5.16 The bottom slope \( S = \frac{h}{l} \) (TWS is constant for a,b,c)

a) \( b_0, \ h_0 \) \( S_0 = \frac{h_0}{b_0} \)

b) \( b > b_0, \ h_0 \) \( S_1 = \frac{h_0}{b_1} < S_0 \)

c) \( b_1 > b_0, \ h_1 \) \( S_2 = \frac{h_1}{b_1} = \frac{h_0}{b_0} \rightarrow h_0 = h_0 \frac{b_1}{b_0} \)

which one allows the greatest storm surge?

Using the results given by 'Example 5.3' at Page 159.

\[
\frac{\eta}{l} = \left(1 - \frac{h+1}{h_0}\right) - A \ln \left( \frac{\frac{h+1}{h_0} - A}{1 - A} \right)  
\]

where \( A = \frac{n \ TWS \ l}{\rho g \ h_0} \)

Since the greatest surge always occur at \( x = l \) (Beach)

\[
\frac{\eta}{l} = 0 \quad A = \frac{n \ TWS \ l}{\rho g \ h_0}
\]

\[
\frac{\eta}{h_0} = -A \ln \left( \frac{\frac{h_0}{h_0} - A}{1 - A} \right) 
\]

a) \[
\eta_a = - \frac{n \ TWS \ h_0}{\rho \ g \ h_0} \cdot \ln \left( \frac{\eta_a}{h_0} - \frac{n \ TWS \ h_0}{\rho \ g \ h_0} \right) 
\]

b) \[
\eta_b = - \frac{n \ TWS \ l_1}{\rho \ a \ l_1} \cdot \ln \left( \frac{\eta_b}{l_1} - \frac{n \ TWS \ l_1}{\rho \ a \ l_1} \right) 
\]
c) \[ \eta_c = - \frac{n \tau_w \xi}{\rho g \eta} \times \left( \frac{\eta_c - \frac{n \tau_w \xi}{\rho g \eta_i}}{\eta_i - \frac{n \tau_w \xi}{\rho g \eta}} \right) \]

Noticing \( \frac{\eta_i}{\eta} = \frac{\eta_0}{\eta_0} \),

\[ \eta_c = - \frac{n \tau_w \xi}{\rho g \eta} \times \left( \frac{\eta_c - \frac{n \tau_w \xi}{\rho g \eta_0}}{\eta_i - \frac{n \tau_w \xi}{\rho g \eta_0}} \right) \]

Comparing Eq. 1 with 3, their right-hand sides are the same except \( \eta_i \). Since \( \eta_i > \eta_0 \), then \( \eta_c > \eta_a \).

This result is expected, for larger \( \eta_i (\xi > \xi_0) \), the wind shear force is bigger.

Comparing Eq. 2 with 1, notice \( \frac{\eta_i}{\eta_0} > \frac{\xi}{\xi_0} \), then we expect \( \eta_b > \eta_c \) (due to smaller slope leads larger \( \eta \) for the same depth \( \eta_0 \)).

Verify:

a) \( A_a = 0.05 \), \( \xi_0, \eta_0 \) using Figure 5.12 \( \eta_a = 0.128 \eta_0 \)

b) \( A_b = 5A_a = 0.25 \), \( 5\xi_0, \eta_0 \)

c) \( A_c = \frac{A_a}{5} = 0.01 \), \( 5\xi_0, 5\eta_0 \)

Since \( A_b > A_a \), we can find \( \eta_b/\eta_0 \) according to Figure 5.12. Using iteration, \( \eta_b = 0.4 \eta_0 \).

Then \( \eta_b > \eta_c > \eta_a \). It is verified.
5.17. \[ h = h_0 \left( 1 - \frac{\eta}{\tau_s} \right)^2 \]

\[ \eta(x) = \sqrt{h^2 + \frac{2\pi\tau_s x}{\rho g}} - h \]

Steady state:

\[(h + \eta) \frac{\partial \eta}{\partial x} = \frac{\pi \tau_s}{\rho g} \]

Proof:

\[ h + \eta = \sqrt{h^2 + \frac{2\pi\tau_s x}{\rho g}} \]

\[ \frac{\partial \eta}{\partial x} = \frac{\frac{\partial h}{\partial x} + \frac{\pi \tau_s}{\rho g}}{\sqrt{h^2 + \frac{2\pi\tau_s x}{\rho g}}} = \frac{\partial h}{\partial x} \]

\[ L.H.S. = (h + \eta) \frac{\partial \eta}{\partial x} = h \frac{\partial h}{\partial x} + \frac{\pi \tau_s}{\rho g} - (h + \eta) \frac{\partial h}{\partial x} \]

\[ = \frac{\pi \tau_s}{\rho g} - \eta \frac{\partial h}{\partial x} = \frac{\pi \tau_s}{\rho g} \]

If \( \eta \frac{\partial h}{\partial x} \approx \frac{\pi \tau_s}{\rho g} \)

This is proved.

Comments on \( h \frac{\partial h}{\partial x} - (h + \eta) \frac{\partial \eta}{\partial x} = 0 \).

1. For \( h \) is large (near \( x = 0 \)), \( \eta \) is small, \( \eta \frac{\partial \eta}{\partial x} \approx 0 \).

2. For \( \eta \) is large, \( \frac{\partial h}{\partial x} = -2\frac{h_0}{\tau_s} (1 - \frac{x}{\ell}) \left( 1 - \frac{x}{\ell} \right) = 0 \) (near the beach).
5.17 \[ h = h_0 (1 - \frac{x}{L})^4 \]

parabolic slope

and \[ h = h_0 (1 - \frac{x}{L}) \]

hot slope

\[ a) \ \eta (x = 0) = \sqrt{\frac{h}{g} + \frac{2 \eta L w x}{\rho g}} \]

b) \[ \eta \] can be determined from Eq. (5.99b)

Check Figure 5.12.

For \[ A = 0.01 \]

\[ \eta_0 = 0.03 \ h_0 \]

\[ \eta_a = 0.141 \ h_0 \]

\[ \eta_a > \eta_0 \]

For \[ A = 0.05 \]

\[ \eta_0 = 0.13 \ h_0 \]

\[ \eta_a = 0.316 \ h_0 \]

\[ \eta_a > \eta_0 \]

With the same \[ h_0 \], \[ \eta_0 \]. The surge for the parabolic slope beach is usually longer than sloping (flat) beach.
Storm Surge: Parabolic and Linear Bottoms

- Linear (q = 0.05)
- Linear (q = 0.1)
- Parabolic (q = 0.05)
- Parabolic (q = 0.1)