1 Eigenvalue Methods for Shear Buildings

Figure 1 shows the general model of a shear building with \( n \) floors. The equation for the undamped dynamic evolution of the building in response to horizontal motion is given by

\[
M \ddot{x} + K x = f(t, x)
\]  

(1)

where \( x = [x_1, x_2, ..., x_n]^T \) is the vector of floor displacements, \( t \) is the time, \( M \) is the mass matrix

\[
M = \begin{bmatrix}
m_1 & 0 & \cdots & 0 \\
0 & m_2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & m_n
\end{bmatrix}
\]  

(2)

and \( K \) is the stiffness matrix

\[
K = \begin{bmatrix}
(k_1 + k_2) & -k_2 & \cdots & \cdots & 0 \\
-k_2 & (k_2 + k_3) & -k_3 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & -k_i & (k_i + k_{i+1}) & -k_{i+1} & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & -k_n & k_n
\end{bmatrix}
\]  

(3)

A certain building has effective spring constants of \( k_1 = 3 \), \( k_2 = k_3 = \ldots = k_{n-1} = 2 \), and \( k_n = 1 \) lb/in and floor masses of \( m_1 = 4 \), \( m_2 = m_3 = \ldots = m_{n-1} = 3 \), and \( m_n = 2 \) lb-s^2/in. Answer the following questions:

1. Find the two mode shapes and natural frequencies when \( n = 2 \) by hand using the matrix determinant method to solve for the eigenvalues.

2. Use the Matlab function `eig` to find the modal shapes and natural frequencies when \( n = 2 \). Why are the values of the modal shapes different than you calculated by hand?

3. Use the Matlab function `eig` to calculate the mode shapes and frequencies when \( n = 10 \). Be careful to pay attention to the order that Matlab sorts the modes. Remember that in our notation, mode 1 is the smallest eigenvalue and mode \( n \) is the largest.
Figure 1: Schematic diagram of an $n$-story shear building.
4. For the 10-story solution in the last step, plot the mode shapes as follows. On one page, create five sub-figures on row 1 and five sub-figures on row 2. Plot modes 1 through 10 with the floor number on the vertical axis. Title each mode and label your axes appropriately.

5. For the 10-story building, compare modes 1, 6, and 10. How are they similar? How are they different?

2 Boundary Value Problems

Problems 22.1, 22.3, 22.4.