How to identify partial differential equations.

**General Form:**

\[ A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D = 0 \]

**Discriminant:** \( B^2 - 4AC \)

- \( < 0 \) Elliptic
- \( = 0 \) Parabolic
- \( > 0 \) Hyperbolic

**Example:**

The Laplace equation

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]

Comparing to general form:

- \( \phi = u \)
- \( A = 1 \)
- \( B = 0 \)
- \( C = 1 \)
- \( D = 0 \)
- \( x = x \)
- \( y = y \)

\[ B^2 - 4AC = 0 - 4(1)(1) = -4 < 0 \]

\[ \therefore \text{Elliptic} \]
EX] **DIFFUSION EQUATION**

\[
\frac{\partial \bar{c}}{\partial t} + u \frac{\partial \bar{c}}{\partial x} = \bar{D} \frac{\partial^2 \bar{c}}{\partial x^2}
\]

\(\bar{D} : \text{DIFFUSION COEFFICIENT} \quad \text{(POSITIVE)}\)

**COMPARING TO GENERAL FORM:**

\(\bar{c} = u\)

\(A = \bar{D}\)

\(B = 0\)

\(C = 0\)

\(X = x\)

\(y = t\)

EX] **WAVE EQUATION**

\[
\frac{\partial^2 h}{\partial t^2} - \bar{c}^2 \frac{\partial^2 h}{\partial x^2} = 0
\]

\(\bar{c} : \text{SPEED} \quad \text{(POSITIVE)}\)

**COMPARING TO GENERAL FORM:**

\(u = 1\)

\(A = -\bar{c}^2\)

\(B = 0\)

\(C = 1\)

\(X = x\)

\(y = t\)

\(X = x\)

\(y = t\)

\(\therefore \text{HYPERBOLIC}\)