October 9, 2001

- You can use only one page (8.5”*11”) of notes.
- If you make any assumption, write it down and explain why you made it.
- Answers should have units and be enclosed in a box.
- Assume that the kinematic viscosity of water is $10^{-6}$ m$^2$/s.
- Assume that the specific weight of water is 9810 N/m$^3$.

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Flow: $Q = 0.075$ m$^3$/s
Pipe diameter: $D = 0.300$ m
Pipe roughness: $\varepsilon = 0.25$ mm (UNITS!!)
First pipe length: $L_1 = 7.45$ m (from the entrance at the reservoir to the pump)
Second pipe length: $L_2 = 30.00$ m (from the pump to the outlet)
Pump speed: $N = 1500$ RPM
1. Calculate the velocity (V) in the pipes

\[ V = \frac{0.075 \text{ m}^3/\text{s}}{\frac{\pi}{4} (0.30 \text{ m})^2} = 1.06 \frac{\text{m}}{\text{s}} \]

2. Calculate the pipe friction factor (f)

\[ Re = \frac{1.06 \text{ m/s} \times 0.30 \text{ m}}{1 \times 10^{-6} \text{ m}^2/\text{s}} = 3.2 \times 10^5 \]

\[ \frac{\varepsilon}{D} = \frac{0.25 \times 10^{-3}}{0.30} = 0.00083 \]

**Moody diagram**: \( f = 0.020 \)

3. Calculate the head at A (H_A)

\[ H_A = 105.13 \text{ m} + \frac{0}{g} + \frac{0^2}{2g} = 105.13 \text{ m} \]

4. Calculate the head at B (H_B)

\[ H_B = 112.58 \text{ m} + \frac{0}{g} + \frac{1.06^2}{2 \times 9.81} = 112.64 \text{ m} \]
5. Calculate the friction head losses in pipe 1 \((H_{L1})\)

\[
H_{L1} = 0.020 \times \frac{7.45 \text{ m}}{0.30 \text{ m}} \times \frac{(1.06 \text{ m/s})^2}{2g} = 0.03 \text{ m}
\]

6. Calculate the minor head losses in pipe 1 \((H_{M1})\)

\[
H_{M1} = (0.8 + 0.5) \left( \frac{1.06 \text{ m/s}}{2g} \right)^2 = 0.07 \text{ m}
\]

7. Calculate the friction head losses in pipe 2 \((H_{L2})\)

\[
H_{L2} = 0.020 \times \frac{30 \text{ m}}{0.30 \text{ m}} \times \frac{(1.06 \text{ m/s})^2}{2g} = 0.11 \text{ m}
\]

8. Calculate the head supplied by the pump \((E_p)\)

\[
105.13 - 0.03 - 0.07 + E_p - 0.11 = 112.64
\]

\[
E_p = 7.72 \text{ m}
\]
9. Calculate the pressure at point X (P_x) (just upstream of the pump)

\[ H_x = 105.13 \text{ m} - 0.03 \text{ m} - 0.07 \text{ m} = 105.03 \text{ m} \]

\[ 105.03 \text{ m} = 112.58 \text{ m} + \frac{P_x}{\gamma} + \frac{(1.06 \text{ m/s})^2}{2g} \]

\[ \frac{P_x}{\gamma} = -7.61 \text{ m} \]

\[ P_x = -7.61 \text{ m} \times 9810 \frac{N}{m^3} = -74,627 \frac{N}{m^2} = -75 \text{ kPa} \]

10. Calculate the pressure at point Y (P_y) (just downstream of the pump)

\[ H_y = H_x + E_p = 105.03 + 7.72 = 112.75 \text{ m} \]

\[ 112.75 = 112.58 + \frac{P_y}{\gamma} + \frac{(1.06)^2}{2g} \]

\[ \frac{P_y}{\gamma} = 0.11 \text{ m} \]

\[ P_y = 0.11 \times 9810 = 1106 \frac{N}{m^2} = 1.11 \text{ kPa} \]

11. (Bonus: 3 points) Is there a risk of cavitation in the system? If yes, where and what would you recommend to decrease it?

Yes, at point X, because the pressure is negative. However, it is far above the vapor pressure of water at "room" temperature.

To decrease the risk of cavitation, move the pump as close as possible to the reservoir, and place it below the water lever.
12. If the pump-characteristic curve is $E_p = -4 Q^2 - 3.7 Q + 8.1$ (in m and $Q$ in m³/s) what would the **actual** head supplied by the pump and flow be? Is the pump adequate? (i.e., can it supply the head and pump the flow required?) If not, should the pump speed increase or decrease?

Energy equation: (assuming $f$ does not change)

$$105.13 - 0.8 \frac{v^2}{2g} - 0.5 \frac{v^2}{2g} - 0.020 \frac{7.45}{0.30} \frac{v^2}{2g} + E_p$$

$$- 0.020 \frac{30}{0.30} \frac{v^2}{2g} = 112.58 + \frac{v^2}{2g}$$

$$\Rightarrow E_p = 4.80 \frac{v^2}{2g} + 7.45$$

But: $$\frac{v^2}{2g} = \left(\frac{Q}{\frac{\pi}{4} \times 0.30^2}\right)^2 = 10.20 Q^2$$

$$\Rightarrow E_p = 48.96 Q^2 + 7.45$$

Intersection of the system curve and pump characteristic curve:

$E_p = -4Q^2 - 3.7Q + 8.1 = 48.96Q^2 + 7.45$

$Q = 0.081 m^3/s \quad E_p = 7.77 m \quad **actual** values$

Yes, the pump is adequate.