Pressure-Velocity Coupling Methods

Compressible Flows

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0
\]

\[
\rho \frac{D\vec{V}}{Dt} = \rho \ddot{\vec{V}} - \nabla p + \frac{1}{3} \mu \nabla (\nabla \cdot \vec{V}) + \mu \nabla^2 \vec{V}
\]

\[
\rho = \rho(p, T) \quad \text{e.g.,} \quad p = \rho RT
\]

- Five equations for five unknowns (\(\rho, u, v, w, p\))
- Solve density using the continuity equation
- Determine the pressure from the equation of state
**Pressure-Velocity Coupling**

- **Incompressible Flows**
  \[ \nabla \cdot \vec{V} = 0 \]
  \[
  \rho \frac{D\vec{V}}{Dt} = \rho \vec{f} - \nabla p + \mu \nabla^2 \vec{V}
  \]

- Four equations for four unknowns (u, v, w, p)
- Continuity equation becomes a compatibility condition (constraint) on the velocity field
- Three momentum equations for (u, v, w)
- Pressure must be determined indirectly from the continuity equation

**Artificial Compressibility**

- **Artificial sound speed** \(a\), such that \(p = a^2 \rho\)
  \[
  \frac{\partial p}{\partial t} + a^2 \nabla \cdot \vec{V} = 0
  \]

- The continuity equation is satisfied when the pressure converged
  \[
  \frac{\partial p}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{V} = 0
  \]
**Direct Poisson Solver**

- **Take divergence of Navier-Stokes equations**

\[
D = \nabla \cdot \vec{V} = 0
\]

\[
\nabla \cdot \left( \rho \frac{\partial \vec{V}}{\partial t} + \rho \vec{V} \cdot \nabla \vec{V} + \nabla p - \mu \nabla^2 \vec{V} \right) = 0
\]

\[
\nabla^2 p = -\rho \nabla \cdot (\vec{V} \cdot \nabla \vec{V}) - \rho \frac{\partial D}{\partial t} + \mu \nabla^2 D
\]

- Retain $\partial D/\partial t$ term to provide a restoring mechanism ($D^n \neq 0$, but requires $D^{n+1} = 0$)

- Ineffective for high-Re flows

---

**Regular Grid Arrangement**

- Both the velocities and pressure were evaluated at the primary nodes
**Uniform Cartesian Grids**

\[ \begin{align*}
  \frac{d q_p}{d t} + \frac{F_e - F_w}{\Delta x} + \frac{G_n - G_s}{\Delta y} &= 0 \\
  \frac{d q_p}{d t} (\Delta x \Delta y) + \frac{\Delta y}{2} (F_e - F_w) + \frac{\Delta x}{2} (G_n - G_s) &= 0 \\
  \frac{d q_p}{d t} + \frac{F_e - F_w}{2 \Delta x} + \frac{G_n - G_s}{2 \Delta y} &= 0
\end{align*} \]

**Pressure Gradient**

\[ \frac{\partial p}{\partial x} = \frac{p_e - p_w}{x_e - x_w} = \frac{p_E - p_W}{2(x_e - x_w)} \]

Contain pressure difference between \(j-1\) and \(j+1\) nodes, not adjacent ones
### Continuity Equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{u_c - u_w}{x_c - x_w} + \frac{v_n - v_s}{y_n - y_s} = 0
\]

Contain velocity differences between alternate nodes, not adjacent nodes

\[
\begin{align*}
u_c &= \frac{(u_p + u_E)}{2} \\
u_w &= \frac{(u_p + u_W)}{2} \\
v_n &= \frac{(v_p + v_N)}{2} \\
v_s &= \frac{(v_p + v_S)}{2}
\end{align*}
\]

### Regular (Collocated) Grid

- Both the velocities and pressure were evaluated at the primary nodes
- **Zigzag pressure field** (zero pressure gradient)
- **Wavy velocity field**, but still satisfy the continuity equation (zero mass source)
**Regular Grid Arrangement**

- Checkerboard pressure field
- If a smooth pressure field is obtained as a solution, any number of additional solutions can be constructed by adding a checkerboard pressure field to that solution
- Momentum equations unaffected by checkerboard pressure

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**Staggered Grid Arrangement**

- Pressure nodes lie at the center of the control volume
- Velocities are evaluated at the control volume face
Staggered Grids

- **Continuity**
  \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{u_{j,k} - u_{j-1,k}}{x_e - x_w} + \frac{v_{j,k} - v_{j,k-1}}{y_n - y_s} = 0 \]

  *Always contain adjacent velocity and pressure gradients*

Momentum Equations

- **x-momentum (u_{j,k})**
  \[ \frac{\partial p}{\partial x} = \frac{p_E - p_P}{x_e - x_w} = \frac{p_{j+1,k} - p_{j,k}}{x_e - x_w} \]

- **y-momentum (v_{j,k})**
  \[ \frac{\partial p}{\partial y} = \frac{p_N - p_P}{y_n - y_s} = \frac{p_{j,k+1} - p_{j,k}}{y_n - y_s} \]

- **Equation**
  \[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]

  \[ u_{j,k} = C_{j+1,k} u_{j+1,k} + C_{j,k+1} u_{j,k+1} + C_{j,k} u_{j,k+1} + C_{j+1,k} u_{j,k+1} + C_{j,k} u_{j+1,k+1} + C_{j,k+1} u_{j+1,k+1} - C_{j,k} S_{j,k} \]

  \[ S_{j,k} = \frac{p_{j+1,k} - p_{j,k}}{x_e - x_w} + \frac{u_{j,k} - u_{j+1,k}}{\Delta t} \]

Chapter 9C Pressure-Velocity Coupling
Staggered Grid Arrangement

- Control volume for x-momentum equation
- Different control volumes for u, v, w and p

Staggered Grid Arrangement

- Control volume for y-momentum equation
- Different control volumes for u, v, w and p
Momentum Equations

Staggered grid arrangement

\[ u_{j,k} = C_{j-1,k} u_{j-1,k} + C_{j,k-1} u_{j,k-1} + C_{j,k+1} u_{j,k+1} + C_{j+1,k} u_{j+1,k} + C_{j+1,k-1} u_{j+1,k-1} + C_{j+1,k+1} u_{j+1,k+1} + C_{j,k} S_{j,k} \]

\[ u_e = \sum C_{ab} u_{ab} - C_t \left( \frac{p_E - p_P}{x_E - x_P} + \frac{u_e - u_e^*}{\Delta t} \right) \]

\[ u_e = \frac{1}{1 + C_t / \Delta t} \left\{ \sum C_{ab} u_{ab} - C_t \left( \frac{p_E - p_P}{x_E - x_P} \right) \right\} \]

\[ \Rightarrow u_e = \hat{u}_e - d_e (p_E - p_P) \]

Decompose the momentum equations into pseudovelocities and pressure gradients

Discretized momentum equations

\[ \begin{aligned}
  u_e &= \hat{u}_e - d_e (p_E - p_P) \\
  v_n &= \hat{v}_n - d_n (p_N - p_P) \\
  w_t &= \hat{w}_t - d_t (p_T - p_P)
\end{aligned} \]
Continuity Equations
Staggered grid arrangement

\[ D = \frac{u_p - u_w + v_p - v_s + w_p - w_b}{\Delta x_p} + \frac{\hat{u}_p - \hat{u}_w + \hat{v}_p - \hat{v}_s + \hat{w}_p - \hat{w}_b}{\Delta y_p} + \frac{\hat{v}_p - \hat{v}_w + \hat{w}_p - \hat{w}_s}{\Delta z_p} = 0 \]

\[ \begin{align*}
& D = \frac{u_p - u_w + v_p - v_s + w_p - w_b}{\Delta x_p} + \frac{\hat{u}_p - \hat{u}_w + \hat{v}_p - \hat{v}_s + \hat{w}_p - \hat{w}_b}{\Delta y_p} + \frac{\hat{v}_p - \hat{v}_w + \hat{w}_p - \hat{w}_s}{\Delta z_p} \\
& - \frac{d}{\Delta x_p} (p_N - p_p) + \frac{d}{\Delta y_p} (p_p - p_s) - \frac{d}{\Delta z_p} (p_p - p_b) = \hat{D} \\
\Rightarrow & \quad D = \hat{D} + a_p p_w - a_p p_e + a_p p_N - a_p p_S - a_p p_b - a_p p_b = 0
\end{align*} \]

Pressure equation (Poisson equation)

\[ D = \nabla \cdot \hat{V} = 0 \quad \Rightarrow \quad \nabla \cdot \nabla p = \nabla^2 p = -\rho \nabla \cdot (\hat{V} \cdot \nabla \hat{V}) \]

\[ a_p = \frac{d}{\Delta x_p}, \quad a_w = \frac{d}{\Delta y_p}, \quad a_s = \frac{d}{\Delta z_p}, \quad a_b = \frac{d}{\Delta x_p}, \quad a_b = \frac{d}{\Delta y_p}, \quad a_b = \frac{d}{\Delta z_p} \]

\[ a_p = a_w + a_s + a_b + a_b + a_b + a_b = \sum a_{ab} \]
**SIMPLE Algorithm**

- **Semi-Implicit Method for Pressure-Linked Equation**
- The initial guesses \((u^*, v^*, w^*, p^*)\), in general, do not satisfy the continuity equation
- Correct the pressure field to satisfy the continuity equation

\[
\begin{align*}
\nu_n &= \tilde{\nu}_n - d_n(p_N - p_p) \\
\nu_n &= \tilde{\nu}_n - d_n(p_N - p_p) \\
\nu_n &= \tilde{\nu}_n - d_n(p_N - p_p)
\end{align*}
\]

\[
\begin{align*}
\tilde{u}_c &= \tilde{\tilde{u}}_c - d_c(p_E - p_p) \\
\tilde{v}_n &= \tilde{\tilde{v}}_n - d_n(p_N - p_p) \\
\tilde{w}_i &= \tilde{\tilde{w}}_i - d_i(p_i - p_p)
\end{align*}
\]

\[
\begin{align*}
\nu_n &= \tilde{\nu}_n - d_n(p_N - p_p) \\
\nu_n &= \tilde{\tilde{v}}_n - d_n(p_N - p_p) \\
\nu_n &= \tilde{\tilde{w}}_i - d_i(p_i - p_p)
\end{align*}
\]

- The velocity-correction is implicit in \((u', v', w')\), i.e., \(u'_e\) depends on \(u'_{nb}\) etc.
- Needs to be solved simultaneously (or iteratively)
- SIMPLE (Semi-implicit!) – Omit the velocity-corrections \(u'_{nb}\), \(v'_{nb}\), and \(w'_{nb}\) on the right-hand-side
- When the solution converges: \(u = v = w = 0 \Rightarrow p' = 0\)
- SIMPLE algorithm does not affect the final converged solution, but produced over-correction during the iterative process
- Underrelaxation of pressure-correction equation needed

**Velocity-Correction Equations**

\[
\begin{align*}
\tilde{u}_c &= \tilde{\tilde{u}}_c - d_c(p_E - p_p) \\
\tilde{v}_n &= \tilde{\tilde{v}}_n - d_n(p_N - p_p) \\
\tilde{w}_i &= \tilde{\tilde{w}}_i - d_i(p_i - p_p)
\end{align*}
\]
Pressure-Correction Equation

- Omit $\sum b_{ab}u'_{ab}$, $\sum b_{ab}v'_{ab}$, $\sum b_{ab}w'_{ab}$, or equivalently $\hat{u}'_c = (\hat{u}'_c - \hat{u}'_c^*) - d_s(p'_E - p'_r)$, $\hat{v}'_n = (\hat{v}'_n - \hat{v}'_n^*) - d_s(p'_N - p'_r)$, $\hat{w}'_t = (\hat{w}'_t - \hat{w}'_t^*) - d_s(p'_T - p'_r)$

- Explicit velocity-correction equation

  - But the pressure-correction equation is still implicit

  
  $$
  \begin{align*}
  a_p p'_p &= a_w P'_w + a_s P'_s + a_t p'_t + a_b p'_b - D^* \\
  D^* &= \frac{u'_c - u'_w}{\Delta x_p} + \frac{v'_n - v'_s}{\Delta y_p} + \frac{w'_t - w'_b}{\Delta z_p}
  \end{align*}
  $$

- SIMPLE: Semi-Implicit, Pressure-Linked

  - $D^* = 0$ when the solution is fully converged with $p' = u' = v' = w' = 0$

SIMPLEC (Consistent SIMPLE)


  - Retain $\sum b_{ab}u'_{ab}$, $\sum b_{ab}v'_{ab}$, $\sum b_{ab}w'_{ab}$

  - But assume $u'_{ab} = u'_c$, $v'_{nb} = v'_n$, $w'_{nb} = w'_t$

  - Omit $(u'_{ab} - u'_c)$, $(v'_{nb} - v'_n)$, $(w'_{nb} - w'_t)$

  - Consistent correction (SIMPLE: over-correction)

  $$
  \begin{align*}
  \hat{u}'_c - \hat{u}'_c^* &= \hat{u}'_c = \sum b_{nb} u'_{nb} = (\sum b_{nb}) u'_c = \alpha u'_c \\
  \hat{v}'_n - \hat{v}'_n^* &= \hat{v}'_c = \sum b_{nb} v'_{nb} = (\sum b_{nb}) v'_n = \alpha v'_n \\
  \hat{w}'_t - \hat{w}'_t^* &= \hat{w}'_t = \sum b_{nb} w'_{nb} = (\sum b_{nb}) w'_t = \alpha w'_t \\
  \alpha &= \sum b_{nb}
  \end{align*}
  $$
**SIMPLEC**

- **Consistent SIMPLE**
  
  \[
  \begin{align*}
  u'_n &= (\hat{u}'_n - \hat{u}_n) - d_x (p'_E - p'_P) \\
  v'_n &= (\hat{v}'_n - \hat{v}_n) - d_n (p'_N - p'_P) \\
  w'_n &= (\hat{w}'_n - \hat{w}_n) - d_z (p'_Z - p'_P)
  \end{align*}
  \]

  \[
  \begin{align*}
  u'_n &= -d_x (p'_E - p'_P) / (1 - \alpha) \\
  v'_n &= -d_n (p'_N - p'_P) / (1 - \alpha) \\
  w'_n &= -d_z (p'_Z - p'_P) / (1 - \alpha)
  \end{align*}
  \]

- **Velocity-correction equations remain explicit**

- **Pressure-correction equation**

  \[
  a_p p'_p = a_e p'_e + a_n p'_n + a_n p'_n + a_e p'_e + a_s p'_s + a_s p'_s + a_h p'_h - (1 - \alpha) D^* \\
  \alpha = \sum b_{nb}
  \]

- **Consistent correction – replace \( D^* \) by \((1 - \alpha) D^*\)**

- **No underrelaxation needed**

---

**SIMPLE, SIMPLEC, SIMPLER**

- **SIMPLE** – solve the pressure-correction equation

  \[
  a_p p'_p = a_e p'_e + a_n p'_n + a_n p'_n + a_e p'_e + a_s p'_s + a_s p'_s + a_h p'_h - D^* \\
  \text{guess } p^* \rightarrow (u^*, v^*, w^*) \rightarrow D^* \rightarrow p'
  \]

  \[
  p = p^* + \alpha p'
  \]

- **SIMPLEC** – consistent SIMPLE

  Better approximation for slow-varying \((u', v', w')\)

  \[
  a_p p'_p = a_e p'_e + a_n p'_n + a_n p'_n + a_e p'_e + a_s p'_s + a_s p'_s + a_h p'_h - (1 - \alpha) D^* \\
  \text{guess } p^* \rightarrow (u^*, v^*, w^*) \rightarrow (1 - \alpha) D^* \rightarrow p'
  \]

  \[
  p = p^* + p'
  \]

- **Many iterations are needed even if the correct \( p^* \) was used initially (both SIMPLE and SIMPLEC)**

- **SIMPLER** – Revised SIMPLE

  Solve both the pressure and pressure-correction equations

  Obtain the correct pressure in one iteration with correct \((u, v, w)\)
**SIMPLER (Revised SIMPLE)**

- Solve both the pressure and pressure-correction
- Obtain exact solution with the correct initial \((u,v,w)\)

\[
a_p p_p = a_s p_e + a_a p_w + a_e p_N + a_p p_T + a_s p_R - \hat{D} \\
a_p p'_p = a_s p'_e + a_a p'_w + a_e p'_N + a_p p'_T + a_s p'_R - D^*
\]

- Original SIMPLER – no pressure-corrections (1.5 steps)
  
  \[
  \text{guess } p^* \rightarrow (u^*, v^*, w^*) \rightarrow D^* \rightarrow p' \rightarrow (u', v', w') \rightarrow \\
  (u, v, w) = (u^* + u', v^* + v', w^* + w') \rightarrow \hat{D} \rightarrow p
  \]

- Modified SIMPLER – include both pressure and velocity corrections (2 full steps)
  
  \[
  \text{guess } (u^*, v^*, w^*) \rightarrow (\hat{u}, \hat{v}, \hat{w}) \rightarrow \hat{D} \rightarrow p = p^* \rightarrow (u^*, v^*, w^*) \\
  \rightarrow D^* \rightarrow p' \rightarrow p = p^* + p' \rightarrow (u', v', w') \rightarrow (u, v, w)
  \]

---

**PISO Algorithm**

- Pressure-Implicit with Splitting of Operators

Compressible:

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) &= -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} + S_i \\
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i) &= 0 \\
\frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x_j} (\rho u_i \varepsilon) &= -\frac{\partial q_j}{\partial x_j} - \frac{\partial}{\partial x_j} (pu_j) + \frac{\partial}{\partial x_j} (u_i \sigma_{ij}) + Q \\
\rho &= p / RT
\end{align*}
\]

Incompressible:

\[
\begin{align*}
\rho \left( \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) \right) &= -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ii}}{\partial x_j} + S_i \\
\frac{\partial u_i}{\partial x_i} &= 0
\end{align*}
\]
Chapter 9C Pressure-Velocity Coupling

**PISO Algorithm**

- Discretization of Momentum and Continuity equations
- Euler Implicit (Fully-Implicit) scheme

\[
\frac{P}{\Delta t} (u_{i}^{n+1} - u_{i}^{n}) = H(u_{i}^{n+1}) - \Delta_{t}p_{i}^{n+1} + S_{i}
\]

\[
\Delta_{i}u_{i}^{n+1} = 0
\]

\[
H(u_{i}) = A_{n}u_{i,m} = A_{p}u_{i,p} - \sum A_{nk}u_{i,nk}
\]

- \(\Delta_{i}\): first derivative (FDM, FEM, FVM, etc.)
- Pressure equation (take divergence of momentum eqn)

\[
\frac{P}{\Delta t} \Delta_{i}(u_{i}^{n+1} - u_{i}^{n}) = \Delta_{i}H(u_{i}^{n+1}) + \Delta_{i}S_{i} - \Delta_{i}^{2}p_{i}^{n+1}
\]

\[
\Delta_{i}u_{i}^{n+1} = 0, \text{ but } \Delta_{i}u_{i}^{n} \neq 0 \text{ in general}
\]

\[
\Rightarrow \Delta_{i}^{2}p_{i}^{n+1} = \Delta_{i}H(u_{i}^{*}) + \Delta_{i}S_{i} + \frac{P}{\Delta t} \Delta_{i}u_{i}^{n}
\]

**PISO Algorithm**

- Operator splitting (ADI, approximate factorization, predictor-corrector, etc.)
- Introduce intermediate solutions, \(u^{*}, u^{**}, u^{***}, \ldots\)

\[
\text{Momentum eqn : } \quad \frac{P}{\Delta t} (u_{i}^{n+1} - u_{i}^{n}) = H(u_{i}^{n+1}) - \Delta_{i}p_{i}^{n+1} + S_{i}
\]

\[
\text{Pressure eqn : } \quad \Delta_{i}p_{i}^{n+1} = \Delta_{i}H(u_{i}^{n+1}) + \Delta_{i}S_{i} + \frac{P}{\Delta t} \Delta_{i}u_{i}^{n}
\]

- (a) **Predictor step** (Explicit in \(p^{n}\), implicit in \(u_{i}^{*}\))

\[
\text{Given } p^{n}, \text{ solve } \frac{P}{\Delta t} (u_{i}^{*} - u_{i}^{n}) = H(u_{i}^{*}) - \Delta_{i}p^{n} + S_{i}
\]

\text{Note: } \Delta_{i}u_{i}^{*}, \Delta_{i}u_{i}^{n} \neq 0, \text{ in general}

- \(u_{i}^{*}\) may not satisfy the continuity equation
(b) First corrector step (explicit in $u_i^{**}$, implicit in $p^*$)

\[
\begin{align*}
\Delta_i u_i^{**} &= 0 \\
\frac{\rho}{\Delta t} (u_i^{**} - u_i^{*}) &= H(u_i^*) - \Delta_i p^* + S_i
\end{align*}
\]

eliminate $u_i^{**}$

Take divergence of momentum equation

\[\Rightarrow \Delta_i^2 p^* = \Delta_i H(u_i^*) + \Delta_i S_i + \frac{\rho}{\Delta t} \Delta_i u_i^{**} \text{ (implicit in } p^* \text{)}\]

Once $p^*$ is solved

\[\Rightarrow u_i^{**} = u_i^{*} + \frac{\Delta t}{\rho} [H(u_i^*) - \Delta_i p^* + S_i] \text{ (explicit in } u_i^{**} \text{)}\]

(c) Second corrector step (explicit in $u_i^{***}$, implicit in $p^{**}$)

\[
\begin{align*}
\Delta_i u_i^{***} &= 0 \\
\frac{\rho}{\Delta t} (u_i^{**} - u_i^{*}) &= H(u_i^*) - \Delta_i p^* + S_i
\end{align*}
\]

eliminate $u_i^{***}$

\[\Rightarrow \Delta_i^2 p^{**} = \Delta_i H(u_i^{**}) + \Delta_i S_i + \frac{\rho}{\Delta t} \Delta_i u_i^{**} \text{ (implicit in } p^{**} \text{)}\]

\[\Rightarrow u_i^{***} = u_i^{*} + \frac{\Delta t}{\rho} [H(u_i^{**}) - \Delta_i p^{**} + S_i] \text{ (explicit in } u_i^{***} \text{)}\]

Third and higher corrector steps

$p^{***}$, $u_i^{****}$, $p^{****}$, $u_i^{*****}$, etc

Not necessary in practice because splitting error are $O(\Delta t^3)$ and $O(\Delta t^4)$ for first and second corrector steps

Splitting errors smaller than the discretization errors
**PISO Summary**

(a) **Predictor step** (given $p^n$)

\[
\frac{\rho}{\Delta t} (u_i^* - u_{i}^{n}) = H(u_i^*) - \Delta_i p^n + S_i \quad \text{…… implicit in } u_i^*
\]

(b) **First corrector step** (explicit in $u_i^{**}$, implicit in $p^*$)

\[
\begin{align*}
\Delta_i^* p^* &= \Delta_i H(u_i^*) + \Delta_i S_i + \frac{\rho}{\Delta t} \Delta_i u_{i}^{n} \quad \text{…… implicit in } p^* \\
\frac{\rho}{\Delta t} u_i^{**} &= \frac{\rho}{\Delta t} u_{i}^{n} + H(u_i^*) - \Delta_i p^* + S_i \quad \text{…… explicit in } u_i^{**}
\end{align*}
\]

(c) **Second corrector step** (explicit in $u_i^{***}$, implicit in $p^{**}$)

\[
\begin{align*}
\Delta_i^* p^{**} &= \Delta_i H(u_i^{**}) + \Delta_i S_i + \frac{\rho}{\Delta t} \Delta_i u_{i}^{n} \quad \text{…… implicit in } p^{**} \\
\frac{\rho}{\Delta t} u_i^{***} &= \frac{\rho}{\Delta t} u_{i}^{n} + H(u_i^{**}) - \Delta_i p^{**} + S_i \quad \text{…… explicit in } u_i^{***}
\end{align*}
\]

**Simpler Algorithm**

- **Afterbody11**
- **Afterbody12**
- **Afterbody15**
- **Modified Spheroid**
- **Lowdrag Body**

Axisymmetric submarine hulls
**SIMPLER Algorithm**

Re = 9.3 \times 10^6

Staggered-grid arrangement

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**PISO / SIMPLER Algorithm**

Appendage / Flat-Plate Junction
Hybrid PISO / SIMPLER

Re = $5 \times 10^5$

Regular-grid arrangement