PROBLEM 2.66

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that \( E = 200 \text{ GPa} \) and \( \nu = 0.29 \), determine the internal force in the bolt, if the diameter is observed to decrease by 13 \( \mu \text{m} \).

**SOLUTION**

\[
\begin{align*}
\sigma_y &= -13 \times 10^{-6} \text{ m} \quad d = 60 \times 10^{-3} \text{ m} \\
\varepsilon_y &= \frac{\sigma_y}{E} = -\frac{13 \times 10^{-6}}{200 \times 10^9} = -216.67 \times 10^{-6} \\
\nu &= -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{216.67 \times 10^{-6}}{0.39} = 747.13 \times 10^{-6} \\
\sigma_x &= E \varepsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa} \\
A &= \frac{\pi}{4} d^2 = \frac{\pi}{4}(60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2 \\
F &= \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N} \\
&= 422 \text{ kN}
\end{align*}
\]

PROBLEM 2.67

An aluminum plate \( (E = 74 \text{ GPa}, \nu = 0.33) \) plate is subjected to a centric axial load which causes a normal stress \( \sigma \). Knowing that before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when \( \sigma = 125 \text{ MPa} \).

**SOLUTION**

The slope after deformation is

\[
\tan \theta = \frac{2(1 + \varepsilon_y)}{1 + \varepsilon_x}
\]

\[
\varepsilon_x = \frac{\sigma_x}{E} = \frac{125 \times 10^6}{74 \times 10^9} = 1.6892 \times 10^{-3}
\]

\[
\varepsilon_y = -\nu \varepsilon_x = -(0.33)(1.6892 \times 10^{-3}) = 0.5574 \times 10^{-3}
\]

\[
\tan \theta = \frac{2(1 - 0.0005574)}{1 + 0.0016892} = 1.99551
\]
PROBLEM 2.71

2.71 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses \( \sigma_x = 120 \times 10^6 \text{ Pa} \), \( \sigma_y = 0 \), \( \sigma_z = 160 \times 10^6 \text{ Pa} \). Knowing that the properties of the fabric can be approximated as \( E = 87 \text{ GPa} \) and \( \nu = 0.34 \), determine the change in length of (a) side \( AB \), (b) side \( BC \), (c) diagonal \( AC \).

**SOLUTION**

\[
\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}
\]

\[
e_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{E} \left[ 120 \times 10^6 - (0.34)(160 \times 10^6) \right] = 754.02 \times 10^{-6}
\]

\[
e_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{87 \times 10^9} \left[ -(0.34)(120 \times 10^6) + 160 \times 10^6 \right] = 1.3701 \times 10^{-3}
\]

(a) \( S_{AB} = (\overline{AB}) e_x = (100 \text{ mm}) (754.02 \times 10^{-6}) = 0.0754 \text{ mm} \)

(b) \( S_{BC} = (\overline{BC}) e_x = (75 \text{ mm}) (1.3701 \times 10^{-3}) = 0.1028 \text{ mm} \)

(c) Label sides of right triangle \( ABC \) as \( a \), \( b \), and \( c \)

\[
c^2 = a^2 + b^2
\]

Obtain differentials by calculus

\[
2c \, dc = 2a \, da + 2b \, db
\]

\[
dc = \frac{a}{c} \, da + \frac{b}{c} \, db
\]

But \( a = 100 \text{ mm}, \quad b = 100 \text{ mm}, \quad c = \sqrt{100^2 + 75^2} = 125 \text{ mm} \)

\[
da = S_{AB} = 0.0754 \text{ mm}, \quad db = S_{BC} = 0.1028 \text{ mm}
\]

\[
S_{AC} = dc = \frac{100}{125} (0.0754) + \frac{75}{125} (0.1028) = 0.1220 \text{ mm}
\]
2.72 The brass rod $AD$ is fitted with a jacket that is used to apply an hydrostatic pressure of 48 MPa to the 250-mm portion $BC$ of the rod. Knowing that $E = 105$ GPa, and $\nu = 0.33$, determine (a) the change in the total length $AD$, (b) the change in diameter of portion $BC$ of the rod.

**SOLUTION**

\[ \sigma_x = \sigma_z = -p = -48 \times 10^6 \text{ Pa}, \quad \sigma_y = 0 \]

\[ \varepsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \]

\[ = \frac{1}{105 \times 10^9} \left[ -48 \times 10^6 - (0.33)(0) - (0.33)(-48 \times 10^6) \right] \]

\[ = -306.29 \times 10^{-6} \]

\[ \varepsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) \]

\[ = \frac{1}{105 \times 10^9} \left[ -0.33(-48 \times 10^6) + 0 - (0.33)(-48 \times 10^6) \right] \]

\[ = -801.71 \times 10^{-6} \]

(a) Change in length: Only portion $BC$ is strained, $L = 240 \text{ mm}$

\[ \Delta L = L \varepsilon_y = (240 \times 0.301.71 \times 10^{-6}) = -0.0724 \text{ mm} \]

(b) Change in diameter:

\[ \Delta d = d \varepsilon_y = (50 \times 0.296.29 \times 10^{-6}) = -0.01531 \text{ mm} \]

2.75 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as plane stress, show that if the strains $\varepsilon_x$ and $\varepsilon_y$ have been determined experimentally, we can express $\sigma_x$, $\sigma_y$, and $\sigma_z$ as follows:

\[ \sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x - \nu \varepsilon_y) \quad (1) \quad \sigma_y = \frac{E}{1-\nu^2} (\varepsilon_y - \nu \varepsilon_x) \quad (2) \quad \varepsilon_z = -\frac{\nu}{1-\nu} (\varepsilon_x + \varepsilon_y) \]

Multiplying (2) by $\nu$ and adding to (1):

\[ \varepsilon_x + \nu \varepsilon_y = \frac{1-\nu^2}{E} \sigma_x \quad \text{ or } \quad \sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y) \]

Multiplying (1) by $\nu$ and adding to (2):

\[ \varepsilon_y + \nu \varepsilon_x = \frac{1-\nu^2}{E} \sigma_y \quad \text{ or } \quad \sigma_y = \frac{E}{1-\nu^2} (\varepsilon_y + \nu \varepsilon_x) \]

\[ \varepsilon_z = \frac{1}{E} (\nu \sigma_z - \nu \sigma_y) = -\frac{\nu}{E} \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y + \varepsilon_y + \nu \varepsilon_x) \]

\[ = -\frac{\nu(1+\nu)}{1-\nu^2} (\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu} (\varepsilon_x + \varepsilon_y) \]
2.77 Two blocks of rubber, each of width \( w = 60 \text{ mm} \), are bonded to rigid supports and to the movable plate \( AB \). Knowing that a force of magnitude \( P = 19 \text{ kN} \) causes a deflection \( \delta = 3 \text{ mm} \), determine the modulus of rigidity of the rubber used.

**SOLUTION**

Consider upper block of rubber. The force carried is \( \frac{1}{2}P \).

The shearing stress is

\[
\tau = \frac{\frac{1}{2}P}{A}
\]

where

\[
A = (180 \text{ mm})(60 \text{ mm}) = 10.8 \times 10^3 \text{ mm}^2 = 10.8 \times 10^{-3} \text{ m}^2
\]

\[
P = 19 \times 10^3 \text{ N}
\]

\[
\tau = \frac{\frac{1}{2}(19 \times 10^3)}{10.8 \times 10^{-3}} = 0.87963 \times 10^6 \text{ Pa}
\]

\[
\frac{\delta}{h} = \frac{3 \text{ mm}}{35 \text{ mm}} = 0.085714
\]

\[
G = \frac{\tau}{\frac{\delta}{h}} = \frac{0.87963 \times 10^6}{0.085714} = 10.26 \times 10^6 \text{ Pa} = 10.26 \text{ MPa}
\]

2.81 An elastomeric bearing \((G = 0.9 \text{ MPa})\) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than \( 10 \text{ mm} \) when a \( 22 \text{ kN} \) lateral load is applied as shown. Determine (a) the smallest allowable dimension \( b \), (b) the smallest required thickness \( a \) if the maximum allowable shearing stress is \( 420 \text{ kPa} \).

**SOLUTION**

Shearing force \( P = 22 \times 10^3 \text{ N} \)

Shearing stress \( \tau = 420 \times 10^3 \text{ Pa} \)

\[
\tau = \frac{P}{A} \Rightarrow A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3} \text{ m}^2
\]

\[
A = (200 \text{ mm})(b)
\]

\[
b = \frac{A}{200} = \frac{52.381 \times 10^{-3}}{200} = 262 \text{ mm}
\]

\[
\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}
\]

But \( \gamma = \frac{S}{a} \Rightarrow a = \frac{S}{\gamma} = \frac{10 \text{ mm}}{466.67 \times 10^{-3}} = 21.4 \text{ mm}
\]
PROBLEM 2.99

2.99 (a) Knowing that the allowable stress is 140 MPa, determine the maximum allowable magnitude of the centric load \( P \). (b) Determine the percent change in the maximum allowable magnitude of \( P \) if the raised portions are removed at the ends of the specimen.

**SOLUTION**

\[
\frac{D}{d} = \frac{75 \text{ mm}}{50 \text{ mm}} = 1.50 \quad \frac{r}{d} = \frac{6 \text{ mm}}{50 \text{ mm}} = 0.12
\]

From Fig 2.64 \( \kappa = 2.10 \)

\[
A_{\text{min}} = \pi d = (15 \times 50) = 750 \text{ mm}^2 = 7.5 \times 10^{-6} \text{ m}^2
\]

(a) \[
G_{\text{max}} = \frac{KP}{A_{\text{min}}} \quad P = \frac{A_{\text{min}}G_{\text{max}}}{\kappa} = \frac{(750 \times 10^{-6})(140 \times 10^3)}{2.10} = 50 \times 10^3 \text{ N} = 50 \text{ kN}
\]

(b) Without raised section. \( \kappa = 1.00 \)

\[
P = A_{\text{min}}G_{\text{max}} = (750 \times 10^{-6})(140 \times 10^3) = 105 \times 10^3 = 105 \text{ kN}
\]

\% change = \( \frac{(105 - 50)}{50} \times 100\% = 110\% \)
PROBLEM 2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which \( \alpha = 345 \) MPa and \( E = 200 \) GPa. A force \( P \) is applied to the rigid bar \( ABC \) until the bar has moved downward a distance \( \delta = 2 \) mm. Knowing that the cables were initially taut, determine (a) the maximum value of \( P \), (b) the maximum stress that occurs in cable \( AD \), (c) the final displacement of the bar after the load is removed. (Hint: In part c, cable \( BE \) is not taut.)

**SOLUTION**

For each cable \( A = \pi (0.006)^2 = 28.274 \times 10^{-6} \) m²

Strain at initial yielding:

\[
\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{345 \times 10^6}{200 \times 10^9} = 1.725 \times 10^{-3}
\]

Strain in cables \( AD \) and \( CF \):

\[
\varepsilon_{AD} = \varepsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2 \text{ mm}}{1600 \text{ mm}} = 1.25 \times 10^{-3}
\]

Strain in cable \( BE \):

\[
\varepsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2 \text{ mm}}{800 \text{ mm}} = 2.50 \times 10^{-3}
\]

Since \( \varepsilon_{AD} < \varepsilon_Y \), \( \sigma_{AD} = E \varepsilon_{AD} = (200 \times 10^9)(1.25 \times 10^{-3}) = 250 \times 10^6 \) Pa

Since \( \varepsilon_{BE} > \varepsilon_Y \), \( \sigma_{BE} = \sigma_Y = 345 \times 10^6 \) Pa

Forces:

\[
P_{AD} = P_{CF} = A \sigma_{AD} = (28.274 \times 10^{-6})(250 \times 10^6) = 7.0685 \times 10^3 \text{ N}
\]

\[
P_{BE} = A \sigma_{BE} = (28.274 \times 10^{-6})(345 \times 10^6) = 9.7545 \times 10^3 \text{ N}
\]

For equilibrium of bar \( ABC \) \( P_{AD} + P_{BE} + P_{CF} = P = 0 \)

(a) \( P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685) \times 10^3 \text{ N} = 23.9 \times 10^3 \text{ N} = 23.9 \text{ kN} \)

(b) \( \sigma_{AD} = 250 \times 10^6 \text{ Pa} = 250 \text{ MPa} \)

After unloading \( P = 0 \)

Cable \( BE \) is not taut \( P_{BE} = 0 \)

By symmetry \( P_{AD} = P_{CF} \)

For equilibrium \( P_{AD} = P_{CF} = 0 \)

(c) Final displacement \( \delta \) is controlled by the final lengths of cables \( AD \) and \( CF \). Since these cables were never permanently deformed, the final displacement is

\[
\delta = \delta_{AD} = \delta_{CF} = 0
\]
2.113 Bar AB has a cross-sectional area of 1200 mm\(^2\) and is made of a steel that is assumed to be elasto-plastic with \(E = 200 \text{ GPa}\) and \(\sigma_y = 250 \text{ MPa}\). Knowing that the force \(F\) increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C, (b) the residual stress in the bar.

**SOLUTION**

\[
 A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2
\]

Force to yield portion AC:

\[
P_{AC} = A \sigma_y = (1200 \times 10^{-6})(250 \times 10^6) = 300 \times 10^3 \text{ N}
\]

For equilibrium:

\[
 F + P_{CB} - P_{AC} = 0
\]

\[
P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3
\]

\[
= -220 \times 10^3 \text{ N}
\]

\[
S_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.29333 \times 10^{-3} \text{ m}
\]

\[
\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}
\]

**Unloading**

\[
S_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}')(L_{CB})}{EA}
\]

\[
P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N}
\]

\[
P_{CB}' = P_{AC}' - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N}
\]

\[
\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa}
\]

\[
\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-141.818 \times 10^3}{1200 \times 10^{-6}} = -118.182 \times 10^6 \text{ Pa}
\]

\[
S_C' = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.189091 \times 10^{-3} \text{ m}
\]

(a) \(S_{CP} = S_C - S_C' = 0.29333 \times 10^{-3} - 0.189091 \times 10^{-3} = 0.1042 \times 10^{-3} \text{ m}
\]

(b) \(\sigma_{AC, res} = \sigma_y - \sigma_{AC}' = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \times 10^6 \text{ Pa}
\]

\(\sigma_{CB, res} = \sigma_{CB}' = -118.182 \times 10^6 \)
2.117 A uniform steel rod of cross-sectional area $A$ is attached to rigid supports and is unstressed at a temperature of 8°C. The steel is assumed to be elastoplastic with $\sigma_y = 250$ MPa and $G = 200$ GPa. Knowing that $\alpha = 11.7 \times 10^{-6}$°C, determine the stress in the bar (a) when the temperature is raised to 165°C, (b) after the temperature has returned to 8°C.

**SOLUTION**

Determine temperature change to cause yielding

$$S = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_y L}{E} + L\alpha(\Delta T)$$

$$\Delta T = \frac{\sigma_y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = 106.838 \degree C$$

But $\Delta T = 165 - 8 = 157 \degree C$

(a) Yielding occurs $= \sigma = -\sigma_y = -250$ MPa

Cooling $(\Delta T)' = 157 \degree C$

$$S' = S_p' + S_T' = -\frac{P'}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(200 \times 10^9)(11.7 \times 10^{-6})(157) = -367.38 \times 10^6$$

(b) $\sigma_{res} = -\sigma_y - \sigma' = -250 \times 10^6 + 367.38 \times 10^6 = 117.38 \times 10^6$ MPa

$$= 117.4$$ MPa
2.127 The block shown is made of a magnesium alloy for which \( E = 6.5 \times 10^6 \) psi and \( \nu = 0.35 \). Knowing that \( \sigma_y = -20 \) ksi, determine (a) the magnitude of \( \sigma_y \) for which the change in the height of the block will be zero, (b) the corresponding change in the area of the face \( ABCD \), (c) the corresponding change in the volume of the block.

**SOLUTION**

\[
S_y = 0 \quad \varepsilon_y = 0
\]

\[
\varepsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = 0
\]

\[(a) \quad \sigma_y = \nu \sigma_x = (0.35)(-20 \times 10^3)
\]

\[= -7 \times 10^3 \text{ psi} = -7 \text{ ksi}
\]

\[(b) \quad \varepsilon_z = \frac{1}{E} \left(-\nu \sigma_x - \nu \sigma_y\right) = -\frac{\nu (\sigma_x + \sigma_y)}{E}
\]

\[= \frac{(0.35)(-20 \times 10^3 - 7 \times 10^3)}{6.5 \times 10^6} = 1.4538 \times 10^{-3}
\]

\[\varepsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{-20 \times 10^3}{6.5 \times 10^6} - \frac{(0.35)(-7 \times 10^3)}{6.5 \times 10^6}
\]

\[= -2.7 \times 10^{-3}
\]

\[A_o + \Delta A = L_x (1 + \varepsilon_x) L_z (1 + \varepsilon_z) = L_x L_z (1 + \varepsilon_x + \varepsilon_z + \varepsilon_x \varepsilon_z)
\]

But \( A_o = L_x L_z \)

\[\Delta A = L_x L_z (\varepsilon_x + \varepsilon_z + \varepsilon_x \varepsilon_z)
\]

\[= (4.0)(1.0)(1.4538 \times 10^{-3} - 2.7 \times 10^{-3} + \text{small term})
\]

\[= -4.98 \times 10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2
\]

\[(c) \quad \text{Since } L_y \text{ is constant}
\]

\[\Delta V = L_y (\Delta A) = (1.375)(-4.98 \times 10^{-3}) = -6.85 \times 10^{-3} \text{ in}^3
\]

\[= -0.00685 \text{ in}^3
\]