1 Introduction

One important reason for imposing tolls on roadways is to help reduce traffic congestion by giving an incentive to drivers to carpool, drive at non-toll times, or to seek alternate routes. While there are correlations between traffic volumes and toll rates, many other factors also influence driver decisions, including gas prices, availability of alternate routes, road construction, family income, and many others. Hence, management and analysis of a toll structure for a roadway can require a complex analysis.

In Seoul, South Korea, tolls were imposed on two roadways between the Han River and the Central Business District (CBD) beginning in November, 1996. These were at the Namsan 1st and Namsan 3rd tunnels. Both roadways are supplied by similar suburb regions and are the main gateways to the CBD.

The toll structure provided several different incentives to drivers. Vehicles with more than two passengers were admitted toll-free. In addition, vehicles participating in a “do not drive one day in seven” program received coupons for some tolls. Other vehicles paid a nearly constant toll rate of 2000 won (approximately US $2.50).

An important measure indicating to what degree the toll price affects driver decision is the elasticity of demand, defined as the percentage change in demand resulting from a percentage change in price. Although the toll rate has remained constant since 1996, the value of the toll to users has changed due to inflation. In order to take this effect into account, a common practice is to convert historical prices to equivalent present market prices, and for the data presented here, this has been done using the consumer price index (CPI) for Seoul. Thus, all prices in the database are Korean Won in 2007 equivalent market values, and the elasticity of demand can be estimated based on these prices.

If the elasticity is assumed constant, then the demand function can be specified as

\[ V = \alpha t^\beta \]  

where \( V \) is the volume of traffic in vehicles per workday, \( t \) is the toll price in 2007 market values, and \( \alpha \) and \( \beta \) are curve-fitting parameters. The elasticity is then evaluated as \( \beta \) (Lim & Burris 2009).

The purpose of this assignment is to analyze a set of time series data for the toll usage at the Namsan 1st and Namsan 3rd tunnels and to compare the toll data with other economic factors, including the price of gas, the average household income, and the cost of public transportation.
2 Dataset

Three datasets are provided describing the toll usage in Seoul, South Korea from January, 1997, through December, 2007. The first two datasets are named Tunnel_1.txt and Tunnel_3.txt, and present data for the Namsan 1st and Namsan 3rd tunnels, respectively. The columns in these two datafiles present data for the year, the month (1 through 12), number of vehicles paying cash, number of vehicles paying with credit card, number of vehicles using coupons, number of vehicles admitted toll-free, total number of vehicles, and the toll price in 2007 market value.

A third dataset, Other_data.txt, presents several other economic measures for the same time period. The columns in this dataset are the year, month (1 through 12), number of work days per month (accounting for holidays, etc.), average income per household in 2007 market value, the price of gas in 2007 market value, the price of public transportation in 2007 market value, and the number of registered vehicles in the Seoul metropolitan area.

3 Assignment

Use the provided data to complete several different analyses of traffic usage of these tollways in Seoul, South Korea, as described in the following sections.

3.1 Montly Statistics for Each Tunnel

Tollway and traffic volume are often correlated with seasonal variations, especially as is related to holidays. In order to visualize the seasonal variation of the toll volume at each tunnel, plot the mean, maximum, minimum, and standard deviation of total number of vehicles for each month in the dataset separately for each tunnel. Figure 1 presents the desired result of this step of the analysis.

In order to accomplish this task, the data need to first be organized by month (group all January data together, etc.). Once the data are grouped by month, compute the statistics using the following equations (do not use the built-in Matlab functions). The mean of an array $x_i$ is defined by

$$\bar{x} = \frac{\sum x_i}{n} \quad (2)$$

where $n$ is the number of values in $x_i$. The sample standard deviation is given by

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} \quad (3)$$

For the figure, the mean values should be plotted versus the month number and connected by a solid line. The ± one standard deviation should be plotted using errorbars. Note that Matlab does not have a built-in ability to plot errorbars; hence, these must be drawn using a series of PLOT commands joined using the HOLD ON function. Finally, the maximum and minimum value for each month should be plotted as starred data points. Refer to the figure for added detail.
3.2 Time Series of Tunnel Usage

While the monthly statistics help identify seasonal variations, they do not uncover long-term trends. In order to understand year-to-year variations, the actual data time series must be analyzed. A second plot should summarize tollway usage for each tunnel by plotting the timeseries of total number of vehicles and of numbers of vehicles paying cash or coupons or admitted toll-free. One plot for each tunnel should be created. For each plot, use different line types for each variable and provide a legend in one of the figures.

3.3 Factors Influencing Toll Usage

Several factors influence a driver’s decision to pay a toll. For the third figure, plot the timeseries of gas price, average income per household, and cost of public transportation in a single figure. Because the household income is much greater than the other values, the datasets must be normalized in order to compare them in a single figure. To do this, divide each timeseries by the value on January 1, 1997. Hence, each line in the figure should originate from 1.0.
3.4 Elasticity of Demand

The elasticity of demand is calculated using Equation 1. Because the elasticity is not defined when the tollway first opens (change in price goes from zero to a fixed value) it takes a few years before the elasticity is well-behaved. For this reason, only consider the data from January 1, 1998 through December 31, 2007.

First, plot the total number of vehicles per workday paying tolls (cash or credit, but not coupons or free) versus 2007 market price. Plot data for both tunnels in a single figure. Use different data point types to distinguish the two datasets, and do not connect any of the points with a line.

Second, fit Equation 1 to each dataset separately and plot the fitted line in the figure. Record the exponent $\beta$: this value is the elasticity.

Lim & Burris (2009) computed the elasticity using a multi-component regression taking all of the economic and seasonal variations into account to compute the elasticity of demand of the toll price. They report values of -1.47 for the Namsan 1st tunnel and -0.89 for the Namsan 3rd tunnel. You should obtain a result in this range, but with slightly different values since only the toll price is taken into account.

3.5 Dependence on Other Factors

In order to evaluate which other factors may contribute the most to drivers’ decisions to pay tolls, a cross-correlation analysis will be performed between the toll usage and the other data. The correlation coefficient is defined in Chapra (2008) in Equation (13.21). Compute the correlation coefficient between the following sets of data for each of the tunnels separately:

- The number of vehicles admitted toll-free (having more than two passengers) and the price of gas.
- The total number of vehicles paying tolls (cash and credit) versus the price of the toll

4 Write-up

The results of your analyses will be reported in the form of a technical memorandum with supporting appendices where the details of the analysis are presented.

For the main body of the memorandum, present and discuss each of the four figures listed above. Use good technical English and present a critical analysis of the results. Your text should both state facts about the figure (e.g., “...in 2004 the price of gas increase showing a corresponding decrease in toll volume...”) and interpret the figure to draw conclusions (e.g., “...thus, gas price is an important predictor of traffic volume...”).

The memorandum should also present the numerical values of the elasticity of demand calculated for each tunnel with a corresponding, brief interpretation of their meaning.

Finally, the results of the correlation analysis should be presented, reporting the numerical values of each correlation coefficient obtained and a brief discussion of the significance of both the value and its sign (positive or negative).
The appendices should present all Matlab codes created to perform the analysis, relevant output from the Matlab Command Window, and any other analyses performed that support the main body of the memorandum.

5 Challenge

The data presented here comprise a wealth of information related to the question, “When and why do drivers choose to pay tolls.” For the challenge problem perform three additional analyses of the data and include them in a separate section of the technical memorandum. These could include additional correlation studies, multi-variable curve-fitting to obtain more accurate elasticity of demand, and any other data analysis tool designed to understand trends in the toll usage data. You may also consult other data sets (Google Earth geographic information, other online data, etc.) for your analysis.

6 References