_4) \ldots \} \). Each \( \pi_i(t_k, t_{k+1}) \) denotes a green indication for phase \( i \in \Phi \) for the time interval \((t_k, t_{k+1})\) except for the all-red \( \pi_R \). Be aware that all phases in \( \Phi \) are mutually exclusive, meaning conflict of ROW between any two of them. Under a policy \( \theta \), \( \pi_i(t_1, t_{k+1}) \) and \( \pi_k(t_k, t_{k+1}) \) are not considered two phases but one \( \pi_i(t_k, t_{k+1}) \). Note that phase \( \pi_R \) is specially for the all-red interval, an interval that almost every signal switch from other phases. Or equivalently, a control policy may be represented by a series of time points for signal switches between phases, e.g. \([t_1, t_2, \ldots, t_n]\). The key is to find the signal switch points in order to determine the control policy. In the modeling that follows, we assume the time goes backward, which means the time clock goes from time \( t \) backward to time zero, where time zero represents the end of the control horizon. This also means that in a time interval \((t_k, t_{k+1})\), we have \( t_k > t_{k+1} \). In Fig. 2, a policy \( \theta \) is illustrated, in which \( t^\Phi \) is a switch point for phase \( \pi_k \) to switch to phase \( \pi_{k+1} \). Note that our developed policy is general, which allows, but does not require, an all-red interval for a phase change. Additionally, we use \( s(n, \tau) \) for an intersection state to start with, in which \( n \) represents queues for all approaches at time \( \tau \) while \( \tau \) is the time prior to the end of the control horizon. Theoretically, a control policy \( \theta \) is the function of the current state \( s(n, \tau) \), and should be written as \( \theta(n, \tau) \). To simplify the presentation, we denote it simply by \( \theta \). In addition, all terms hereafter with a superscript \( \theta \) mean being under policy \( \theta \).

The intersection signal control problem may be described in Eq. (1) below, which assumes policy \( \theta \) as illustrated in Fig. 2.
\[ w^\theta(n, t) = \sum_{\mathcal{W}} \int_0^t \left( n_i + \int_0^t \lambda^\theta_i (\tau) d\tau - \int_0^t d_i^\theta (\tau_1) d\tau_1 + \int_0^t \lambda^\theta_i (\tau_1) d\tau_1 - \int_T^t d_i^\theta (\tau_1) d\tau_1 \right) \, d\tau + \int_0^t \left( n_i + \int_0^t \lambda^\theta_i (\tau_1) d\tau_1 - \int_T^t d_i^\theta (\tau_1) d\tau_1 \right) \, d\tau + w^\theta \left( \left( n + \int_0^t \lambda^\theta_i (\tau_1) d\tau_1 - \int_T^t d_i^\theta (\tau_1) d\tau_1 \right), T \right) \]  

Where, \( w^\theta(n, t) \) is the total intersection vehicle waiting time from time \( t \) to the end of control time horizon, given a green phase \( \pi_k \) at time \( t \), which will change to phase \( \pi_{k+1} \) at time point \( t' \), where \( t' \) is within \([t, T]\). Here, \( t' \) is called a pivotal point of signal. Additionally green phase \( \pi_{k+1} \) changes to \( \pi_{k+2} \) at time \( T \). \( w^\theta(n_0, T) \) is the salvage waiting time at time \( T \) with a resultant state \( (n_0, T) \). For modeling, we assume \( t \) and \( T \) are both given so that we may study the conditions for \( t' \). \( \lambda^\theta_i (\tau) \) is the arrival rate from approach \( i \) under policy \( \theta \), usually equal to the actual arrival rate \( \lambda_i (\tau) \), unless the intersection queueing capacity restricts arriving vehicles from entering the queue so that those vehicles are lost to elsewhere. \( d_i^\theta (\tau) \) is the discharge rate under policy \( \theta \) for approach \( i \). \( d_i^\theta \) takes the following situational values.

Case I: \( d_i^\theta (\tau) = s_i \), the saturation flow rate when the signal is green AND when a queue exists for approach \( i \).

Case II: \( d_i^\theta (\tau) = 0 \) when the signal is red for approach \( i \).

Case III: \( d_i^\theta (\tau) = \lambda^\theta_i (\tau) \). the arrival rate when the signal is green and when no vehicle queue is present at the intersection for approach \( i \).

Because this methodology is developed for a full information environment, the departure rates are assumed to be continuously monitored and be known through sensors. If technologies are able to monitor the departure situation to tell if the departure line is blocked by downstream vehicles, more situational values may be assigned to \( d_i^\theta \), which is beyond the normal discussion of this paper. Another notion, under any control policy \( \theta \), the departure rate \( d_i^\theta \) is set such that the cumulative departures is no more than the cumulative arrivals at any given time, that is, \( n_i + \int_0^t \lambda^\theta_i (\tau) d\tau - \int_0^t d_i^\theta (\tau) d\tau \geq 0 \), \( \forall \tau < t \), \( \forall i \), starting from an initial state \( (n_0, t) \). Readers shall accordingly understand the valuation of \( d_i^\theta \). For simplicity, no notation for how to get values for \( d_i^\theta (\tau) \) is used in Eq. (1).

In Eq. (1), the first term is for waiting time after the pivotal point \( t' \) till \( T \) when green phase switches from \( \pi_k \) towards \( \pi_{k+1} \). The big parenthesis within integral is for the total waiting vehicles at time \( \tau \in [t', T] \). The second integral is for the time before the pivotal point \( t' \) till \( t \). The third term is a salvage value term, which also has to do with the choice of pivotal point \( t' \) because \( t' \) results in the queue \( n_0 \) at time \( T \).

It appears that a solution approach is to find the optimal pivotal point \( t' \) backward from the very end of the solution horizon, assuming a total cost of queues at the end of the optimization horizon is zero according to the traditional dynamic programming technique, which mimics that in Gartner (1982) and Sen and Head (1997). First, we start by examining the signal switch point. Optimal pivotal point \( t' \) is a time point such that shifting it backward or forward would both increase the total waiting time, which leads to the optimal signal condition below.

Taking first-order derivative of Eq. (1) over \( t' \) gives the following result,

\[
\frac{\partial w^\theta(n_t, t)}{\partial t'} = \sum_{\mathcal{W}} \int_0^t \left[ -\lambda^\theta_i (t' +) + d_i^\theta (t' +) \right] d\tau + \frac{\partial w^\theta(n_0, T)}{\partial t'} \tag{2}
\]

In the above, we use \( t' + \) and \( t' - \) as defined earlier. Because both \( \lambda \) and \( d \) are assumed continuous, \( w^\theta(n_t, t) \) is differentiable at \( t' \). Naturally, we denote the arrival/departure rates at both sides of time \( t' \) under policy \( \theta \) by \( \lambda^\theta_i (t' +) \) vs. \( \lambda^\theta_i (t' -) \) and \( d_i^\theta (t' +) \) vs. \( d_i^\theta (t' -) \), respectively. Eq. (2) becomes Eq. (3), which essentially establishes the equivalence between maximizing intersection (net) throughput and minimizing intersection vehicle delay.

\[
\frac{\partial w^\theta(n_t, t)}{\partial t'} = \sum_{\mathcal{W}} \left[ -\lambda^\theta_i (t' +) + d_i^\theta (t' +) \right] (t' - T) + \frac{\partial w^\theta(n_0, T)}{\partial t'} \tag{3}
\]

Clearly, Eq. (3) is identical to Eq. (4).

\[
\sum_{\mathcal{W}} \left[ d_i^\theta (t' +) - \lambda^\theta_i (t' +) \right] = \sum_{\mathcal{W}} \left[ d_i^\theta (t' -) - \lambda^\theta_i (t' -) \right] - \frac{w^\theta(n_0, T)}{t' - T} \tag{4}
\]

Eq. (4) represents a necessary condition for the general case of optimal signal switch. The left side of Eq. (4) is the net discharge rate under the current phase while the right hand side is the equivalent net discharge rate of the switch-to-phase. By the term equivalent, we mean to include the converted rate from the salvage function \( w^\theta(n_0, T) \), the last term in Eq. (4). The pivotal time \( t' \) shall be chosen such that the net gain of the total vehicle discharge rate from the before to after switch shall sum up to off set the resulting marginal change in the subsequent intersection waiting time. To better understand it, we look at two cases below.

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3.1. A special case

Just for obtaining some insights, assume that the last term in Eq. (3), the salvage value \( \frac{w^\theta(n_0, T)}{t_{i}-T} \) = 0, which gives rise to the following equation.

\[
\frac{\partial w^\theta(n, t)}{\partial t_{i}} = \sum_{\forall i} \left[ -\left( \lambda^\theta_i (t_{i}^{+}) - d^\theta_i (t_{i}^{+}) \right) + \lambda^\theta_i (t_{i}^{-}) - d^\theta_i (t_{i}^{-}) \right] \left( t_{i} - T \right)
\]

(5)

In this special case, when the first order derivative is zero, the second order derivative is also zero, which means indefinite regarding whether \( t_{i} \) is a minimum point. Therefore, we have to further examine the first order derivative. Assume the arrivals \( \lambda^\theta_i (t_{i}^{+}) = \lambda^\theta_i (t_{i}^{-}) \) for all \( i \) without losing much generality. Whenever we have \( \sum_{\forall i} d^\theta_i (t_{i}^{+}) < \sum_{\forall i} d^\theta_i (t_{i}^{-}) \), where \( \delta \rightarrow 0 \) and \( \delta > 0 \), signal switch at time \( t_{i} \) will lead to less total waiting time than otherwise. Therefore, \( \sum_{\forall i} d^\theta_i (t_{i}^{+}) < \sum_{\forall i} d^\theta_i (t_{i}^{-}) \) for very small \( \delta > 0 \), is a necessary and sufficient condition for signal switch at time \( t_{i} \). This concludes that a switch shall result in a higher intersection discharge rate.

3.2. The general case

In the general case, the following is a sufficient and necessary condition in light of the insights from the special case above:

\[
\sum_{\forall i} \left[ d^\theta_i (t_{i}^{+}) - \lambda^\theta_i (t_{i}^{+}) \right] - \sum_{\forall i} \left[ d^\theta_i (t_{i}^{-}) - \lambda^\theta_i (t_{i}^{-}) \right] + \frac{w^\theta(n_0, T)}{t_{i}-T} < 0.
\]

(6)

\[\text{where } \lim \delta = +0.\]

Translate this observation into the case in which an effective time loss exists in the form of an equivalent all-red period due to signal switch. In this case, the switch-to phase is All-Red, implying immediate switch-to efficiency is zero, i.e. \( \sum_{\forall i} d^\theta_i (t_{i}^{-}) = 0 \). Within a certain time period to come, the average discharge rate equivalent shall be larger than that prior to signal switch when the objective function is to minimize the total vehicle delay, which is expressed in Eq. (7) below as further simplified equation from Eq. (6).

\[
\sum_{\forall i} d^\theta_i (t_{i}^{+}) < -\frac{w^\theta(n_0, T)}{t_{i}-T}.
\]

(7)

We call \(-\frac{w^\theta(n_0, T)}{t_{i}-T}\) as switch-to efficiency equivalent. Note that \(-\frac{w^\theta(n_0, T)}{t_{i}-T}\) is usually negative (considering the time is counted backwards), which means earlier switch allows earlier release of waiting vehicles at subsequent phases. \(-\frac{w^\theta(n_0, T)}{t_{i}-T}\) is therefore the (marginal) net gain from signal switch. Equivalently, if the current phase maintains a higher rate, the green signal shall not switch to the next phase. We therefore have Proposition 1 to summarize the findings as follows.

**Proposition 1.** At roadway intersection signal control, where all vehicles have equal weight and where the objective is to minimize the total vehicle waiting time at the intersection, the green indication switches if and only if the current (e.g. ongoing) service rate is less than the switch-to-efficiency equivalent.

**Proposition 1** illustrates the optimal control structure. In other words, green signal switch shall lead to a higher intersection service rate equivalent.

In **Section 4**, we will discuss the calculation of the current rate and switch-to rate equivalent in details. Our derivations have used continuous arrivals and departures because we believe they approximate the actual discrete processes well enough. In other words, we believe any discrete process may be approximated by a continuous one of vehicles without causing nontrivial differences. In the subsequent algorithms proposed, we will have to resort to approximations using discrete measures for practical implementations.

4. Efficiency equivalent marginal effect

This section discusses the intersection efficiency equivalent, the term \( \frac{\partial w^\theta(n_0, T)}{\partial t_{i}} \) in Eq. (3), which represents the marginal effect of the pivotal point \( t_{i} \) on subsequent phases’ waiting time. Here we treat the entire remaining section of the full signal cycle starting from, but not including, the current phase in green as a big virtual phase considering a fixed sequence of phases. The marginal effect of \( t_{i} \) on vehicle waiting as calculated below is over this period also with the last all-red at the end of the cycle being included. The marginal effect beyond the current cycle is assumed to be unclear and is expectedly zero. However, one can extend in a similar fashion this period of consideration till a certain phase beyond the current cycle as long as the marginal effect on the phases considered is non-trivial and is quantifiable, which is beyond the scope of this paper.
Simply put, the marginal effect of $t_i^1$, which can be certain at $t_i^1$, is mainly due to the (earlier release of) queued vehicles. If a signal switch point is made earlier by a small amount $\Delta$, the total reduced vehicle delay at the subsequent phases in the case of a fixed sequence of phases may be approximated by $\sum_{i=0}^{\Delta} q_i^0(t_i^1) \ast \Delta$, where $q_i^0$ represents the total (potentially) queued vehicles to be cleared for phase $i$ during a normally run cycle under policy $\theta$. $\sum_{i=0}^{\Delta} q_i^0(t_i^1) \ast \Delta$ means that all the observed and forecast queues for approaches in the current cycle forward that are cleared during the phases may be cleared earlier by $\Delta$ if the signal switches earlier by this amount $\Delta$ and if each subsequent phase runs as it should be running. Note that policy $\theta$ does not necessarily guarantee clearance of queued vehicles for all approaches. On the other hand, $\theta$ may dictate clearance of more vehicles than those queued in order for a higher intersection efficiency. However, $q_i^0$ normally only represents a subset of queued vehicles that are cleared during the phasing under policy $\theta$.

We show the marginal effect of $\Delta$ in an approximate way as follows. $w_i^0$ in the right hand side of Eq. (2) may be re-written as follows.

$$w_i^0\left(\left(n + \int_T^{t_i^1} \lambda_i^0(\tau_1)d\tau_1 - \int_T^{t_i^1} \bar{d}_i^0(\tau_1)d\tau_1\right) \cdot T\right)$$

$$= w_i^0\left(\left(n + \int_T^{t_i^1} \lambda_i^0(\tau_1)d\tau_1 - \int_T^{t_i^1} \bar{d}_i^0(\tau_1)d\tau_1 + \int_T^{t_i^1} \lambda_i^0(\tau_1)d\tau_1 - \int_T^{t_i^1} \bar{d}_i^0(\tau_1)d\tau_1\right) \cdot T\right)$$

$$\left(\frac{\partial w_i^0(n_0, T)}{\partial t} \approx \sum_{\forall i_c, \phi', \phi \in \Phi} \frac{\partial w_i^0(n_0, T)}{\partial n_0} \times \frac{\partial n_0}{\partial t} + \sum_{\forall i} \bar{d}_i^0(t^\dagger) \ast C(t^\dagger)\right)$$

$$= \left(\frac{\partial n_0}{\partial t} \times \sum_{\forall i_c, \phi', \phi \in \Phi} \frac{\partial w_i^0(n_0, T)}{\partial n_0} + \sum_{\forall i} \bar{d}_i^0(t^\dagger) \ast C(t^\dagger)\right)$$

$$= -\sum_{\forall i_c, \phi', \phi \in \Phi} \frac{1}{\sum_{\forall i_c, \phi', \phi \in \Phi}} \times \sum_{\forall i_c, \phi', \phi \in \Phi} q_i, \text{ in which } \frac{1}{\sum_{\forall i_c, \phi', \phi \in \Phi}} q_i \text{ represents the average discharge time for this incremental vehicle in the current phase, meaning that each vehicle in the current phase increases all queued vehicles in all subsequent approaches by an additional time } \frac{1}{\sum_{\forall i_c, \phi', \phi \in \Phi}} q_i^0. \text{ This above relationship is easier to understand if one treats } n_0 \text{ as the discharged vehicles in the current approach. An alternative interpretation, if the current signal is switched by a small amount of time earlier } \Delta, \text{ all queued vehicles in the subsequent approaches will wait approximately for less amount of time } \sum_{\forall i_c, \phi', \phi \in \Phi} q_i^0. \text{ Therefore the marginal effect on reduced waiting time for the subsequent phases is just } \sum_{\forall i_c, \phi', \phi \in \Phi} q_i^0(t_i^1). \text{ Regarding the second term on the right hand side of Eq. (9), the rationale is as follows. Earlier switch of signal by an amount of time } \Delta \text{ would make } \sum_{\forall i_c, \phi', \phi \in \Phi} q_i^0(t_i^1+\Delta) \Delta \text{ vehicles wait for another full signal cycle that they otherwise could have passed the intersection, which dictates a marginal effect represented by the second term in Eq. (9). This explains Eq. (9). The above arguments seem relatively more appropriate in medium to heavy traffic situations, which might justifies the impressive performance of DORAS during peak hours in the later numerical tests.}$$

Eq. (9) offers a perspective to argue for the convexity of $w_i^0(n, t)$ in this approximation method. If one takes another derivative of both sides of Eq. (9), one gets the second order derivative of $w_i^0(n_0, T)$ over $t_i^1$ as follows if the second term on the RHS is ignorable.

$$\frac{\partial^2 w_i^0(n_0, T)}{\partial (t_i^1)^2} = \sum_{\forall i} \xi_i,$$  

where

$$\xi_i = \begin{cases} \frac{\lambda_i(t_i^1)s_i}{\bar{s}_i - \lambda_i(t_i^1)}, & \text{when } q_i \text{ is discharged within (min, max) of phase } i, \\ 0, & \text{otherwise.} \end{cases}$$

With slight notational abuse, here $\lambda_i(t_i^1)$ represents the vehicle arrival rate at the end of green phase of an interval $t_i$. The right side of the above equation represents the rate at which the queues are joined by arriving vehicles. Case I: if $t_i^1$ is made earlier (e.g. larger), the queued vehicles for approach $i$ would be reduced at a rate equal to $\lambda_i$ ‘normally’, the queue for approach $i$ is between the maximum and minimum green times. Case II: if excessive queues exist for approach $i$, or if the queue is too short and is cleared within the minimum green time, the right hand for this approach $i$ of this
subsequent cycle shall be zero. We make a brief argument for Case I here. Suppose in the reference case, the time taken to clear queued vehicles in approach \( i \) is \( t = \frac{q_i^d(0)}{s_i - \lambda_i(t_i)} \), where \( q_i^d(0) \) is the observed queue at the start of the green signal. If \( t \) for the current phase is made earlier by a small time \( \Delta \), everything else unchanged (a strong assumption), the observed queue at approach \( i \) at the start of its green signal would be \( \frac{q_i^d(0) - \lambda_i(t_i) \Delta}{s_i - \lambda_i(t_i)} \). Therefore the change of the total cleared vehicles that had been queued due to the shift of \( t \) is \( \frac{\lambda_i(t_i) \Delta}{s_i - \lambda_i(t_i)} \). The new green time for approach \( i \) is therefore \( t^* = \frac{q_i^d(0) - \lambda_i(t_i) \Delta}{s_i - \lambda_i(t_i)} \). Therefore the change of the total cleared vehicles that had been queued due to the shift of \( t^* \) is \( \frac{\lambda_i(t_i) \Delta}{s_i - \lambda_i(t_i)} \). The rate at which the queued vehicles change due to \( t \) is therefore as in Eq. (10).

Note that so far, we have been discussing about \( \frac{\partial q_i^d(0)}{\partial (t_i)^2} \) and \( \frac{\partial q_i^d(0)}{\partial t_i} \), based on the effect on one subsequent cycle only. If one is technically able to examine the delay effect beyond one cycle, the result shall be more accurate. In this paper, we only consider one cycle in our numerical method because we only have a general confidence on the intermediate phases within the next cycle (but excluding the current phase, which could significantly contribute to suboptimal result). Similarly, the one cycle concept is also adopted in OPAC (Gartner, 1982, 1983, 2002).

Eq. (7) combined with Eq. (9) gives rise to the following:

\[
\sum_{\forall i} d_i^q(t^1 +) \leq \frac{\sum_{\forall i; \phi_i \neq 0} q_i^d}{C - (t^1 - T)} = \frac{\sum_{\forall i; \phi_i \neq 0} q_i^d}{C - AR} \tag{11}
\]

Where \( AR \) is the all-red period due to signal switch. \( t^1 - T \) represents the ‘phase’ right after the signal switch, which by our definition is an All-Red period \( AR \). For situations without a clear All-Red period, readers may also develop an approximation in light of the logic here. The full cycle length \( C \) following the signal switch at \( t^1 \) is estimated. The right hand side of Eq. (11) is considered as intersection efficiency equivalent in the switch-to-phase. Note that in Eq. (9), \( q_i^d \) only accounts for the phases in the current partial cycle starting with but excluding the current phase under green. It does not account for phases after a full cycle from now because the effect of this marginal change \( \Delta \) in signal switch on queuing beyond one cycle is not clear: \( \Delta \) may be too small to see a practical impact. The left side of Eq. (11) is the discharge rate under the current phase because \( d_i^q \) = 0 for \( \forall i \neq \phi_i \) at time \( t^1 \).

The derivation above involves the end of horizon \( T \). The choice of \( T \) value appears arbitrary. However, if one expands Eq. (1) for \( T \) to be such that \( (t^1, T) \) includes all the phases in the coming cycle starting with but excluding the current phase, all the derivations above would remain similar, except that choice \( T \) would have a clearer explanation. Clearly, Eq. (11) means that the average intersection efficiency in the remainder of current cycle, starting with the current phase, shall be larger than the current discharge rate in order to justify a signal switch. Conceptually, the choice of \( T \) shall be such that each phase overall maintains a similar service rate of vehicles.

Note that the result in Eq. (11) may shed light on the queue clearance policy studied in literature such as Dunne and Potts 1964 and 1967. The queue clearance policy switches green signal after the queue is cleared for a current phase. Seemingly, this policy maximizes the intersection efficiency, but not always so. Grafton and Newell (1967) examine this policy by assuming constant and continuous arriving and departure traffic at an intersection between two one-ways. Grafton and Newell identify conditions under which the queue clearance policy is and is not optimal, respectively. Our general results is consistent to Grafton and Newell (1967), and is more general for being applicable to the general heterogeneous traffic. We conjecture that a queue clearance policy may be near optimal in most moderate traffic situations when the intersection is roughly symmetric in terms of the lanes and discharge capacity between phases.

5. Basic mechanism

With the condition of being a pivotal point \( t^1 \) clarified earlier, we are ready to propose an approximate optimal policy for an infinite time horizon. At a current state \((n, t)\) and green signal, the decision is whether to switch the green signal to the next phase while continuously satisfying the set of constraints \( \Omega \).

In our proposed algorithm below, \( N \) is the total number of phases that take turn to get the green signal by following a predefined sequence. The current green phase is indexed zero (e.g. \( \phi_0 \)). We let \( \Delta T = C - AR \). Note that \( C \) shall be the cycle length for the cycle to come\(^2\), which will be estimated numerically.

Practically, Eq. (11) becomes the following:

\[
\varepsilon(\pi_0) < \varepsilon(\pi_1) \tag{12}
\]

where \( \varepsilon(\pi_0) \) and \( \varepsilon(\pi_1) \) are numerical estimates of the current phase efficiency and the switch-to-efficiency equivalent, respectively.

The current efficiency \( \varepsilon(\pi_0) \) may be chosen such that \( \varepsilon(\pi_0) = \max_{\Delta t_0} \left( \frac{\sum_{\phi_i \neq 0} D_i(t_1 + \Delta t_0)}{\Delta t_0} \right) \) while \( \Delta t_0 \) is the additional green time from the current decision point as allowed by the constraints in \( \Omega \).

\(^2\) It is not exactly the cycle length, but a length of time for the combination of phase intervals to get the largest possible switch-to-efficiency.
Choice of $\Delta t_0$ in calculation of $\varepsilon(\pi_0)$ above is a heuristic method. One may examine the average intersection efficiency for up to a certain time length under the current green phase, for example 5 s, into the near future, by comparing the average throughput for each time length shorter than the limit, for example, 1 s, 2 s, 3 s, 4 s, 5 s, respectively. Within each time length, the expected arrivals is known and is divided by the according time length to get a current efficiency. As a special case, if a platoon of 10 vehicles arrive fairly uniformly between 3 s and 5 s from now, and if no vehicle arrives prior to time 3 s, $\varepsilon(\pi_0) = 2$ vehicles per second, assuming the preset time limit is 5 s and assuming the remaining green time allowed is more than five seconds till the maximum green. In our numerical test, 5 s is used to get $\varepsilon(\pi_0)$.

We next explain the switch-to-efficiency. $\Delta T = \sum_{i=1,2,...,N-1} \Delta t_i + (N-1)L$, where $N$ is the total number of phases. Here $\Delta t_i$ is the estimated phase interval $i$. Denote the fixed sequence cycle as $(\phi_0, AR, \phi_1, AR, \phi_2, ..., \phi_{N-1}, AR)$, where $L = AR$. This set of notation repeats after signal switches.

$$\varepsilon(\pi_1) = \max_{\Delta t} \left\{ \frac{\sum_{i=0}^{k} q^0_i}{\Delta t} \right\}$$

Here, $q^0_i$ is the expected number of vehicles to be cleared in approach $i$ during a normally optimized cycle starting from the current phase $\phi_0$, which may include queued and free vehicles, or in other words, include both the currently observed vehicles in queue $q^0_i$ and the additional vehicles expected to clear $\Delta q^0_i$. It is important to explain the calculation of $q^0_i$ in our proposed algorithm DORAS later. We take phase $\phi_1$ as an example. To facilitate understanding, assume only one approach for each phase here so that approach $i$ corresponds to phase $\phi_i$. At time $t$, phase $\phi_1$ has a queue of $q^0_i$ vehicles. When the signal is turned green after a switch loss $L$, the green phase estimation for phase $\phi_1$ in order to maintain a highest phase specific intersection efficiency is calculated to be as follows.

$$\Delta t_1 = \max_{\Delta t_1 \in [0, \bar{\Delta}]} \left\{ \frac{q^0_1}{\Delta t_1 + L} \right\}.$$  

Where $q^0_1$ may be equal to $q^0_1 + \Delta q^0_i(\Delta t_1 + L)$ with $\Delta q^0_i(\Delta t_1 + L)$ being the expected additional discharge vehicles during $\Delta t_1$ in phase 1 when phase 1 is expected to maintain its highest average discharge efficiency. $\Delta t_1$ satisfies the min/max green time constraints. This explains the calculation of $\Delta t_1$.

In a similar fashion, the expected total vehicles to discharge and the total discharge (green) time for each subsequent phase may be sequentially calculated. In a general case in which a phase has multiple approaches, i.e. $|\phi_k| > 1$, we have a general equation as follows.

$$\Delta t_k = \max_{\Delta t_k \in [0, \bar{\Delta}]} \left\{ \frac{\sum_{i=0}^{k} q^0_i}{\Delta t_k + L} \right\}, k \leq N-1.$$  

In Eq. (15), $\sum_{i=0}^{k} q^0_i$ is the number of vehicles discharged during the time $\Delta t_k$ for phase $k$. Those discharged vehicles are $\tilde{q}_k$ and additional ones that have arrived during $(\sum_{i=1,2,...,k} \Delta t_i + kl)$ (the current active phase has an index 0). Note that the green time starts for phase $k(k \geq 1)$ at time $t - \sum_{j=1,k-1} \Delta t_j - kl$. At which time point, the beginning queue for phase $k$ needs to be predicted using the known arrival processes. Last but likely equally important, there is room to further fine tune the process to get the ‘expected’ green time for each subsequent phase so that the entire switch-to-efficiency equivalent $\varepsilon_1$ may be maximized and be compared with the current intersection efficiency $\varepsilon_0$. That is, $\max_{\Delta t_1, \Delta t_2, ..., \Delta t_{N-1}} \left\{ \frac{\sum_{i=0}^{k,N-1} q^0_i(\Delta t_i)}{\sum_{i=0}^{k,N-1} \Delta t_i + (N-1)L} \right\}$. This makes our algorithm inherently share some similarity with the logic of OPAC III.

5.1 Dynamic, optimal, real-time algorithm for signals (DORAS)

We now present our proposed algorithm DORAS, which takes inputs and generates outputs as illustrated in Fig. 3.

Set incremental time step $\Delta$. Along the time clock, after the minimum green time is reached until the green interval maxes out, at each $\Delta$, the currently active green signal is examined for possible switch by calling the algorithm DORAS below. Note that $\Delta$ may be determined empirically such as set to 1.0 s. In the numerical test, we use 1.0 s for it.

**Time Clock Direction** Opposite to before, from now on, we assume the time clock increases in the presentation of DORAS. For simplicity, we first define a function named GreenEst(·) used in the algorithm DORAS to estimate the green time $\Delta t_k$ needed for each subsequent phase $k$. Treating the next phase after the current one as phase 1, we explain GreenEst(·) as follows.

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Function GreenEst(_:)
Initialization. Set \( k = 1 \).
While \( k \) is not the last phase in the cycle
Case:
\[
\text{Case 1: } \Delta t_k = \alpha_k \text{ when } s_i \alpha_k \geq \hat{q}_i + \bar{D}_i \sum_{j=1,2, \ldots, l-1} \Delta t_j + k \cdot L + \alpha_j, \forall i \in \phi_k,
\]
which means green time is set to the minimum when little queue is present.
\[
\text{Case 2: } \Delta t_k = \beta_k \text{ when } s_i \beta_k \leq \hat{q}_i + \bar{D}_i \sum_{j=1,2, \ldots, l-1} \Delta t_j + k \cdot L + \beta_j, \forall i \in \phi_k,
\]
which means heavy traffic maximizes out the green time.
\[\text{Case 3: If } \Delta t_k \in [\alpha_k, \beta_k]. \text{ use Eq. (15) to determine the green interval for phase } k.\]
End Case
\[i = i + 1;\]
Endwhile
END GreenEst(_:).

In the above function, GreenEst(_:)
may be easier to understand to new readers by assuming only one approach in each phase.

Algorithm DORAS
DORAS algorithm applies only when the current green phase has exceeded its minimum green time. There is no need
to run DORAS during the all-red interval or when the current green time is less than the minimum duration.
DORAS is explained in details below.

Step 1: Initialization. Update \( \hat{q}_i, \forall i \).
Step 2: Run GreenEst(_:) to determine \( \Delta t_k \) for each phase \( k \geq 1 \).
Step 3: Update vehicles \( \sum_{i \in \phi_k} q_i \) that will be discharged for each phase \( k, 1 \leq k \leq N - 1 \).
Step 4: Calculate the next phase equivalent efficiency factor \( \varepsilon_1 \) by following Eq. (13).
Step 5: Calculate the current phase efficiency factor \( \varepsilon_0 \) as explained earlier.
Step 6: Make switch decision according to the cases below:
- If \( \varepsilon_1 > \varepsilon_0 \), terminate the current phase and switch to all-red interval to transit to the next phase.
- If \( \varepsilon_1 \leq \varepsilon_0 \), do not switch signal.

5.2. Existing-queue based heuristic DORAS-Q

One may feel too much of a requirement to know vehicle arrivals to implement DORAS. In fact, implementation of DO-
RAS only requires knowledge of vehicle arrivals for the current phase up to a certain limited time length.
In the simulation, we use 5 s. For major arteries, one may be able to easily extend it to up to 10–20 s. For sub-
sequent phases, only some ability to predict total arrivals for a certain time period is required. One may prob-
ably need less and may be aggressive for the prediction of the current phase. Additionally, the queue lengths as rep-
aired for DORAS may be estimated approximately in practice by counting in and out flows of approaches as facili-
tated by video imaging techniques. However, the requirement of predicting vehicle arrivals for each of the sub-
sequent phases within the length of about a full signal cycle may still pose a practical challenge to many prac-
titioners. To further simplify the implementation, we propose a control method that only uses the exist-
ing queues and only requires to know traffic arrivals for the current approach for up to five seconds and to know the
average historical arrival rates for other phases. This method is identical to DORAS except that it uses \( \hat{q}_i \) to
proportionately predict \( q^i \) to calculate \( \varepsilon_1 \). In predicting \( q_i \) with \( \hat{q}_i \), we need to know the lost time within the past cycle that has con-
tributed to the formation of \( \hat{q}_i \), which gives rise to the average queue growth rate for \( q^i \). At this rate, \( \hat{q}_i \) is expected
to continue to grow until phase \( i \) if the signal switches now. This 'expected' duration in which the queue continues to grow is
estimated by utilizing the average phase duration in the past. We term this method DORAS-Q. DORAS-Q is much less data
demanding, but does require knowledge of the existing queues.

Worthy of a notion is that Dunne and Potts (1964,1967) first studied a queue clearance policy, in which the green signal
switches when the existing queue is cleared provided that the green interval has not reached its preset maximum. Dunne
and Potts (1964) studied from the stability point of view while Dunne and Potts (1967) examined from the performance
perspective. Our proposed method here may be considered an extension to the line of literature on queue clearance.
But our result often dictates a signal switch even if the current queue (likely from a minor approach) has not been cleared, a
way consistent to the special analysis in Grafton and Newell (1967).

5.3. An example parsing actuated control

We use an example as in Fig. 4 to illustrate the algorithm DORAS vs. actuated control. In Fig. 4, the intersection has four
phases with queued vehicle information shown. The effective green loss is 3 s for each signal switch. It takes 6, 4, and 6 s
respectively to clear the vehicle queues in phases 1 through 3 hypothetically. The minimum green time for each phase is
4 s and max 15 s. To be simple for presentation, we assume that no vehicles will join the queues for phases 1 through 3.
The current phase \( \phi_0 \) has cleared all queued vehicles and has two arriving vehicles in 1.5 and 2.0 s respectively as their
headway. The decision is whether to switch the green signal to phase 1. By information, it takes 4 s to clear 2 queued
vehicles in the current phase.
If the typical actuated signal control is set for the current phase, for example, with a critical gap of 2 s, the signal would wait till both arriving vehicles to have passed through the intersection. However, according to DORAS, \( \varepsilon_1 = \frac{q_2 + q_3 + q_4}{3 + 2 + 6 + 16} = 1 \) vehicle per second. The largest possible \( \varepsilon_0 = \frac{1}{3} = \frac{2}{3} \) vehicle per second. Obviously, DORAS would suggest a signal at the current decision point and would subsequently suggest a switch of signal after clearance of queued vehicles for each phase. The total vehicle waiting time saved compared to the actuated control due to DORAS is 117.5 out of a total of 371.5 vehicle seconds, a 30% saving.

Note that Fig. 4 is a fairly normal, light traffic example, considering that the queues are for the entire cycle time, and that each phase might serve several approaches, and each approach having multiple lanes. Therefore, the example here may imply only 2-3 vehicles in queue for each lane. A critical gap of 2 s used in actuated control implies, according to DORAS, a constant switch-to efficiency \( \varepsilon_1 = 1800 \) vph. A headway in the current phase smaller than the critical gap implies a current intersection efficiency \( \varepsilon_0 \geq \varepsilon_1 \), which justifies extension of green phase. In fact, the switch-to efficiency \( \varepsilon_1 \) varies with traffic, and is not a constant. Therefore, a fixed critical gap is usually suboptimal in the criterion for DORAS.

### 6. Numerical tests

We use a real intersection in an urban area of several million people for assessing the efficiency of our proposed algorithm. The intersection layout is illustrated as in Fig. 5a, in which each lane is labeled by an index number. There are four phases adopted in the simulation as illustrated in Fig. 5b. We simulate and compare four controls: fully actuated, DORAS, DORAS-Q, and OPAC III. OPAC III is selected because it represents a well established method in literature as elaborated in Gartner (1982). OPAC III conducts myopic optimization within a horizon of about a signal cycle with optimality and optimization horizon of each time similar to those of DORAS. They all repeat on a rolling horizon.

This intersection has vehicle detectors for each approach. The actual vehicle arriving processes are recorded through the detectors. We use those arrival processes recorded in history as input to the simulation. We select traffic data from both peak hours and off peak hours. Traffic volumes are summarized in Tables 1 and 2 on two days. As shown, the East-West direction is the major corridor while the north-south direction is relatively minor. The simulation setting includes: a 10 s minimum green time for each phase mainly to consider pedestrian crossing need, a 50 s maximum green time for phase 1 and 2, and 35 s for phase 3 and 4. For the actuated control, a 2 s critical gap is set up by following the conventional method. Other setups include a saturation flow rate of two seconds for each vehicle on each lane and a point queue policy.
regard, all the simplest the control.

These parameters are generally adopted from field implementation, and remain consistent in the simulation. In the setup for OPAC III, 60 s is used as the horizon for each optimization, which is roughly the average cycle length.

Tables 4 and 5 as equivalently illustrated by Fig. 6 show the intersection performance via simulation by using two day actual traffic arrival data. DORAS, DORAS-Q and OPAC III all consistently and significantly outperform the fully actuated control. Compared to the fully actuated control, DORAS and DORAS-Q may reduce the average delay by up to about 20% in the peak hour. DORAS overall appears slightly better than DORAS-Q, but DORAS-Q is much less data demanding and is the simplest to implement among all the three. OPAC III performs the best among all of them, especially during midnight when the traffic is very sparse.

OPAC III has an impressive performance in the simulation. OPAC III out looks into a cycle length for traffic arrivals for all approaches including the current one. In contrast, DORAS only looks over a period of 4-5 s for the current signal. In this regard, DORAS appears more myopic and local than OPAC III. However, OPAC III resorts to an enumeration method that cost

Table 2  
Traffic count in select hours on Day One (in cars).

<table>
<thead>
<tr>
<th>Time period</th>
<th>Total</th>
<th>Left-Turn/Through volume</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Northbound</td>
<td>Southbound</td>
<td>Eastbound</td>
</tr>
<tr>
<td>7:30–8:30 AM</td>
<td>4369</td>
<td>338/345</td>
<td>75/493</td>
<td>252/1278</td>
</tr>
<tr>
<td>4:30–5:30 PM</td>
<td>4399</td>
<td>330/396</td>
<td>146/588</td>
<td>313/1390</td>
</tr>
<tr>
<td>1:00–2:00 PM</td>
<td>2802</td>
<td>184/213</td>
<td>77/261</td>
<td>159/964</td>
</tr>
<tr>
<td>12:00–1:00 AM</td>
<td>438</td>
<td>49/39</td>
<td>4/49</td>
<td>27/127</td>
</tr>
</tbody>
</table>

Table 3  
Day Two intersection cycles under different signal control (in seconds).

<table>
<thead>
<tr>
<th>Time period</th>
<th>Actuated</th>
<th>DORAS</th>
<th>DORAS-Q</th>
<th>OPAC III</th>
</tr>
</thead>
<tbody>
<tr>
<td>7:30–8:30 AM</td>
<td>66.47</td>
<td>61.90</td>
<td>66.46</td>
<td>65.05</td>
</tr>
<tr>
<td>4:30–5:30 PM</td>
<td>56.05</td>
<td>57.00</td>
<td>61.72</td>
<td>62.63</td>
</tr>
<tr>
<td>1:00–2:00 PM</td>
<td>52.37</td>
<td>54.80</td>
<td>57.48</td>
<td>63.79</td>
</tr>
<tr>
<td>12:00–1:00 AM</td>
<td>48.22</td>
<td>48.74</td>
<td>62.07</td>
<td>68.42</td>
</tr>
</tbody>
</table>

Table 4  
Day Two vehicle delay comparison (in seconds).

<table>
<thead>
<tr>
<th>Time period</th>
<th>Actuated</th>
<th>DORAS</th>
<th>DORAS-Q</th>
<th>OPAC III</th>
</tr>
</thead>
<tbody>
<tr>
<td>7:30–8:30 AM</td>
<td>20.24</td>
<td>17.36</td>
<td>18.17</td>
<td>17.23</td>
</tr>
<tr>
<td>4:30–5:30 PM</td>
<td>17.73</td>
<td>14.95</td>
<td>15.75</td>
<td>15.28</td>
</tr>
<tr>
<td>1:00–2:00 PM</td>
<td>16.66</td>
<td>15.12</td>
<td>14.47</td>
<td>13.33</td>
</tr>
<tr>
<td>12:00–1:00 AM</td>
<td>14.16</td>
<td>12.76</td>
<td>11.39</td>
<td>7.75</td>
</tr>
</tbody>
</table>

These parameters are generally adopted from field implementation, and remain consistent in the simulation. In the setup for OPAC III, 60 s is used as the horizon for each optimization, which is roughly the average cycle length.

Tables 4 and 5 as equivalently illustrated by Fig. 6 show the intersection performance via simulation by using two day actual traffic arrival data. DORAS, DORAS-Q and OPAC III all consistently and significantly outperform the fully actuated control. Compared to the fully actuated control, DORAS and DORAS-Q may reduce the average delay by up to about 20% in the peak hour. DORAS overall appears slightly better than DORAS-Q, but DORAS-Q is much less data demanding and is the simplest to implement among all the three. OPAC III performs the best among all of them, especially during midnight when the traffic is very sparse.

OPAC III has an impressive performance in the simulation. OPAC III out looks into a cycle length for traffic arrivals for all approaches including the current one. In contrast, DORAS only looks over a period of 4-5 s for the current signal. In this regard, DORAS appears more myopic and local than OPAC III. However, OPAC III resorts to an enumeration method that cost

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significantly more computational time. For each of the four select peak hours on the two days, DORAS takes less than 30 s to complete the hourly simulation while OPAC III takes an average of about 1970 s, 60–70 times more than DORAS. The time difference is largely due to the control algorithms.

**Table 3** indicates that the improved controls tend to have a slightly longer cycle length. Compared with actuated control, our test results also show that all the other three controls have consistently and significantly reduced the number of vehicle stops, which shows consistency between the two often mentioned objectives: minimizing vehicle waiting time and vehicle stops at the intersection.

**Control with Partial Data** DORAS-Q uses almost only the queueing data at approaches along with limited statistics such as average phase lengths. However, when the same partial data is applied to OPAC III, OPAC III could perform much worse than even the actuated in our simulation. In summary, if data is available in a longer term as evidenced by the intervehicle communication technologies, OPAC III appears superior. For shorter terms, DORAS-Q appears advantageous. However, the performance of DORAS and DORAS-Q is conditional on how to approximate the salvage effect. If better approximations to the salvage effect are developed, their relative performance may be further improved.

7. Conclusion

This paper attempts to study the optimal policy for isolated, real-time signal control with perfect traffic information. It defines the optimal policy to be one with a series of signal switch points of time that keep the intersection vehicle waiting time to its minimum. A continuous model is developed to characterize the queueing process as a function of control policy and vehicle arrival processes. The continuous model applied to the general intersection appears to be the first in literature and nicely allows to examine the problem in a way that previous literature does not allow.

A simple structure to interpret and intuitively understandable, the optimal condition implied is that signal control shall maximize the intersection efficiency, which deepens the understanding the queue clearance policy that is well studied in literature. An interesting feature is that our model and algorithm allow for a flexible set of rules. The major challenge is to calculate the marginal effect from signal switch. This paper proposes a simple and efficient algorithm DORAS, which is based on projected vehicle clearance for each phase. We simulate an intersection control by using actual intersection traffic to show the efficiency of our proposed algorithm DORAS. Tests show that DORAS significantly reduces delay compared with the fully actuated control. Another easy-to-implement, queue based control called DORAS-Q is also proposed, which requires little advance information except for the exiting queues. DORAS-Q also significantly reduces delay of vehicles. All these controls are compared with an established one in literature, OPAC III. Tests show that OPAC III consistently outperforms DORAS and DORAS-Q, but is computationally much more costly. In addition, if OPAC III is subject to the same data supply as for DORAS-Q, OPAC III performs much worse.

DORAS and DORAS-Q are fairly flexible for implementation. They can flexibly accommodate constraints such as the minimum/maximum green time and phase sequence requirement. In addition, they can conduct differential treatment of vehicles by assigning varying weight factors to vehicles such as buses. The optimal intersection control in turn highlights a social implication of intersection geometric design. Geometrically asymmetric intersections almost always mean that ROW for minor...
streets implies loss of intersection efficiency or increased waiting time for vehicles along the major approaches. This social burden (or cost) due to intersection with a minor street can be relieved by raising the number of pass-through lanes for the minor streets at the intersection, in which case, a queue clearance policy would likely be optimal.

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References


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