PROBLEM 4.43

Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity</th>
<th>Allowable Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>2 x 10^6 psi</td>
<td>2000 psi</td>
</tr>
<tr>
<td>Steel</td>
<td>3 x 10^6 psi</td>
<td>22 ksi</td>
</tr>
</tbody>
</table>

...SOLUTION...

Use wood as the reference material.

\[ n = 1.0 \text{ in wood} \]

\[ n = \frac{E_s}{E_w} = 30/2 = 15 \text{ in steel} \]

For the transformed section:

\[ I_1 = \frac{n_1 b_1 h_1^3}{12} = \frac{10}{12} (3)(10)^3 = 250 \text{ in}^4 \]

\[ I_2 = \frac{n_2 b_2 h_2^3}{12} = \frac{15}{12} (\frac{1}{2})(10)^3 = 625 \text{ in}^4 \]

\[ I_3 = I_1 = 250 \text{ in}^4 \]

\[ I = I_1 + I_2 + I_3 = 1125 \text{ in}^4 \]

\[ M = \frac{6 I}{n y} \]

Wood:

\[ n = 1.0, \quad y = 5 \text{ in}, \quad E = 2000 \text{ psi} \]

\[ M = \frac{(2000)(1125)}{(1.0)(5)} = 450 \times 10^3 \text{ lb} \cdot \text{in} \]

Steel:

\[ n = 15, \quad y = 5 \text{ in}, \quad E = 22 \text{ ksi} = 22 \times 10^3 \text{ psi} \]

\[ M = \frac{(22 \times 10^3)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb} \cdot \text{in} \]

Choose the smaller value:

\[ M = 330 \times 10^3 \text{ lb} \cdot \text{in} = 330 \text{ kip} \cdot \text{in} \]
PROBLEM 4.44

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

<table>
<thead>
<tr>
<th>Modulus of elasticity:</th>
<th>Wood</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 × 10^6 psi</td>
<td>30 × 10^6 psi</td>
</tr>
<tr>
<td>Allowable stress:</td>
<td>2000 psi</td>
<td>22 ksi</td>
</tr>
</tbody>
</table>

SOLUTION

Use wood as the reference material

\[ n = 1.0 \text{ in wood} \]

\[ n = E_s/E_w = 30/2 = 15 \text{ in steel} \]

For the transformed section

\[ I_1 = \frac{n_1}{12} - b_1 h_1^3 + n_1 A_1 d_1^2 \]

\[ = \frac{15}{12} - (5)(\frac{1}{2})^3 + (15)(5)(\frac{.25}{2})(5.25)^2 = 1034.4 \text{ in}^4 \]

\[ I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{10}{12} (6)(10)^3 = 500 \text{ in}^4 \]

\[ I_3 = I_1 = 1034.4 \text{ in}^4 \]

\[ I = I_1 + I_2 + I_3 = 2569 \text{ in}^4 \]

\[ |\sigma| = \frac{|n M Y|}{I} \Rightarrow M = \frac{\sigma I}{ny} \]

Wood: \[ n = 1.0, \ y = .5 \text{ in}, \ \sigma = 2000 \text{ psi} \]

\[ M = \frac{(2000)(2569)}{(1.0)(5)} = 1.028 \times 10^6 \text{ lb-in} \]

Steel: \[ n = 15, \ y = 5.5 \text{ in}, \ \sigma = 22 \text{ ksi} = 22 \times 10^3 \text{ psi} \]

\[ M = \frac{(22 \times 10^3)(2569)}{(15)(5.5)} = 685 \times 10^3 \text{ lb-in} \]

Choose the smaller value \[ M = 685 \times 10^3 \text{ lb-in} = 685 \text{ kip-in} \]
PROBLEM 4.58

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is $3 \times 10^6$ psi for the concrete and $30 \times 10^6$ psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

\[ n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10 \]

\[ A_s = 3 \left( \frac{\pi}{4} d^2 \right) = 3 \left( \frac{\pi}{4} \left( \frac{3}{4} \right)^2 \right) = 1.8040 \text{ in}^2 \]

\[ nA_s = 18.040 \text{ in}^2 \]

Locate neutral axis.

\[ 8x^2 - (18.040)(14-x) = 0 \]

\[ 4x^2 + 18.040x - 252.56 = 0 \]

Solve for \( x \)

\[ x = \frac{-18.040 \pm \sqrt{18.040^2 + 4(4)(252.56)}}{8} \]

\[ x = 6.005 \text{ in} \]

\[ 14 - x = 7.995 \text{ in} \]

\[ I = \frac{1}{12} 8 x^3 + nA_s(14-x)^2 = \frac{1}{12}(8)(6.005)^3 + (18.040)(7.995)^2 \]

\[ = 1730.4 \text{ in}^4 \]

\[ |\sigma| = \left| \frac{nMy}{I} \right| \quad : \quad M = \frac{6I}{ny} \]

Concrete: \( n = 1.0 \), \( |\gamma| = 6.005 \text{ in}, \quad |\sigma| = 1350 \text{ psi} \)

\[ M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 3.89 \times 10^3 \text{ lb-in} = 339 \text{ kip-ft} \]

Steel: \( n = 10 \), \( |\gamma| = 7.995 \), \( \sigma = 20 \times 10^3 \text{ psi} \)

\[ M = \frac{(20 \times 10^3)(1730.5)}{(10)(7.995)} = 433 \times 10^3 \text{ lb-in} = 433 \text{ kip-ft} \]

Choose the smaller value. \( M = 339 \text{ kip-ft} = 32.4 \text{ kip-ft} \). 

\[ \boxed{\text{Choose the smaller value}} \]
**PROBLEM 4.60**

The design of a reinforced concrete beam is said to be balanced if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses \( \sigma_s \) and \( \sigma_c \). Show that to achieve a balanced design the distance \( x \) from the top of the beam to the neutral axis must be

\[
x = \frac{d}{\frac{\sigma_s E_c}{\sigma_c E_s}}
\]

where \( E_s \) and \( E_c \) are the moduli of elasticity of concrete and steel, respectively, and \( d \) is the distance from the top of the beam to the reinforcing steel.

**SOLUTION**

\[
\begin{align*}
\frac{E_s}{E_c} &= \frac{\frac{n (d-x)}{x}}{\frac{M x}{I}} \\
\frac{E_s}{E_c} &= \frac{n (d-x)}{x} = n \frac{d}{x} - n
\end{align*}
\]

\[
\frac{d}{x} = 1 + \frac{1}{n} \frac{E_s}{E_c} = 1 + \frac{E_c}{E_s} \frac{E_s}{E_c}
\]

\[
x = \frac{d}{1 + \frac{E_c}{E_s} \frac{E_s}{E_c}}
\]

**PROBLEM 4.117**

Knowing that the magnitude of the horizontal force \( P \) is 8-kN, determine the stress at (a) point \( A \), (b) point \( B \).

**SOLUTION**

\[
\begin{align*}
A &= (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2 \\
\epsilon &= 4.5 - 12 = 33 \text{ mm} = 0.033 \text{ m} \\
I &= \frac{1}{12} bh^3 = \frac{1}{12} (30)(24)^3 = 34.56 \times 10^3 \text{ mm}^4 = 3.456 \times 10^{-6} \text{ m}^4 \\
C &= 24 \text{ mm} = 0.12 \text{ m} \\
P &= 8 \times 10^3 \text{ N} \\
M &= Pe = (8 \times 10^3)(0.033) = 264 \text{ N} \cdot \text{m} \\
\text{At } A \quad \sigma_A &= -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-6}} \\
&= -102.8 \times 10^6 \text{ Pa} = -102.8 \text{ MPa} \\
\text{At } B \quad \sigma_B &= -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-6}} = 80.6 \times 10^6 \text{ Pa} = 80.6 \text{ MPa}
\end{align*}
\]
PROBLEM 4.122

4.122 An offset $h$ must be introduced into a solid circular rod of diameter $d$. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

SOLUTION

For centric loading $\sigma_e = \frac{P}{A}$

For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Ph_c}{I}$

Given $\sigma_e = 4\sigma_c$

$$\frac{P}{A} + \frac{Ph_c}{I} = 4\frac{P}{A}$$

$$\frac{Ph_c}{I} = 3\frac{P}{A} \quad \therefore \quad h = \frac{3I}{cA} = \frac{(3\frac{1}{12}d^4)(d^4)}{(\frac{1}{12})(\frac{2}{3}d^3)} = \frac{8}{3}d = 0.875d$$
PROBLEM 4.129

Three steel plates, each of 1 × 6-in. cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force \( P \), (a) which could be applied to the original column, (b) which can be applied to the modified column.

**SOLUTION**

(a) **Centric loading**

\[ A = (3)(1)(6) = 18 \text{ in}^2 \]

\[ \sigma = \frac{P}{A} \]

\[ P = \sigma A = (15)(18) = 270 \text{ kips} \]

(b) **Eccentric loading**

**Reduced cross section**

<table>
<thead>
<tr>
<th>( A_i \text{ in}^2 )</th>
<th>( \bar{y}_{i, \text{ in}} )</th>
<th>( A_i \bar{y}_{i, \text{ in}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.5</td>
<td>21.0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-3.5</td>
<td>-14.0</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>7.0</td>
</tr>
</tbody>
</table>

The centroid lies 0.4375 in from the midpoint of the web.

\[ I_1 = \frac{1}{12} (6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4 \]

\[ I_2 = \frac{1}{12} (4)(1)^3 + (4)(0.4375)^2 = 19.148 \text{ in}^4 \]

\[ I_3 = \frac{1}{12} (4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4 \]

\[ I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4 \]

\[ C = 4.4375 \text{ in} \]

\[ M = P e \quad \text{where} \quad e = 0.4375 \text{ in} \]

\[ G = -\frac{P}{A} - \frac{M c}{I} = -\frac{P}{A} + \frac{P e c}{I} = -K P \]

\[ K = \frac{1}{A} + \frac{GC}{I} = \frac{1}{16} + \frac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2} \]

\[ P = -\frac{G}{K} = -\frac{15}{0.076541} = 196.0 \text{ kips} \]
A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (a) the largest force $P$ that can be applied to the element, (b) the corresponding location of the neutral axis. Given: Centroid of the cross section is at $C$ and $I_c = 4195$ mm$^4$.

**Problem 4.130**

**Solution**

(a) $A = (3)(18) + \frac{\pi}{4}(3)^2 = 82.27$ mm$^2 = 82.27 \times 10^{-6}$ m$^2$

$I = 4195$ mm$^4 = 4.195 \times 10^{-12}$ m$^4$

$e = 13.12$ mm = 0.01312 m

Based on tensile stress at $y = -13.12$ mm $= -0.01312$ m

$$\sigma = \frac{P}{A} + \frac{PeC}{I} = \left(\frac{1}{A} + \frac{Ce}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{Ce}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4.195 \times 10^{-12}} = 53.188 \times 10^5 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^5} = 2.538 \times 10^3 \text{ N} = 2.54 \text{ kN}$$

(b) Location neutral axis.

$$\sigma = 0$$

$$\sigma = \frac{P}{A} - \frac{Mv}{I} = \frac{P}{A} - \frac{PeY}{I} = 0 \quad \frac{eY}{I} = \frac{1}{A}$$

$$Y = \frac{I}{Ae} = \frac{4.195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-8} \text{ m} = 3.89 \text{ mm}$$

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.
PROBLEM 4.148

4.148 through 4.150 The couple $M$ is applied to a beam of the cross section shown in a plane forming an angle $\beta$ with the vertical. Determine the stress at (a) point $A$, (b) point $B$, (c) point $D$.

**SOLUTION**

Locate centroid

<table>
<thead>
<tr>
<th></th>
<th>$A_i$ in$^2$</th>
<th>$Z_i$ in</th>
<th>$A_iZ_i$ in$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>-1</td>
<td>-16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>24</td>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

The centroid lies at point C

\[
I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4
\]

\[
I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(4)(4)^3 = 64 \text{ in}^4
\]

\[
y_A = -y_B = 1 \text{ in} \\
z_A = z_B = -4 \text{ in}
\]

\[
M_x = 10 \cos 20^\circ = 9.3969 \text{ kip-in}
\]

\[
M_y = 10 \sin 20^\circ = 3.4202 \text{ kip-in}
\]

(a) $\sigma_A = \frac{M_x y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{(9.3969)(1)}{88} + \frac{(3.4202)(4)}{64} = 0.321 \text{ksi}$

(b) $\sigma_B = \frac{M_x y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{(9.3969)(-1)}{88} + \frac{(3.4202)(4)}{64} = -0.107 \text{ksi}$

(c) $\sigma_D = \frac{M_x y_D}{I_z} + \frac{M_y z_D}{I_y} = \frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64} = 0.427 \text{ksi}$
PROBLEM 4.152

The couple \( M \) acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.

SOLUTION

For S 150 \( \times \) 18.6 rolled steel shape

\[ I_x = 9.11 \times 10^5 \text{ mm}^4 = 9.11 \times 10^{-5} \text{ m}^4 \]
\[ I_y = 0.782 \times 10^6 \text{ mm}^4 = 0.782 \times 10^{-6} \text{ m}^4 \]
\[ z_E = -z_A = -z_B = z_D = \left( \frac{t}{2} \right) (85) = 42.5 \text{ mm} \]
\[ y_A = y_B = -y_D = -y_E = \left( \frac{t}{2} \right) (52) = 76 \text{ mm} \]
\[ M_z = (1.5 \times 10^3) \sin 20^\circ = 0.51303 \times 10^3 \text{ N} \cdot \text{m} \]
\[ M_y = (1.5 \times 10^3) \cos 20^\circ = 1.4095 \times 10^3 \text{ N} \cdot \text{m} \]

(a) \[ \tan \varphi = \frac{I_x}{I_y} \tan \theta = \frac{9.11 \times 10^{-6}}{0.782 \times 10^{-6}} \tan (90^\circ - 20^\circ) = 32.007 \]
\[ \varphi = 88.21^\circ \]
\[ \alpha = 88.21^\circ - 70^\circ = 18.21^\circ \]

(b) Maximum tensile stress occurs at point D

\[ \sigma_D = \frac{-M_y y_D}{I_x} + \frac{M_z z_D}{I_y} = \frac{-0.51303 \times 10^3 \cdot 76 \cdot 10^{-3}}{9.11 \times 10^{-5}} + \frac{(1.4095 \times 10^3)(42.5 \times 10^{-3})}{0.782 \times 10^{-6}} \]
\[ = 80.9 \times 10^6 \text{ Pa} = 80.9 \text{ MPa} \]
PROBLEM 4.164

An axial load $P$ of magnitude $50 \text{kN}$ is applied as shown to a short section of a W $150 \times 24$ rolled steel member. Determine the largest distance $a$ for which the maximum compressive stress does not exceed $90 \text{ MPa}$.

SOLUTION

Add $y$- and $z$- axes.

For W $150 \times 24$ rolled steel section

$A = 3060 \text{ mm}^2 = 3.06 \times 10^{-6} \text{ m}^2$

$I_x = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$

$d = 160 \text{ mm}, \quad b_f = 102 \text{ mm}$

$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}.$

$P = 50 \times 10^3 \text{ N}$

$M_z = - (50\times10^3)(75\times10^{-3}) = -3.75 \times 10^3 \text{ N\cdot m}$

$M_y = - Pa$

$G_A = - \frac{P}{A} - \frac{M_y y_A}{I_x} + \frac{M_z z_A}{I_y}$

$G_A = -90 \times 10^6 \text{ Pa}$

$M_y = \frac{I_y}{z_A} \left\{ \frac{M_z y_A}{I_x} + \frac{P}{A} + G_A \right\}$

$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ \frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6) \right\}$

$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ -22.388 + 16340 - 90 \right\} \times 10^6$

$= -1.839 \times 10^3 \text{ N\cdot m}$

$\alpha = \frac{-M_y}{P} = - \frac{1.839 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-5} \text{ m} = 36.8 \text{ mm}$
4.165 An axial load $P$ of magnitude 30 kN is applied as shown to a short section of a
C 150 x 12.2 rolled-steel channel. Determine the largest distance $a$ for which the
maximum compressive stress is 60 MPa.

SOLUTION

Add $y$- and $z$-axes as shown.

For C 150 x 12.2 rolled steel section

\[
A = 1540 \text{ mm}^2 = 1540 \times 10^{-6} \text{ m}^2
\]

\[
d = 152 \text{ mm}
\]

\[
b_p = 48 \text{ mm}
\]

\[
t_w = 5.1 \text{ mm}
\]

\[
I_x = 5.35 \times 10^6 \text{ mm}^4 = 5.35 \times 10^{-6} \text{ m}^4
\]

\[
I_y = 0.276 \times 10^6 \text{ mm}^4 = 0.276 \times 10^{-6} \text{ m}^4
\]

\[
x = 12.7 \text{ mm}
\]

Line of action of force $P$

\[
y_p = -a
\]

\[
z_p = \frac{x}{2} t_w = 10.15 \text{ mm}
\]

\[
P = 30 \times 10^8 \text{ N}
\]

\[
M_y = -Pz_p = -(30 \times 10^8)(10.15 \times 10^3) = -304.5 \text{ N.m}
\]

\[
M_z = -Pa
\]

\[
\sigma_x = -60 \times 10^6 \text{ Pa}
\]

\[
y_a = -\frac{d}{2} = -76 \text{ mm}
\]

\[
z_a = x = 12.7 \text{ mm}
\]

\[
\sigma_x = -\frac{P}{A} - \frac{M_z y_a}{I_z} + \frac{M_y z_a}{I_y}
\]

\[
M_z = \frac{I_x}{y_a} \left\{ \frac{M_y z_a}{I_y} + \frac{P}{A} - \sigma_x \right\}
\]

\[
= \frac{5.35 \times 10^6}{-76 \times 10^{-6}} \left\{ \frac{-304.5 (12.7 \times 10^3)}{0.276 \times 10^6} + \frac{30 \times 10^8}{1540 \times 10^{-6}} + 60 \times 10^6 \right\}
\]

\[
= \frac{5.35 \times 10^6}{-76 \times 10^{-6}} \left\{ -14.011 - 19.481 + 60 \times 10^6 = -1.866 \times 10^3 \text{ N.m} \right\}
\]

\[
\alpha = -\frac{M_z}{P} = -\frac{(-1.866 \times 10^3)}{30 \times 10^8} = 62.2 \times 10^{-3} \text{ m} = 62.2 \text{ mm}
\]