3.8.1 The shaft-disk-belt arrangement shown is used to transmit 3 hp from point A to point D. (a) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are respectively 0.75 in. and 0.625 in.

**SOLUTION**

\[ \tau = 9500 \text{ psi}, \quad P = 3 \text{ hp} = (3)(6000) = 19800 \text{ lb-in/s} \]

\[ \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad T = \frac{P}{2} c^3 \tau \]

**Allowable torques**

\( \frac{5}{8} \) in. diameter shaft

\[ c = \frac{5}{8} \text{ in}, \quad T_{a.m} = \frac{2}{\pi} \left(\frac{5}{8}\right)^3 (9500) = 455.4 \text{ lb-in} \]

\( \frac{3}{4} \) in. diameter shaft

\[ c = \frac{3}{4} \text{ in}, \quad T_{a.m} = \frac{2}{\pi} \left(\frac{3}{4}\right)^3 (9500) = 786.9 \text{ lb-in} \]

**Statics:**

\[ T_B = T_e (F_1 - F_2) \quad T_C = T_e (F_1 - F_2) \]

\[ T_B = \frac{P}{2 \pi f} \quad T_C = \frac{1.125}{4.5} T_e = 0.25 T_e \]

(a) **Allowable torques**

\[ T_{a.m} = 455.4 \text{ lb-in}, \quad T_{c.a.m} = 786.9 \text{ lb-in} \]

Assume \( T_C = 786.9 \text{ lb-in} \). Then \( T_B = (0.25)(786.9) = 196.73 \text{ lb-in} \)

\[ P = 2\pi f T \quad f_{ab} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)} = 16.02 \text{ Hz} \]

(b) **Allowable torques**

\[ T_{b.a.m} = 786.9 \text{ lb-in}, \quad T_{c.a.m} = 455.4 \text{ lb-in} \]

Assume \( T_C = 455.4 \text{ lb-in} \). Then \( T_B = (0.25)(455.4) = 113.85 \text{ lb-in} \)

\[ P = 2\pi f T \quad f_{ab} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)} = 27.7 \text{ Hz} \]
PROBLEM 3.84

3.84 A 1.5-in.-diameter steel shaft of length 4 ft will be used to transmit 60 hp between a motor and a pump. Knowing that $G = 11.2 \times 10^6$ psi, determine the lowest speed of rotation at which the shearing stress will not exceed 8500 psi and the angle of twist will not exceed 2°.

SOLUTION

\[ C = \frac{1}{2} d = 0.75 \text{ in} \quad L = 4 \text{ ft} = 48 \text{ in.} \]

Torque based on maximum shearing stress limit $C = 8500$ psi,

\[ T = \frac{2 \pi}{\pi C^2} \cdot \frac{x}{2} = \frac{\pi}{2} (0.75)^2 (8500) = 5.683 \times 10^3 \text{ lb} \cdot \text{in.} \]

Torque based on twist angle limit $\phi = 2^\circ = 34.907 \times 10^3$ rad

\[ \phi = \frac{TL}{C^3}; \quad T = \frac{GJ\phi}{L} = \frac{\pi}{2L} \left( \frac{0.75}{4} \right)^2 (11.2 \times 10^6)(34.907 \times 10^3) \]

\[ = 4.048 \times 10^3 \text{ lb} \cdot \text{in.} \]

Smaller torque governs $T = 4.048 \times 10^3 \text{ lb} \cdot \text{in.}$

\[ P = 2\pi ft \quad \text{where} \quad P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb} \cdot \text{in.} / \text{s} \]

\[ f = \frac{P}{2\pi T} = \frac{396 \times 10^3}{2\pi (4.048 \times 10^3)} = 15.57 \text{ Hz} = 934 \text{ rpm} \]

---

PROBLEM 3.157

3.157 Two solid brass rods $AB$ and $CD$ are brazed to a brass sleeve $EF$. Determine the ratio $d_2/d_1$ for which the same maximum shearing stress occurs in the rods and in the sleeve.

SOLUTION

Let $c_1 = \frac{1}{2} d_1$ and $c_2 = \frac{1}{2} d_2$

Shaft $AB$ $\tau_1 = \frac{Tc_1}{I_1} = \frac{2T}{\pi c_1^3}$

Sleeve $EF$ $\tau_2 = \frac{Tc_2}{I_2} = \frac{2Tc_2}{\pi (c_2^3 - c_1^3)}$

For equal stresses $\frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi (c_2^3 - c_1^3)}$

\[ c_2^4 - c_1^4 = c_1^3 c_2 \]

Let $x = \frac{c_2}{c_1}$, $x^4 - 1 = x$ or $x = \sqrt[4]{1 + x}$

Solve by successive approximations starting with $x_0 = 1.0$

\[ x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216 \quad x_3 = \sqrt[4]{2.216} = 1.220 \]

\[ x_4 = \sqrt[4]{2.220} = 1.221 \quad x_5 = \sqrt[4]{2.221} = 1.221 \quad \text{(converged)} \]

\[ x = 1.221 \quad \frac{c_2}{c_1} = \frac{d_2}{d_1} = 1.221 \]
3.161 The composite shaft shown is twisted by applying a torque \( T \) at end \( A \). Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use \( G = 77 \) GPa for steel and \( G = 27 \) GPa for aluminum.

**SOLUTION**

Let \( G_1 \), \( J_1 \), and \( \tau_1 \) refer to the aluminum core, and \( G_2 \), \( J_2 \), and \( \tau_2 \) refer to the steel shell.

At the outer surface on the steel shell

\[
\tau_2 = \frac{C_2 \phi}{L} \quad \frac{\phi}{L} = \frac{\tau_1}{C_1} = \frac{C_2 \tau_2}{C_1 G_2}
\]

At the outer surface of the aluminum core

\[
\tau_1 = \frac{C_1 \phi}{L} \quad \frac{\phi}{L} = \frac{\tau_1}{C_1} = \frac{C_1 \tau_1}{C_1 G_1}
\]

Matching \( \frac{\phi}{L} \) for both components

\[
\frac{\tau_2}{C_1 G_2} = \frac{\tau_1}{C_1 G_1}
\]

Solving for \( \tau_2 \)

\[
\tau_2 = \frac{C_1}{C_1} \cdot \frac{G_2}{G_1} \cdot \tau_1
\]

\[
= \frac{0.030}{0.040} \cdot \frac{27 \times 10^6}{77 \times 10^6} \cdot 150 \times 10^6
\]

\[
= 39.4 \times 10^6 \text{ Pa} = 39.4 \text{ MPa}
\]

---

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point \( A \), (b) point \( B \).

**SOLUTION**

For rectangle \( I = \frac{1}{12} bh^3 \)

For cross sectional area

\[
I = I_1 + I_2 + I_3 = \frac{1}{12} (2)(1.5)^3 + \frac{1}{12} (2)(5.5)^3 + \frac{1}{12} (2)(1.5)^3 = 28.854 \text{ in}^4
\]

(a) \( y_A = 2.75 \text{ in} \)

\[
\sigma_A = -\frac{M y_A}{I} = -\frac{(25)(2.75)}{28.854} = -2.38 \text{ ksi}
\]

(b) \( y_B = 0.75 \text{ in} \)

\[
\sigma_B = -\frac{M y_B}{I} = -\frac{(25)(0.75)}{28.854} = -0.650 \text{ ksi}
\]
**PROBLEM 4.2**

Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

**SOLUTION**

\[
V_A = \frac{1}{2} d_1 = 15 \text{ mm} \quad V_B = \frac{1}{2} d_0 = 20 \text{ mm}
\]

\[
I = \frac{4}{3}(V_A^4 - V_B^4) = \frac{4}{3}(20^4 - 15^4)
\]

\[
I = 85,903 \times 10^3 \text{ mm}^4 = 85,903 \times 10^{-9} \text{ m}^4
\]

(a) \[y_A = 20 \text{ mm} = 0.020 \text{ m}\]

\[\sigma_A = -\frac{M y_A}{I} = -\frac{(500)(0.020)}{85,903 \times 10^{-9}}\]

\[= -116.4 \times 10^6 \text{ Pa} = -116.4 \text{ MPa}\]

(b) \[y_B = 15 \text{ mm} = 0.015 \text{ m}\]

\[\sigma_B = -\frac{M y_B}{I} = -\frac{(500)(0.015)}{85,903 \times 10^{-9}}\]

\[= -87.3 \times 10^6 \text{ Pa} = -87.3 \text{ MPa}\]

**PROBLEM 4.3**

The wide-flange beam shown is made of a high-strength, low-alloy steel for which \(\sigma_f = 345\) MPa and \(\sigma_y = 450\) MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the \(z\) axis. Neglect the effect of fillets.

**SOLUTION**

\[
I_1 = \frac{1}{12} b h^3 + A d^2
\]

\[
= \frac{1}{12} (250)(18^3) + (250)(18)(171)^2
\]

\[= 131,706 \times 10^6 \text{ mm}^4\]

\[I_2 = \frac{1}{12} (10)(324)^3 = 28,344 \times 10^6 \text{ mm}^4\]

\[I_3 = I_1 = 131,706 \times 10^6 \text{ mm}^4\]

\[I = I_1 + I_2 + I_3 = 291,76 \times 10^6 \text{ mm}^4 = 291,76 \times 10^{-4} \text{ m}^4\]

\[\sigma = \frac{M c}{I} \quad \text{where} \quad c = \frac{360}{2} = 180 \text{ mm} = 0.180 \text{ m}\]

\[\sigma_{fl} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}\]

\[M_{fl} = \frac{\sigma_{fl} I}{c} = \frac{(150 \times 10^6)(291,76 \times 10^{-4})}{0.180} = 243 \times 10^3 \text{ Nm}\]

\[= 243 \text{ kN.m}\]
4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_t = 345$ MPa and $\sigma_y = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the $z$ axis. Neglect the effect of fillets.

4.4 Solve Prob. 4.3, assuming that is bent about the $y$ axis.

**SOLUTION**

$$I_1 = \frac{1}{12} (18)(250)^3 = 23,438 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (324)(10)^3 = 27 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 23,438 \text{ mm}^4$$

$$I_y = I_1 + I_2 + I_3 = 46,903 \times 10^6 \text{ mm}^4 = 46,903 \times 10^6 \text{ m}^4$$

$$C = \frac{250}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

$$G_{sfl} = \frac{G_y}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$G = \frac{Mc}{I} \quad M_y = \frac{G_{sfl} I}{C} = \frac{(150 \times 10^6)(46,903 \times 10^6)}{0.125}$$

$$= 56.3 \times 10^3 \text{ N} \cdot \text{m} = 56.3 \text{ kN} \cdot \text{m}$$
PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple \( M \) that can be applied.

### SOLUTION

<table>
<thead>
<tr>
<th>( A, \text{mm}^2 )</th>
<th>( \bar{y}, \text{mm} )</th>
<th>( A\bar{y}, \text{mm}^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. solid rectangle</td>
<td>46.03</td>
<td>48</td>
</tr>
<tr>
<td>2. square cutout</td>
<td>-12.96</td>
<td>-30</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>38.12</td>
<td>182.04</td>
</tr>
</tbody>
</table>

\[
\bar{Y} = \frac{182.04}{38.12} = 55.04 \quad \text{mm}
\]

Neutral axis lies 55.04 mm above bottom.

\[
Y_{\text{top}} = 96 - 55.04 = 40.96 \quad \text{mm} = 0.04096 \quad \text{m}
\]
\[
Y_{\text{bot}} = -55.04 \quad \text{mm} = -0.05504 \quad \text{m}
\]

\[
I_e = \frac{1}{12} b h e^3 + A e \bar{y}^2 = \frac{1}{12} (48)(96)^3 + (48)(96)(7.04)^2 = 8.7673 \times 10^6 \quad \text{mm}^4
\]

\[
I_x = \frac{1}{12} b h e^3 + A_2 e x^2 = \frac{1}{12} (36)(86)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \quad \text{mm}^4
\]

\[
I = I_e - I_x = 2.8147 \times 10^6 \quad \text{mm}^4 = 2.8147 \times 10^{-4} \quad \text{m}^4
\]

\[
|G| = \left| \frac{My}{I} \right| : M = + \left| \frac{Gy}{I} \right|
\]

Top: tension side
\[
M = \frac{(120 \times 10^6)(2.8147 \times 10^{-4})}{0.04096} = 8.25 \times 10^3 \quad \text{N}\cdot\text{m}
\]

Bottom: compression
\[
M = \frac{(150 \times 10^6)(2.8147 \times 10^{-4})}{0.05504} = 7.67 \times 10^3 \quad \text{N}\cdot\text{m}
\]

\( M_{\text{ult}} \) is the smaller value
\[
M = 7.67 \times 10^3 \quad \text{N}\cdot\text{m} = 7.67 \quad \text{kN}\cdot\text{m}
\]
PROBLEM 4.20

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.

SOLUTION

\[ \begin{array}{c|c|c|c}
A_i, \text{ mm}^2 & \bar{y}_i, \text{ mm} & A_i \bar{y}_i, \text{ mm}^3 \\
\hline
1 & 2160 & 27 & 58820 \\
2 & 1080 & 36 & 38880 \\
\Sigma & 3240 & 97200 \\
\end{array} \]

\[ \overline{y} = \frac{97200}{3240} = 30 \text{ mm} \]

The neutral axis lies 30 mm above the bottom.

\[ y_{nt} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \]
\[ y_{nf} = -30 \text{ mm} = -0.030 \text{ m} \]

\[ I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^8 \text{ mm}^4 \]

\[ I_2 = b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.24 \times 10^8 \text{ mm}^4 \]

\[ I = I_1 + I_2 = 758.16 \times 10^8 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4 \]

\[ |M| = |\frac{My}{I}| \]

Top: tension side \[ M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N-m} \]

Bottom: compression \[ M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N-m} \]

Choose the smaller as \( M_{ul} \) \[ M_{ul} = 3.7908 \times 10^3 \text{ N-m} = 3.79 \text{ kN-m} \]
PROBLEM 4.32

4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple \( \mathbf{M} \). Considering the case where \( h = 0.9h_0 \), express the maximum stress in the bar in the form \( \sigma = k \sigma_0 \), where \( \sigma_0 \) is the maximum stress that would have occurred if the original square bar had been bent by the same couple \( \mathbf{M} \), and determine the value of \( k \).

**SOLUTION**

\[
I = 4I_1 + 2I_t = (4)(\frac{1}{12})h_0^3 + (2)(\frac{1}{2})(2h_0 - 2h)(h^3) = \frac{1}{3}h_0^4 + \frac{4}{3}h_0^3 - \frac{1}{3}h^3 = \frac{4}{3}h_0^3 - h^3
\]

\[
c = h
\]

\[
G = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0^3 - h^3} = \frac{3M}{(4h_0 - 3h)h^2}
\]

For the original square \( h = h_0 \), \( c = h_0 \)

\[
G_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}
\]

\[
\frac{G}{G_0} = \frac{h_0^3}{(4h_0 - 3h_0)h_0^2} = \frac{h_0^3}{(4h_0 - 3)(0.9)h_0(0.9h_0^2)} = 0.950
\]

\[
G = 0.950 G_0 \Rightarrow k = 0.950
\]

PROBLEM 4.35

4.35 For the bar and loading of Example 4.01, determine (a) the radius of curvature \( \rho \), (b) the radius of curvature \( \rho' \) of a transverse cross section, (c) the angle between the sides of the bar which were originally vertical. Use \( E = 29 \times 10^6 \) psi and \( \nu = 0.29 \).

**SOLUTION**

From Example 4.01

\[ M = 30 \text{ kip \cdot in} \quad I = 1.042 \text{ in}^4 \]

(a) \( \rho = \frac{M}{EI} = \frac{(30 \times 10^3)}{(29 \times 10^6)(1.042)} = 993 \times 10^{-6} \text{ in}^{-1} \quad \rho = 1007 \text{ in}^{-1} \)

(b) \( \varepsilon' = \nu \varepsilon = \frac{\nu E}{\rho} = \nu \frac{E}{\rho'} \)

\[ \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(993 \times 10^{-6}) \text{ in}^{-1} = 288 \text{ in}^{-1} \quad \rho' = 3470 \text{ in}^{-1} \]

(c) \( \theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230 \times 10^{-6} \text{ rad} = 0.01320^\circ \)