CVEN303 ENGINEERING MEASUREMENT

Lectures 3&4– Statistical Analysis of Random Errors in Engineering Measurements (Sections 1-15 & 1-16 & Appendix A)

2013

Developed by Nasir G. Gharaibeh, Ph.D., P.E.
Zachry Department of Civil Engineering
Texas A&M University

1

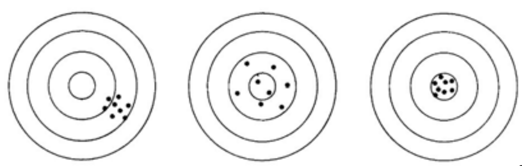
Random Errors vs. Systematic Errors

- Systematic (also called cumulative): the magnitude and arithmetic sign (direction) of error are fixed.
- Random (also called accidental): the magnitude and arithmetic sign (direction) of error fluctuate randomly.

2

Goodness of Measurements - Accuracy & Precision

- Accuracy: refers to the agreement between a measurement and the true value.
- Precision: refers to the closeness of one measurement to another. It has nothing to do with the true value.
- Describe the accuracy and precision of the shooting practice below.
- Let's represent these cases as normal distributions….



3

Measuring Accuracy & Precision

- Accuracy: The accuracy of an instrument is tested in the lab/field or provided by the manufacturer.
- Precision:
  - Residual = Measurement – Mean

$$\nu = (x - \mu_x)$$

  - Residuals (ν) are then turned to a statistic called "standard deviation".

Note: In some surveying literature, the terms "standard deviation" and "standard error" are used interchangeably.

4

Standard Deviation

- Standard deviation (σ) for a large sample size (say n ≥ 30 values):

$$\sigma_x = \sqrt{\frac{\sum (X - \mu_x)^2}{n}}$$

Where *n* is the sample size (i.e., number of measurements) and μ is the mean.

- Standard deviation (s) for a small sample size (say n < 30 values):

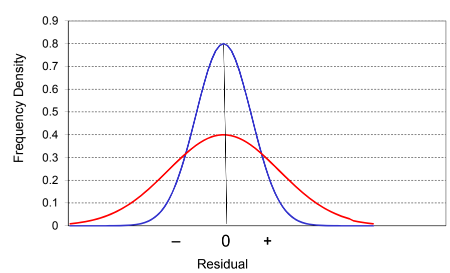
$$s_x = \sqrt{\frac{\sum (X - \bar{x})^2}{n-1}}$$

Where *n* is the sample size (i.e., number of measurements) and $\bar{x}$ is the sample mean.

5

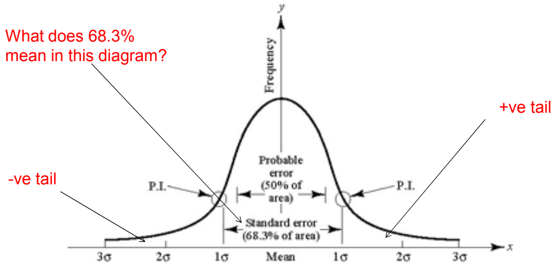
Probability Curve of Residuals

Which set of measurements is more precise (the blue or the red one)? Why?



6

Normal Probability Distribution of Measurements



Probability of Error

Error range	Associated Probability, %
$\pm 0.50\sigma$	38.3
$\pm 0.6745\sigma$	**50 (Probable Error)**
$\pm 1.00\sigma$	**68.3 (Standard Deviation)**
$\pm 1.6449\sigma$	90
$\pm 1.9599\sigma$	**95 (Maximum Anticipated Error)**
$\pm 2.00\sigma$	95.4
$\pm 3.00\sigma$	99.7
$\pm 3.29\sigma$	99.9

Note: In some surveying literature, the terms "standard deviation" and "standard error" are used interchangeably.

Example

- Determine the standard error, probable error, and the maximum anticipated error for the following distance measurements:
- To be solved in class

Measurement, ft
152.93
153.01
152.87
152.98
152.78
152.89

Answers: SE=0.0827 ft, $E_{50} = \pm 0.056$ ft, and $E_{95} = \pm 0.162$ ft

Standard Error of the Mean (SE_m)

- Each value is an average of *n* measurements.
- The standard deviation of the mean values (not the individual measurements) is called the Standard Error of the Mean (SE_m):

$$SE_m = SE_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

Standard Error of the Mean-Example

Suppose that the measurements in the previous example were to be averaged. What's the SE_m?

To be solved in class

$$SE_m = SE_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

Answer: 0.0338 ft

Cumulative Random Error in Sum of Multiple Measurements

- Random errors tend to accumulate in proportion to the square root of the number of measurements
- For *m* independent sub-measurements, the error in the total measurement is:

$$E_{total} = \pm\sqrt{E_1^2 + E_2^2 + E_3^2 + ... + E_m^2}$$

Cumulative Random Error in Sum of Multiple Measurements – Example

Four sides of a land tract were measured. The probable errors in these measurements are:

Side 1: 0.09 ft   Side 2: 0.013 ft

Side 3: 0.18 ft   Side 4: 0.15 ft.

What is the cumulative probable error for the perimeter of the tract?

To be solved in class.

Answer: ±0.25 ft

13

Cumulative Random Error in Sum of Multiple Measurements with Equal Error

- If a quantity is measured as the sum of multiple sub-measurements (m) that have equal error, the total random error in that quantity is computed as:

$$E_{total} = \pm E_{sub}\sqrt{m}$$

14

Cumulative Random Error in Sum of Multiple Measurements with Equal Error - Example 1

- If a distance is measured in 9 sub-measurements and the estimated standard error in each sub-measurement is 0.02 ft, what is the estimated total standard error in the cumulative measurement?

To be solved in class.

$$E_{total} = \pm E_{sub}\sqrt{m}$$

Answer: ±0.06 ft

15

Cumulative Random Error in Sum of Multiple Measurements with Equal Error - Example 2

- It is desired to tape a distance of 2000 ft with a total error of no more than ±0.10ft. How much error can we accept in each 100-ft measurement so that the desired total error is not exceeded?

To be solved in class.

Answer: ±0.022 ft

16

Cumulative Random Error in the Product of Two Measurements

For M_1 and M_2 measurements that have corresponding E_1 and E_2 errors, the error in the multiplication of these measurements ($E_{product}$) is computed as follows:

$$E_{product} = \pm\sqrt{M_2^2 E_1^2 + M_1^2 E_2^2}$$

Example: Consider a rectangular field having side A=250 ft ± 0.04ft and side B = 100 ft ± 0.02 ft. Find the area of the field and the SE of the area.

17

Answer: Area = 25,000ft^2 and $SE_{product}$= ±6.40ft^2

Cumulative Random Error in the Product of Three Measurements

For M_1, M_2, M_3 measurements and corresponding E_1, E_2, E_3 errors in each measurement, the error in the multiplication of these measurements ($E_{product}$) is computed as follows:

$$E_{product} = \pm\sqrt{M_2^2 M_3^2 E_1^2 + M_1^2 M_3^2 E_2^2 + M_1^2 M_2^2 E_3^2}$$

18