HW #7

1. 1 (1), (2) and (3) (5.0)
2. 2.2 (2.5)
3. 2.3 (2.5)

Due date Oct. 28 2014, Wednesday
1. If the progressive wave number $k_p$ and the standing wave numbers $k_S(n)$ satisfy the following equations respectively:

$$\sigma^2 = g k_p \tanh (k_p h)$$
$$\sigma^2 = -g k_S(n) \tan [k_S(n) h], \quad (n=1, 2, \ldots)$$

where $\sigma$ is the frequency of the wavemaker and $h$ is the depth of the water in the wave tank. $h = \text{const}$.

Prove that $\cosh[k_p(z+h)], \cos[k_S(n)(h+z)]$ form a set of orthogonal functions at $[0, -h]$.

i.e. Prove

$$\begin{align*}
(1) \int_{-h}^{0} \cosh[k_p(z+h)] \cos[k_S(n)(z+h)] dz &= 0 \\
(2) \int_{-h}^{0} \cosh^2[k_p(z+h)] dz &= \frac{1}{4k_p} \left[ 2k_p h + \sinh(2k_p h) \right] \\
(3) \int_{-h}^{0} \cos[k_S(n)(h+z)] \cos[k_S(m)(h+z)] dz &= \begin{cases} 0 & \text{if } n \neq m \\ \frac{1}{4k_S(n)} \left[ (2k_S(n) h + \sin[2k_S(n) h]) \right] & \text{if } n = m. \end{cases}
\end{align*}$$
2. It is known that the potential function $\phi$ in a wave tank is

$$\phi = \frac{A_p g \cosh[k_p(z+h)]}{\sigma} \sin(k_p x - \sigma t)$$

$$+ \sum_{n=1}^{\infty} \frac{C_n g}{\sigma} e^{-k_s(n)x} \cos[k_s(n)(h+z)] \cos \omega t$$

and $\sigma^2 = g k_p \tanh(k_p h)$,

2.1 Prove that $\phi$ satisfies:

1. Laplace Eq. \( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \) for \( 0 \leq x < \infty \)

- \(-h \leq z \leq 0\)

2. Free Surface B.C. \( \frac{1}{g} \frac{\partial^2 \phi}{\partial t^2} = -\frac{\partial \phi}{\partial z} \) at \( z = 0 \)

3. Bottom B.C. \( \frac{\partial \phi}{\partial z} = 0 \) at \( z = -h \)

2.2 If the motion of a paddle is known, i.e.

\( \frac{\partial \phi}{\partial x} \bigg|_{x=0} = \frac{\zeta(z)}{2} \sigma \cos \omega t \) at \( x = 0 \)

For a piston-type wavemaker \( \zeta(z) = \zeta = \text{const.} \)

Please find \( A_p \) and \( C_n \) \( (n=1,2,3) \).

2.3 Show that when \( x \) is large \( (x k_s(1) >> 1) \), then the only wave observed in the tank is the right-going progressive wave.