8.2 (a) \([A] = 3 \times 2\) 
\([B] = 3 \times 3\)  
\([C] = 3 \times 1\)  
\([D] = 2 \times 4\)  
\([E] = 3 \times 3\)  
\([F] = 2 \times 3\)  
\([G] = 1 \times 3\) 

(b) Square: \([B]\) and \([E]\)  
Column: \([C]\)  
Row: \([G]\) 

c) \(a_{12} = 7\)  
\(b_{23} = 7\)  
\(d_{12}\) does not exist  
\(e_{22} = 2\)  
\(f_{12} = 0\)  
\(g_{12} = 6\) 

d) 
(1) \([E] + [B] = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}\) 
(2) \([A] + [F]\) not possible 
(3) \([B] \cdot [E] = \begin{bmatrix} 3 & -2 & -1 \\ 6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}\) 
(4) \(7[B] = \begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 28 \end{bmatrix}\) 
(5) \([E] \times [B] = \begin{bmatrix} 25 & 13 & 74 \\ 36 & 25 & 75 \\ 28 & 12 & 52 \end{bmatrix}\) 
(6) \(\{C\}^T = \begin{bmatrix} 3 & 6 & 1 \end{bmatrix}\) 
(7) \([D] \times [A] = \begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 38 \end{bmatrix}\) 
(8) \(\{D\}^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}\) 
(9) \([A] \times [C]\) not possible 
(10) \([I] \times [B] = [B]\) 
(11) \([E]^T \cdot [E] = \begin{bmatrix} 66 & 19 & 53 \\ 19 & 29 & 46 \\ 53 & 46 & 109 \end{bmatrix}\) 
(12) \(\{C\}^T \cdot [C] = 46\)
8.3 The terms can be collected to give

\[
\begin{bmatrix}
0 & -7 & 5 \\
0 & 4 & 7 \\
-4 & 3 & -7
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
50 \\
-30 \\
40
\end{bmatrix}
\]

Here is the MATLAB session:

\[
\text{>> } A = [0, -7, 5; 0, 4, 7; -4, 3, -7]; \\
\text{>> } b = [50; -30; 40]; \\
\text{>> } x = A \backslash b \\
\]

\[
x = \\
-15.1812 \\
-7.2464 \\
-0.1449
\]

\[
\text{>> } A^T = A'; \\
\]

\[
A^T = \\
\begin{bmatrix}
0 & 0 & 4 \\
-7 & 4 & 3 \\
5 & 7 & -7
\end{bmatrix}
\]

\[
\text{>> } A^{-1} = \text{inv}(A) \\
\]

\[
A^{-1} = \\
\begin{bmatrix}
-0.1775 & -0.1232 & -0.2500 \\
-0.1014 & 0.0725 & 0 \\
0.0500 & 0.1014 & 0
\end{bmatrix}
\]

8.7

\[
\text{>> } k1 = 10; k2 = 30; k3 = 30; k4 = 10; \\
\text{>> } m1 = 1; m2 = 1; m3 = 1; \\
\text{>> } km = [(1/m1)*(k2+k1), -(k2/m1), 0; -(k2/m2), (1/m2)*(k2+k3), -(k3/m2); 0, (k3/m3), (1/m3)*(k3+k4)]; \\
\text{>> } x = [0.05; 0.04; 0.03]; \\
\text{>> } kmx = km*x
\]

\[
kmx = \\
0.8000 \\
0 \\
0
\]

Therefore, \( \dot{x}_1 = -0.8, \dot{x}_2 = 0 \), and \( \dot{x}_3 = 0 \text{ m/s}^3 \).
9.5 (a) The equations can be expressed in a format that is compatible with graphing $x_2$ versus $x_1$:

$$x_2 = 0.5x_1 + 9.5$$
$$x_2 = 0.51x_1 + 9.4$$

The resulting plot indicates that the intersection of the lines is difficult to detect:

Only when the plot is zoomed is it at all possible to discern that solution seems to lie at about $x_1 = 14.5$ and $x_2 = 10$. 
(b) The determinant can be computed as

\[
\begin{vmatrix}
0.5 & 1 \\
1.02 & -2
\end{vmatrix} = 0.5(-2) - (-1)(1.02) = 0.02
\]

which is close to zero.

(c) Because the lines have very similar slopes and the determinant is so small, you would expect that the system would be ill-conditioned.

(d) Multiply the first equation by 1.02/0.5 and subtract the result from the second equation to eliminate the \( x_1 \) term from the second equation,

\[
0.5x_1 - x_2 = -9.5
\]

\[
0.04x_2 = 0.58
\]

The second equation can be solved for

\[
x_2 = \frac{0.58}{0.04} = 14.5
\]

This result can be substituted into the first equation which can be solved for

\[
x_1 = \frac{-9.5 + 14.5}{0.5} = 10
\]

(e) Multiply the first equation by 1.02/0.52 and subtract the result from the second equation to eliminate the \( x_1 \) term from the second equation,

\[
0.52x_1 - x_2 = -9.5
\]

\[
-0.03846x_2 = -0.16538
\]

The second equation can be solved for

\[
x_2 = \frac{-0.16538}{-0.03846} = 4.3
\]

This result can be substituted into the first equation which can be solved for

\[
x_1 = \frac{-9.5 + 4.3}{0.52} = -10
\]

Interpretation: The fact that a slight change in one of the coefficients results in a radically different solution illustrates that this system is very ill-conditioned.
9.8 Multiply the first equation by $-0.4/0.8$ and subtract the result from the second equation to eliminate the $x_1$ term from the second equation.

\[
\begin{bmatrix}
0.8 & -0.4 \\
0.6 & -0.4 \\
-0.4 & 0.8
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
41 \\
45.5 \\
105
\end{bmatrix}
\]

Multiply pivot row 2 by $-0.4/0.6$ and subtract the result from the third row to eliminate the $x_2$ term.

\[
\begin{bmatrix}
0.8 & -0.4 \\
0.6 & -0.4 \\
0.533333 & 0.4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
41 \\
45.5 \\
135.3333
\end{bmatrix}
\]

The solution can then be obtained by back substitution

\[x_3 = \frac{135.3333}{0.533333} = 253.75\]
\[x_2 = \frac{45.5 - (-0.4)253.75}{0.6} = 245\]
\[x_1 = \frac{41 - (-0.4)245}{0.8} = 173.75\]

(b) Check:

\[0.8(173.75) - 0.4(245) = 41\]
\[-0.4(173.75) + 0.8(245) - 0.4(253.75) = 25\]
\[-0.4(245) + 0.8(253.75) = 105\]

9.11 Let \(c_i = \text{component } i\). Therefore, the following system of equations must hold

\[15c_1 + 17c_2 + 19c_3 = 3890\]
\[0.30c_1 + 0.40c_2 + 0.55c_3 = 95\]
\[1.0c_1 + 1.2c_2 + 1.5c_3 = 282\]

These can then be solved for \(c_1 = 90\), \(c_2 = 60\), and \(c_3 = 80\).