CVEN 311 – Fluid Dynamics
Fall Semester 2008
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Final Exam

<u>Open-book, Open-notes (8 pages, front & back, not including reference sheets; 25 questions)</u>

An excerpt from the NCEES *Fundamentals of Engineering Supplied-Reference Handbook* is attached to this exam. This excerpt is <u>only</u> for the use of students during this exam and must be returned at the conclusion of the exam.

Questions 1 to 16 are written in the format of the F.E. Exam Morning Section and should require on average 2 minutes per question to complete. Each question is worth *2.5 points*. ***<u>Clearly</u>* <u>write the letter corresponding to the best answer in the blank provided on the answer sheet.</u>**

1. The density of a fluid is 133 000 N/m^3. What is the value of its specific gravity?

   (A) 2.13
   (B) 13.6
   (C) 68.6
   (D) 133

2. Water flowing through a circular cross-section, 3 inch diameter, cast iron pipe has a velocity of 2.4 ft/sec. The water temperature is 60°F. The Reynolds number for the pipe flow is most nearly:

   (A) 4.93×10^4
   (B) 5.92×10^5
   (C) 1.59×10^6
   (D) 2.54×10^7

3. Oxygen flows in a 10 mm diameter tube at a velocity of 0.24 m/s. At a point in the tube the pressure is 141 kPa (abs) and the temperature of the oxygen is 20°C. The gas constant for oxygen $R = 2.598 \times 10^2$ J/(kg·K). What is the mass flowrate of the oxygen at this point?

   (A) 3.49×10^{-3} kg/s
   (B) 5.12×10^{-4} kg/s
   (C) 3.49×10^{-5} kg/s
   (D) 3.49×10^{-8} kg/s

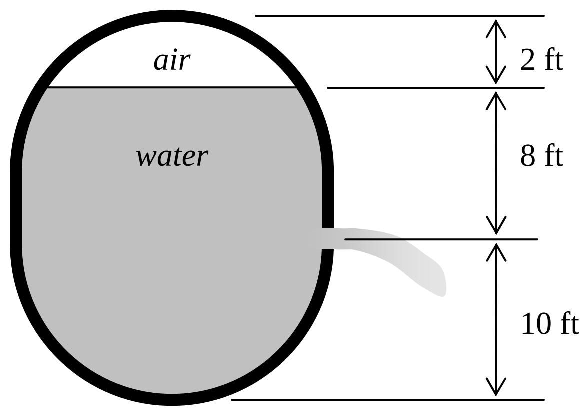
4. Which of the following statements regarding laminar pipe flow are true?

    I. The velocity profile is always symmetric about the pipe centerline.
    II. Kinetic energy and inertial forces dominate viscous energy and force in the flow.
    III. Decreasing the pipe diameter by one-half will cause the flow to become turbulent.
    IV. Fully developed flow will have a parabolic velocity profile.

    (A) I and IV
    (B) I, II, and IV
    (C) II and III
    (D) IV only

5. A pressurized tank is partially filled with water as shown in the figure below. Gage pressure of the air above the water surface is 45 lb/in^2. Water will flow out of the tank through the sharp-edged orifice of diameter 0.8 inches at a volumetric flowrate of:

    (A) 0.30 ft^3/sec
    (B) 0.050 ft^3/sec
    (C) 0.10 ft^3/sec
    (D) 0.18 ft^3/sec



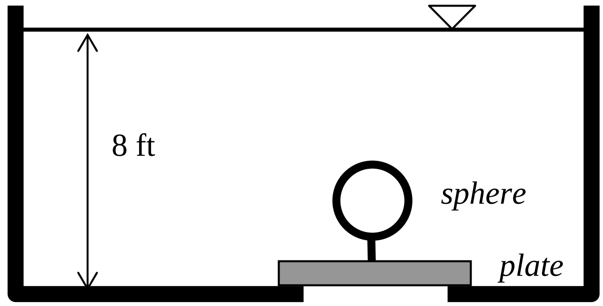
6. Flow of water in a 4 inch diameter galvanized iron pipe has a velocity of 1.2 ft/sec. The head loss (ft) in a 100 foot length of pipe is:

    (A) 0.156
    (B) 0.601
    (C) 0.0151
    (D) 0.183

7. A pitot-static tube is used to measure velocity of ethyl alcohol (ρ = 789 kg/m^3) flowing in a pipe. If pressure on the pitot gage is 78.2 kPa and pressure on the static gage is 54.2 kPa, the velocity of the flow is:

    (A) 0.22 m/s
    (B) 6.9 m/s
    (C) 7.8 m/s
    (D) 0.25 m/s

8. A tank has a 0.8 ft diameter circular hole in its bottom (see figure below). A 1 ft diameter circular plate of weight 35 lb covers the hole. An air-filled sphere of volume 3.4 ft^3 is attached to the circular plate by a 0.5 ft long cord of negligible weight. The depth of water in the tank is 8.0 ft. Which of the following statements are true?



(A) The plate will be held in place over the hole, but lowering the water level will eventually cause the plate to lift off of the hole.
(B) The plate will be held in place over the hole, and lowering the water level will not cause the plate to lift off of the hole.
(C) The plate will not be held in place over the hole, but raising the water level will eventually cause the plate to be held in place over the hole.
(D) The plate will not be held in place over the hole, and raising the water level will not cause the plate to be held in place over the hole.

9. A pump is required to add 15.3 m of head to water flowing at 1.9 m/s in a 250 mm diameter pipe. Assuming perfect efficiency, the power input to the pump is:

(A) 14 W
(B) 89 W
(C) 14 000 W
(D) 56 000 W

10. A mercury (ρ = 13 600 kg/m^3; p_{vapor} = 0.160 Pa [abs]) barometer measures the current atmospheric pressure as 764 mm of mercury. How tall (mm) would a barometer need to be to measure this pressure if the barometer fluid were carbon tetrachloride (ρ = 1590 kg/m^3; p_{vapor} = 13.0 kPa [abs])?

(A) 6 530
(B) 7 370
(C) 1 640
(D) 5 700

11. A pipe connects two reservoirs, and fluid flows from the upper to the lower reservoir through this pipe. Which of the following parameters would you need to know before you could calculate the flowrate in the pipe?

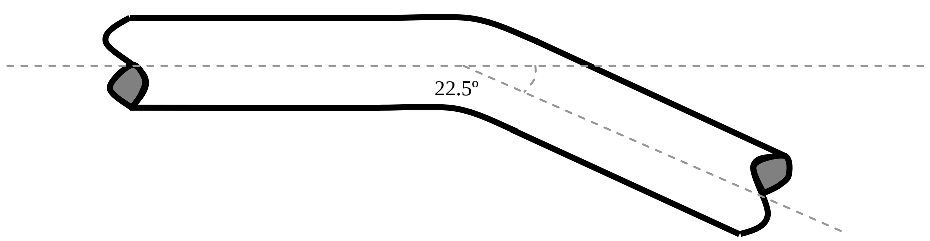
I. Pipe material
II. Fluid viscosity
III. Fluid surface tension
IV. Whether flow is turbulent or laminar
V. Pipe diameter

(A) II and IV
(B) I, II, and V
(C) II, III, and V
(D) I, II, III, IV, and V

12. Water flows through a 6 inch diameter cast iron pipe at a velocity of 6.5 ft/sec. Within a 1000 ft length of pipe there are 5 45° bends (minor loss coefficient $K_L = 0.4$) and one fully open globe valve (minor loss coefficient $K_L = 10$). What will be the total head loss for this section of pipe and fittings?

(A) 38.1 ft
(B) 30.2 ft
(C) 37.0 ft
(D) 125 ft

13. Water flowing in a 200 mm pipe at a velocity of 2.40 m/s passes through a 22.5° bend as shown in the drawing below. The total force (N) required to hold the bend in place is:



(A) 87.7
(B) 1150
(C) 449
(D) 1280

14. At 1:00 PM, the inflow of seawater ($\rho = 1.99$ slugs/ft^3) to a storage tank is 0.134 ft^3/sec, the outflow from the tank is 0.102 ft^3/sec, and the weight of seawater in the tank is 1,370 lb. Assuming that the inflow and outflow are both steady, what will be the weight of seawater in the tank at 2:45 PM?

(A) 14,300 lb
(B) 12,900 lb
(C) 1,770 lb
(D) 5,310 lb

15. A square plate of area 2.00 m^2 is dragged across the surface of 3 mm thick film of motor oil (μ = 0.380 $N{\cdot}s/m^2$). If the film of motor oil is on top of a stationary surface, what force will be required to drag the plate at a velocity of 0.240 m/s?

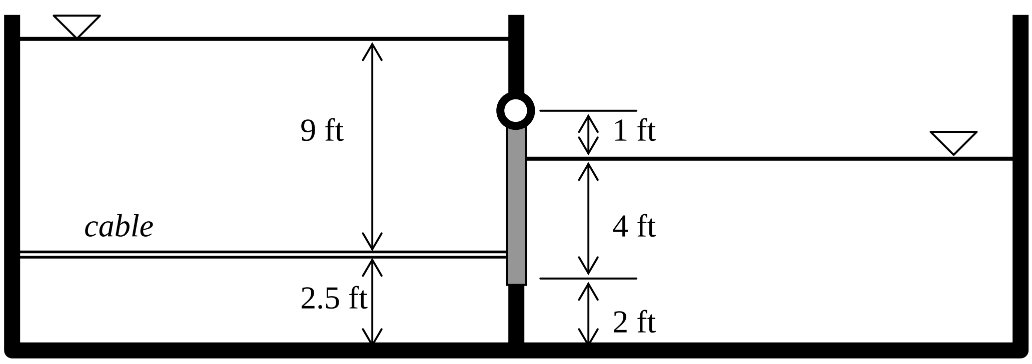
(A) 421 N
(B) 0.182 N
(C) 15.2 N
(D) 60.8 N

16. An inviscid fluid (ρ = 3.09 slugs/ft^3) flowing in a pipe has velocity 3.4 ft/sec and absolute pressure 35 lb/in^2 at a specific point in the pipe. If the fluid's vapor pressure is 1.9 lb/in^2 (abs) and the pipe diameter does not change, how much elevation could the pipe gain up a slope before cavitation would occur?

(A) 76 ft
(B) 48 ft
(C) 0.53 ft
(D) 0.33 ft

Questions 17 to 24 are written in the format of the F.E. Exam Afternoon Section and should require on average 4 minutes per question to complete. Each question is worth *5.0 points*. ***Clearly*** **write the letter corresponding to the best answer in the blank provided on the answer sheet.**

17. A hinged gate between two reservoirs is held stationary by a cable attached as shown in the drawing below. The fluid in the reservoirs is water. The tensile force (lb) in the cable is most nearly:



(A) 691
(B) 805
(C) 636
(D) 312

18. A Froude number-similar physical model of an open channel flow is to be built with the flow depth as the characteristic length. If the prototype flow depth is 2 ft and prototype flow velocity is 3 ft/sec, what is the minimum length scaling ratio (i.e., model : prototype) possible if the Reynolds number of the model is to be greater than 15,000?

(A) 0.03
(B) 0.25
(C) 0.10
(D) 0.67

19. Two reservoirs are connected by a concrete (roughness height = 2.0 mm) pipe of length 2000 m and diameter 500 mm. The water surface elevations in the upper and lower reservoirs are 254 m and 212 m, respectively. The volumetric flowrate in the pipe is most nearly:

(A) 0.53 m^3/s
(B) 2.71 m^3/s
(C) 1.06 m^3/s
(D) 0.97 m^3/s

20. Water flows through a tube of diameter 10 mm that then contracts to 5 mm diameter. In the 10 mm diameter section the velocity at a point 3mm from the tube centerline is 0.3 mm/s. What will be the drop in pressure along each 1 m of length in the 5mm diameter tube?

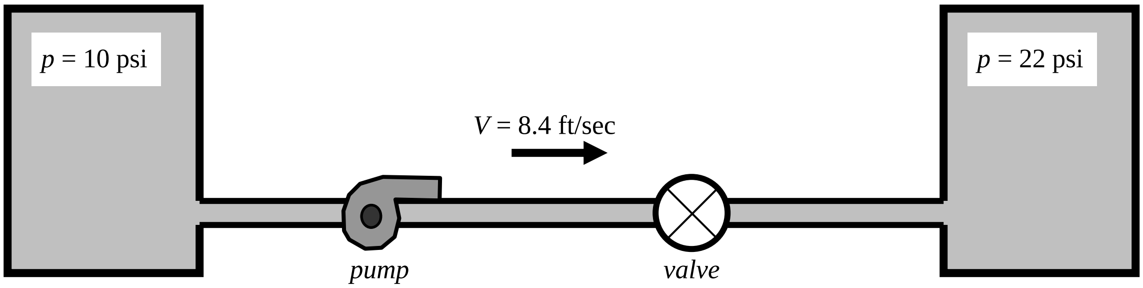
(A) 2.40 Pa
(B) 1.92 Pa
(C) 1.54 Pa
(D) 1.20 Pa

21. A flow of water through a cast iron pipe has Reynolds number equal to 3200. The value of relative roughness for the flow is 0.017. Which of the following statements is/are <u>not</u> true?

I. If the pipe material is changed to galvanized iron, the flow velocity may be increased by 70% with the same head loss per unit length of pipe.
II. Flow velocity can be increased in this pipe by as much as a factor of 22 with no increase in head loss per unit length of pipe.
III. If flow velocity is decreased by 38%, the head loss per unit length of pipe will decrease by approximately 43%.
IV. If flow velocity is decreased by 38%, the head loss per unit length of pipe will decrease by approximately 78%.

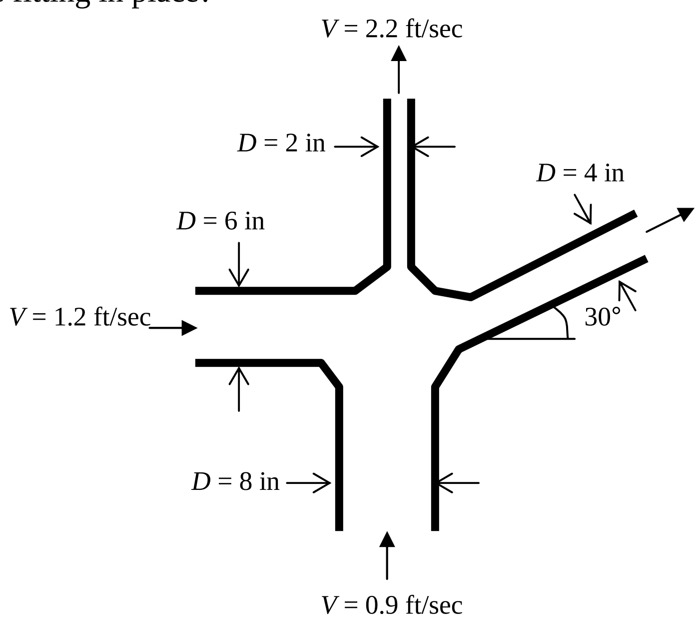
(A) I, II, and III
(B) I and IV
(C) II and III
(D) IV only

22. A 50 ft long smooth plastic pipe of 1.5 inch diameter connects two pressurized tanks of water as shown in the drawing below. The pump has a power input of 241 ft·lb/sec with perfect efficiency. The tanks are at the same elevation. What is the minor loss coefficient of the valve?



(A) 1.67
(B) 26.5
(C) 1.52
(D) 4.30

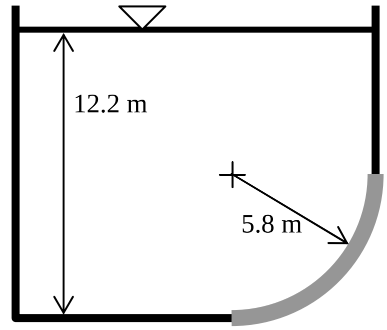
23. A pipe fitting with 4 circular cross-section branches is shown in the figure below. If water flows in and out of the fitting as indicated and the static pressure at all 4 branches is 2.0 psi, what will be the total net force required to hold the fitting in place?



(A) 4.95 lb
(B) 80.4 lb
(C) 84.8 lb
(D) 92.7 lb

24. A quarter-circular plate has been welded into the corner of the tank drawn below. The dimension of the plate perpendicular to the page is 3.75 m. What is the total force acting on the plate?

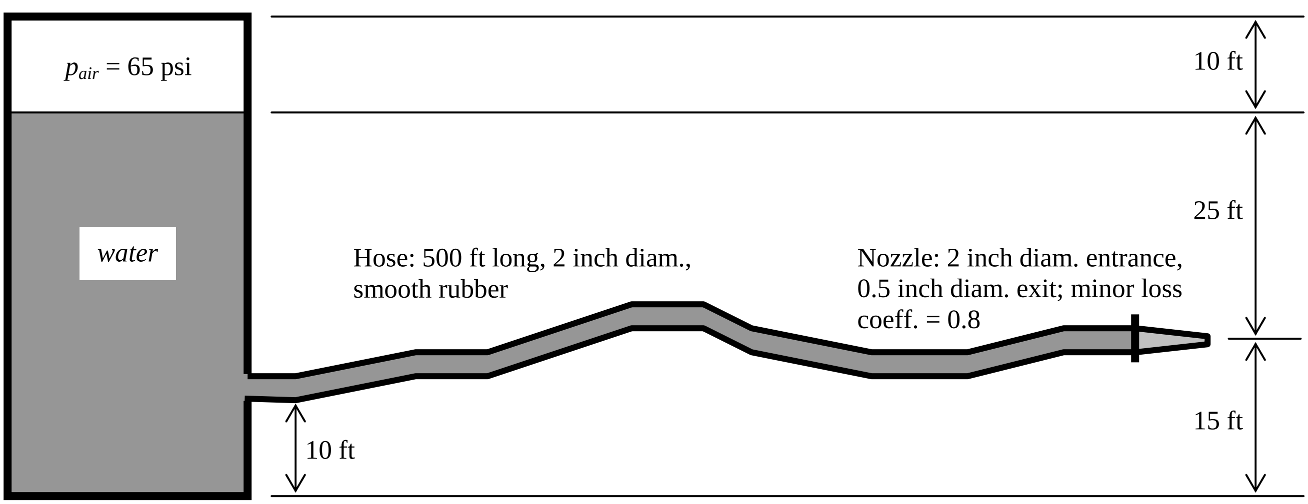
(A) 0.817 MN
(B) 3.07 MN
(C) 1.98 MN
(D) 2.21 MN



Questions 25 is a "work-out" problem for which partial credit may be given. You should answer the question on the back side of the answer sheet ***exactly as asked*** to receive credit as indicated.

25. A hose is connected to a pressurized tank is partially filled with water as shown in the drawing below. Water flows through the hose and the nozzle at the hose end.

(i) Note the entrance and exit locations for an appropriate control volume, and write the full energy equation across that control volume. (4 pts)
(ii) Remove terms in the energy equation that may be neglected. (2 pts)
(iii) Expand all remaining terms to the expressions from which you will determine their values. (3 pts)
(iv) Re-arrange the equation from (iii) to one where velocity or volumetric flowrate is isolated on one side. Circle any Reynolds number-dependent terms on the other side of the equation. (3 pts)
(v) Carry out an appropriate iterative solution to determine flowrate through the hose. In each iteration explicitly state: beginning value of velocity, Reynolds number, friction factor, calculated velocity. (6 pts)
(vi) State your final solution for flowrate through the hose. (2 pts)



FLUID MECHANICS

DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

$$\rho = \lim_{\Delta V \to 0} \Delta m/\Delta V$$

$$\gamma = \lim_{\Delta V \to 0} \Delta W/\Delta V$$

$$\gamma = \lim_{\Delta V \to 0} g \cdot \Delta m/\Delta V = \rho g$$

also $SG = \gamma/\gamma_w = \rho/\rho_w$, where

ρ = *density* (also *mass density*),

Δm = mass of infinitesimal volume,

ΔV = volume of infinitesimal object considered,

γ = *specific weight,*

ΔW = weight of an infinitesimal volume,

SG = *specific gravity*, and

ρ_w = mass density of water at standard conditions = 1,000 kg/m^3 (62.43 lbm/ft^3).

STRESS, PRESSURE, AND VISCOSITY

Stress is defined as

$$\tau(P) = \lim_{\Delta A \to 0} \Delta F/\Delta A, \text{ where}$$

$\tau(P)$ = surface stress vector at point P,

ΔF = force acting on infinitesimal area ΔA, and

ΔA = infinitesimal area at point P.

$$\tau_n = -p$$

$$\tau_t = \mu\,(dv/dy) \quad \text{(one-dimensional; i.e., } y\text{), where}$$

τ_n and τ_t = the normal and tangential stress components at point P,

p = the pressure at point P,

μ = *absolute dynamic viscosity* of the fluid N·s/m^2 [lbm/(ft-sec)],

dv = velocity at boundary condition, and

dy = normal distance, measured from boundary.

$v = \mu/\rho$, where

v = *kinematic viscosity*; m^2/s (ft^2/sec).

For a thin Newtonian fluid film and a linear velocity profile,

$v(y) = Vy/\delta$; $dv/dy = V/\delta$, where

V = velocity of plate on film and

δ = thickness of fluid film.

For a power law (non-Newtonian) fluid

$\tau_t = K\,(dv/dy)^n$, where

K = consistency index, and

n = power law index.

$n < 1 \equiv$ pseudo plastic

$n > 1 \equiv$ dilatant

SURFACE TENSION AND CAPILLARITY

Surface tension σ is the force per unit contact length

$\sigma = F/L$, where

σ = surface tension, force/length,

F = surface force at the interface, and

L = length of interface.

The *capillary rise* h is approximated by

$h = 4\sigma \cos\beta/(\gamma d)$, where

h = the height of the liquid in the vertical tube,

σ = the surface tension,

β = the angle made by the liquid with the wetted tube wall,

γ = specific weight of the liquid, and

d = the diameter of the capillary tube.

THE PRESSURE FIELD IN A STATIC LIQUID

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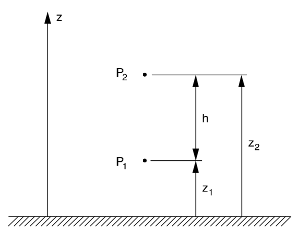

The difference in pressure between two different points is

$$p_2 - p_1 = -\gamma\,(z_2 - z_1) = -\gamma h$$

For a simple manometer,

$$p_o = p_2 + \gamma_2 h_2 - \gamma_1 h_1$$

Absolute pressure = atmospheric pressure + gage pressure reading

Absolute pressure = atmospheric pressure – vacuum gage pressure reading

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, John Wiley & Sons, Inc., 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

FORCES ON SUBMERGED SURFACES AND THE CENTER OF PRESSURE

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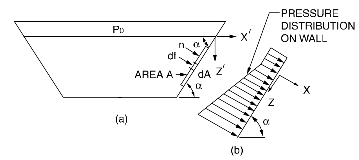


Forces on a submerged plane wall. (a) Submerged plane surface. (b) Pressure distribution.

The pressure on a point at a distance Z' below the surface is

$$p = p_o + \gamma Z', \text{ for } Z' \geq 0$$

If the tank were open to the atmosphere, the effects of p_o could be ignored.

The coordinates of the *center of pressure CP* are

$$y^* = \left(\gamma I_{y_c z_c} \sin\alpha\right)/\left(p_c A\right) \quad \text{and}$$

$$z^* = \left(\gamma I_{y_c} \sin\alpha\right)/\left(p_c A\right), \text{ where}$$

y^* = the y-distance from the centroid (C) of area (A) to the center of pressure,

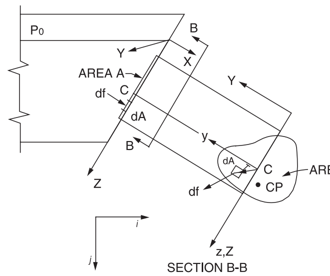
z^* = the z-distance from the centroid (C) of area (A) to the center of pressure,

I_{y_c} and $I_{y_c z_c}$ = the moment and product of inertia of the area,

p_c = the pressure at the centroid of area (A), and

Z_c = the slant distance from the water surface to the centroid (C) of area (A).

♦



If the free surface is open to the atmosphere, then

$p_o = 0$ and $p_c = \gamma Z_c \sin\alpha$.

$$y^* = I_{y_c z_c}/\left(A Z_c\right) \quad \text{and} \quad z^* = I_{y_c}/\left(A Z_c\right)$$

The force on a rectangular plate can be computed as

$$\boldsymbol{F} = [p_1 A_v + (p_2 - p_1)\,A_v/2]\,\mathbf{i} + V_f \gamma_f\,\mathbf{j}, \text{ where}$$

$\boldsymbol{F}$ = force on the plate,

p_1 = pressure at the top edge of the plate area,

p_2 = pressure at the bottom edge of the plate area,

A_v = vertical projection of the plate area,

V_f = volume of column of fluid above plate, and

γ_f = specific weight of the fluid.

ARCHIMEDES PRINCIPLE AND BUOYANCY

1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the displaced fluid volume.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

ONE-DIMENSIONAL FLOWS

The Continuity Equation So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1V_1 = A_2V_2$.

$$Q = AV$$

$$\dot{m} = \rho Q = \rho AV, \text{ where}$$

Q = volumetric flow rate,

$\dot{m}$ = mass flow rate,

A = cross section of area of flow,

V = average flow velocity, and

ρ = the fluid density.

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, John Wiley & Sons, Inc., 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

The Field Equation is derived when the energy equation is applied to one-dimensional flows.

Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\text{, where}$$

p_1, p_2 = pressure at sections 1 and 2,

V_1, V_2 = average velocity of the fluid at the sections,

z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),

γ = the specific weight of the fluid (ρg), and

g = the acceleration of gravity.

FLUID FLOW

The velocity distribution for ***laminar flow*** **in circular tubes or between planes** is

$$v=v_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\text{, where}$$

r = the distance (m) from the centerline,

R = the radius (m) of the tube or half the distance between the parallel planes,

v = the local velocity (m/s) at r, and

v_{max} = the velocity (m/s) at the centerline of the duct.

v_{max} = 1.18V, for fully turbulent flow (Re > 10,000)

v_{max} = $2V$, for circular tubes in laminar flow and

v_{max} = $1.5V$, for parallel planes in laminar flow, where

V = the average velocity (m/s) in the duct.

The shear stress distribution is

$$\frac{\tau}{\tau_w}=\frac{r}{R}\text{, where}$$

τ and τ_w are the shear stresses at radii r and R respectively.

The *drag force* F_D on **objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid** is

$$F_D=\frac{C_D\rho V^2A}{2}\text{, where}$$

C_D = the *drag coefficient* (see page 55),

V = the velocity (m/s) of the undisturbed fluid, and

A = the *projected area* (m^2) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

For **flat plates placed parallel with the flow**

$C_D = 1.33/\text{Re}^{0.5}$ ($10^4 < \text{Re} < 5 \times 10^5$)

$C_D = 0.031/\text{Re}^{1/7}$ ($10^6 < \text{Re} < 10^9$)

The characteristic length in the Reynolds Number (Re) is the length of the plate parallel with the flow. For blunt objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) which is perpendicular to the flow.

AERODYNAMICS

Airfoil Theory

The lift force on an airfoil is given by

$$F_L=\frac{C_L\rho V^2A_P}{2}$$

C_L = the lift coefficient

V = velocity (m/s) of the undisturbed fluid and

A_P = the projected area of the airfoil as seen from above (plan area). This same area is used in defining the drag coefficient for an airfoil.

The lift coefficient can be approximated by the equation

$C_L = 2\pi k_1\sin(\alpha+\beta)$ which is valid for small values of α and β.

k_1 = a constant of proportionality

α = angle of attack (angle between chord of airfoil and direction of flow)

β = negative of angle of attack for zero lift.

The drag coefficient may be approximated by

$$C_D=C_{D\infty}+\frac{C_L^2}{\pi AR}$$

$C_{D\infty}$ = infinite span drag coefficient

$$AR=\frac{b^2}{A_p}=\frac{A_p}{c^2}$$

The aerodynamic moment is given by

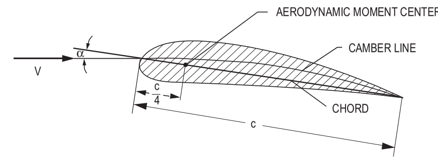
$$\text{M}=\frac{\text{C}_\text{M}\rho\text{V}^2\text{A}_\text{p}\text{c}}{2}$$

where the moment is taken about the front quarter point of the airfoil.

C_M = moment coefficient

A_p = plan area

c = chord length



Reynolds Number

$$\text{Re}=VD\rho/\mu=VD/\nu$$

$$\text{Re}'=\frac{V^{(2-n)}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n8^{(n-1)}}\text{, where}$$

ρ = the mass density,

D = the diameter of the pipe or dimension of the fluid streamline,

μ = the dynamic viscosity,

ν = the kinematic viscosity,

Re = the Reynolds number (Newtonian fluid),

Re′ = the Reynolds number (Power law fluid), and

K and n are defined on page 44.

The critical Reynolds number $(\text{Re})_c$ is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Hydraulic Gradient (Grade Line)

The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the "total head line" above a horizontal datum.

The difference between the hydraulic grade line and the energy line is the $V^2/2g$ term.

STEADY, INCOMPRESSIBLE FLOW IN CONDUITS AND PIPES

The energy equation for incompressible flow is

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+h_f$$

h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $V_1 = V_2$. The pressure drop $p_1 - p_2$ is given by the following:

$p_1 - p_2 = \gamma\, h_f$

The *Darcy-Weisbach equation* is

$$h_f=f\frac{L}{D}\frac{V^2}{2g}\text{, where}$$

f = f(Re, e/D), the Moody or Darcy friction factor,

D = diameter of the pipe,

L = length over which the pressure drop occurs,

e = roughness factor for the pipe, and all other symbols are defined as before.

A chart that gives f versus Re for various values of e/D, known as a *Moody* or *Stanton diagram*, is available at the end of this section.

Friction Factor for Laminar Flow

The equation for Q in terms of the pressure drop Δp_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region.

$$Q=\frac{\pi R^4\Delta p_f}{8\mu L}=\frac{\pi D^4\Delta p_f}{128\mu L}$$

Flow in Noncircular Conduits

Analysis of flow in conduits having a noncircular cross section uses the *hydraulic diameter* D_H, or the *hydraulic radius* R_H, as follows

$$R_H=\frac{\text{cross-sectional area}}{\text{wetted perimeter}}=\frac{D_H}{4}$$

Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+h_f+h_{f,\text{ fitting}}\text{, where}$$

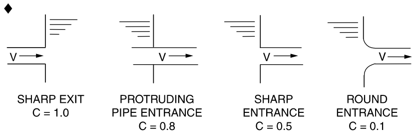
$$h_{f,\text{ fitting}}=C\frac{V^2}{2g}$$

Specific fittings have characteristic values of C, which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\ \text{fitting}} = 0.04\ V^2/\ 2g$$

The *head loss* at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f,\ \text{fitting}}$ equation. Values for C for various cases are shown as follows.

♦



PUMP POWER EQUATION

$$\dot{W} = Q\gamma h/\eta, \text{ where}$$

Q = quantity of flow (m^3/s or cfs),

h = head (m or ft) the fluid has to be lifted,

η = efficiency, and

$\dot{W}$ = power (watts or ft-lbf/sec).

Additonal information on fans, pumps, and compressors is included in the **MECHANICAL ENGINEERING** section of this handbook.

THE IMPULSE-MOMENTUM PRINCIPLE

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

$$\Sigma \boldsymbol{F} = Q_2\rho_2\boldsymbol{V}_2 - Q_1\rho_1\boldsymbol{V}_1, \text{ where}$$

$\Sigma \boldsymbol{F}$ = the resultant of all external forces acting on the control volume,

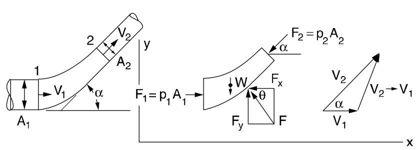
$Q_1\rho_1\boldsymbol{V}_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and

$Q_2\rho_2\boldsymbol{V}_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.

Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipe line may be computed using the impulse-momentum principle.

•



$$p_1A_1 - p_2A_2\cos\alpha - F_x = Q\rho\ (V_2\cos\alpha - V_1)$$

$$F_y - W - p_2A_2\sin\alpha = Q\rho\ (V_2\sin\alpha - 0), \text{ where}$$

$\boldsymbol{F}$ = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign), F_x and F_y are the x-component and y-component of the force,

p = the internal pressure in the pipe line,

A = the cross-sectional area of the pipe line,

W = the weight of the fluid,

V = the velocity of the fluid flow,

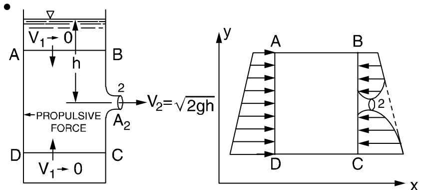
α = the angle the pipe bend makes with the horizontal,

ρ = the density of the fluid, and

Q = the quantity of fluid flow.

Jet Propulsion

•



$$F = Q\rho(V_2 - 0)$$

$$F = 2\gamma hA_2, \text{ where}$$

F = the propulsive force,

γ = the specific weight of the fluid,

h = the height of the fluid above the outlet,

A_2 = the area of the nozzle tip,

Q = $A_2\sqrt{2gh}$, and

V_2 = $\sqrt{2gh}$.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, John Wiley & sons, Inc., 1980. Diagram reprinted by permission of William Bober & Richard A. Kenyon.

• Vennard, J.K., *Elementary Fluid Mechanics*, J.K. Vennard, 1954. Diagrams reprinted by permission of John Wiley & Sons, Inc.

Deflectors and Blades

Fixed Blade

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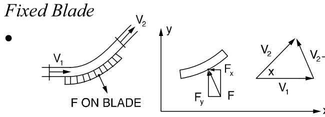
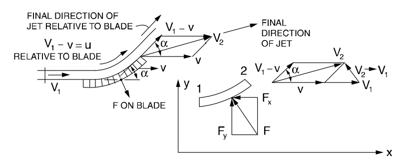


$$-F_x = Q\rho(V_2\cos\alpha - V_1)$$

$$F_y = Q\rho(V_2\sin\alpha - 0)$$

Moving Blade

•



$$-F_x = Q\rho(V_{2x} - V_{1x})$$

$$= -Q\rho(V_1 - v)(1 - \cos\alpha)$$

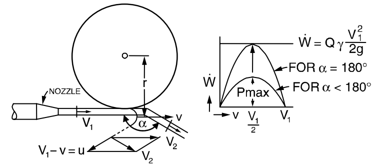
$$F_y = Q\rho(V_{2y} - V_{1y})$$

$$= +Q\rho(V_1 - v)\sin\alpha, \text{ where}$$

v = the velocity of the blade.

Impulse Turbine

•



$$\dot{W} = Q\rho\ (V_1 - v)(1 - \cos\alpha)\ v, \text{ where}$$

$\dot{W}$ = power of the turbine.

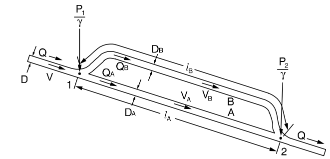
$$\dot{W}_{max} = Q\rho\ (V_1^2/4)(1 - \cos\alpha)$$

When $\alpha = 180°$,

$$\dot{W}_{max} = (Q\rho V_1^2)/2 = (Q\gamma V_1^2)/2g$$

MULTIPATH PIPELINE PROBLEMS

•



The same head loss occurs in each branch as in the combination of the two. The following equations may be solved simultaneously for V_A and V_B:

$$h_L = f_A\frac{l_A}{D_A}\frac{V_A^2}{2g} = f_B\frac{l_B}{D_B}\frac{V_B^2}{2g}$$

$$\left(\pi D^2/4\right)V = \left(\pi D_A^2/4\right)V_A + \left(\pi D_B^2/4\right)V_B$$

The flow Q can be divided into Q_A and Q_B when the pipe characteristics are known.

OPEN-CHANNEL FLOW AND/OR PIPE FLOW

Manning's Equation

$$V = (k/n)R^{2/3}S^{1/2}, \text{ where}$$

k = 1 for SI units,

k = 1.486 for USCS units,

V = velocity (m/s, ft/sec),

n = roughness coefficient,

R = hydraulic radius (m, ft), and

S = slope of energy grade line (m/m, ft/ft).

Hazen-Williams Equation

$$V = k_1CR^{0.63}S^{0.54}, \text{ where}$$

C = roughness coefficient,

k_1 = 0.849 for SI units, and

k_1 = 1.318 for USCS units.

Other terms defined as above.

• Vennard, J.K., *Elementary Fluid Mechanics*, J.K. Vennard, 1954. Diagrams reprinted by permission of John Wiley & Sons, Inc.

FLUID MEASUREMENTS

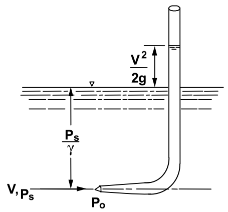
The Pitot Tube – From the stagnation pressure equation for an *incompressible fluid*,

$$V=\sqrt{(2/\rho)(p_o-p_s)}=\sqrt{2g(p_o-p_s)/\gamma}\text{ , where}$$

V = the velocity of the fluid,

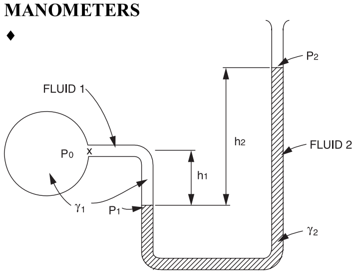
p_o = the stagnation pressure, and

p_s = the static pressure of the fluid at the elevation where the measurement is taken.

• 

For a *compressible fluid*, use the above incompressible fluid equation if the mach number ≤ 0.3.

MANOMETERS

♦ 

For a simple manometer,

$$p_o = p_2 + \gamma_2 h_2 - \gamma_1 h_1$$

If $h_1 = h_2 = h$

$$p_o = p_2 + (\gamma_2 - \gamma_1)h = p_2 + (\rho_2 - \rho_1)gh$$

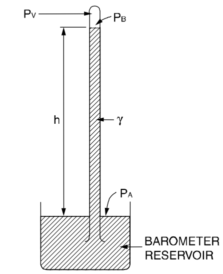
Note that the difference between the two densities is used.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, John Wiley & Sons, Inc., 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

• Vennard, J.K., *Elementary Fluid Mechanics*, J.K. Vennard, 1954. Diagrams reprinted by permission of John Wiley & Sons, Inc.

Another device that works on the same principle as the manometer is the simple barometer.

$$p_{atm} = p_A = p_v + \gamma h = p_B + \gamma h$$

♦ 

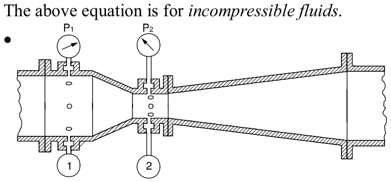
p_v = vapor pressure of the barometer fluid

Venturi Meters

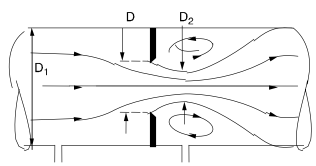
$$Q=\frac{C_vA_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1}{\gamma}+z_1-\frac{p_2}{\gamma}-z_2\right)}\text{, where}$$

C_v = the coefficient of velocity.

The above equation is for *incompressible fluids*.

• 

Orifices The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by C_cA.

• 

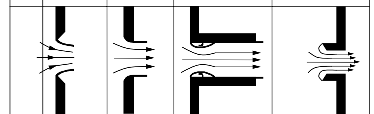
$$Q=CA\sqrt{2g\left(\frac{p_1}{\gamma}+z_1-\frac{p_2}{\gamma}-z_2\right)}$$

where C, the *coefficient of the meter*, is given by

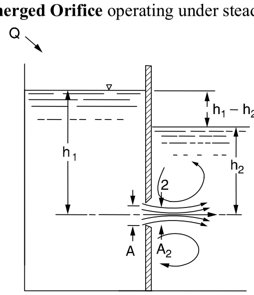
$$C=\frac{C_vC_c}{\sqrt{1-C_c^2(A/A_1)^2}}$$

♦ ORIFICES AND THEIR NOMINAL COEFFICIENTS

	SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
C	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98



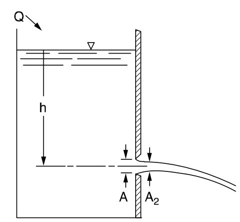
Submerged Orifice operating under steady-flow conditions:

• 

$$Q = A_2V_2 = C_cC_vA\sqrt{2g(h_1-h_2)}$$
$$= CA\sqrt{2g(h_1-h_2)}$$

in which the product of C_c and C_v is defined as the *coefficient of discharge* of the orifice.

Orifice Discharging Freely into Atmosphere

• 

$$Q = CA\sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, John Wiley & Sons, Inc., 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

• Vennard, J.K., *Elementary Fluid Mechanics*, J.K. Vennard, 1954. Diagrams reprinted by permission of John Wiley & Sons, Inc.

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve n variables is equal to the number $(n - \bar{r})$, where $\bar{r}$ is the number of basic dimensions (i.e., M, L, T) needed to express the variables dimensionally.

SIMILITUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically*, *kinematically*, and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_p}\right]_p = \left[\frac{F_I}{F_p}\right]_m = \left[\frac{\rho V^2}{p}\right]_p = \left[\frac{\rho V^2}{p}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{Vl\rho}{\mu}\right]_p = \left[\frac{Vl\rho}{\mu}\right]_m = [\mathrm{Re}]_p = [\mathrm{Re}]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{V^2}{lg}\right]_p = \left[\frac{V^2}{lg}\right]_m = [\mathrm{Fr}]_p = [\mathrm{Fr}]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho V^2}{E_v}\right]_p = \left[\frac{\rho V^2}{E_v}\right]_m = [\mathrm{Ca}]_p = [\mathrm{Ca}]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l V^2}{\sigma}\right]_p = \left[\frac{\rho l V^2}{\sigma}\right]_m = [\mathrm{We}]_p = [\mathrm{We}]_m$$

where

the subscripts p and m stand for *prototype* and *model* respectively, and

F_I = inertia force,
F_P = pressure force,
F_V = viscous force,
F_G = gravity force,
F_E = elastic force,
F_T = surface tension force,
Re = Reynolds number,
We = Weber number,
Ca = Cauchy number,
Fr = Froude number,
l = characteristic length,
V = velocity,
ρ = density,
σ = surface tension,
E_v = bulk modulus,
μ = dynamic viscosity,
ν = kinematic viscosity,
p = pressure, and
g = acceleration of gravity.

$$\mathrm{Re} = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

PROPERTIES OF WATER[f] SI METRIC UNITS

Temperature °C	Specific Weight[a], γ, kN/m³	Density[a], ρ, kg/m³	Viscosity[a], Pa·s	Kinematic Viscosity[a], m²/s	Vapor Pressure[e], p_v, kPa
0	9.805	999.8	0.001781	0.000001785	0.61
5	9.807	1000.0	0.001518	0.000001518	0.87
10	9.804	999.7	0.001307	0.000001306	1.23
15	9.798	999.1	0.001139	0.000001139	1.70
20	9.789	998.2	0.001002	0.000001003	2.34
25	9.777	997.0	0.000890	0.000000893	3.17
30	9.764	995.7	0.000798	0.000000800	4.24
40	9.730	992.2	0.000653	0.000000658	7.38
50	9.689	988.0	0.000547	0.000000553	12.33
60	9.642	983.2	0.000466	0.000000474	19.92
70	9.589	977.8	0.000404	0.000000413	31.16
80	9.530	971.8	0.000354	0.000000364	47.34
90	9.466	965.3	0.000315	0.000000326	70.10
100	9.399	958.4	0.000282	0.000000294	101.33

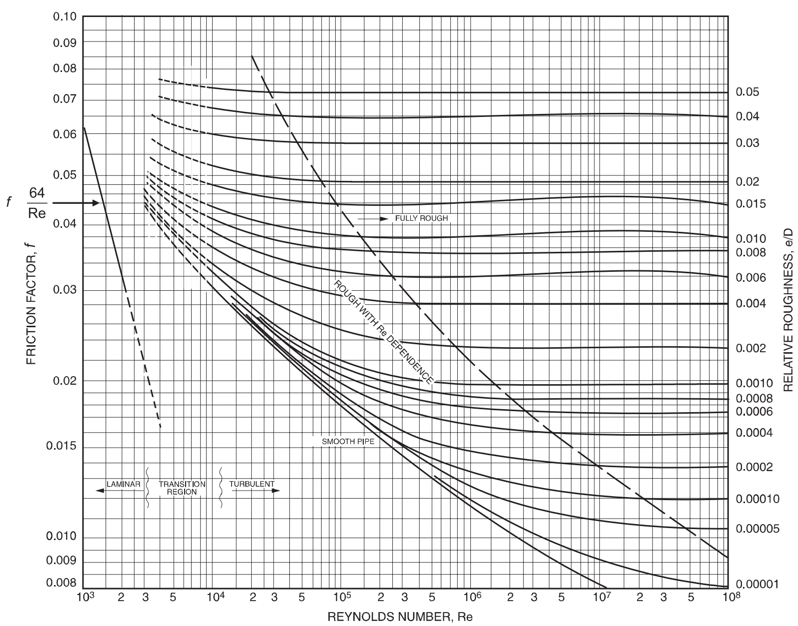
[a]From "Hydraulic Models," *A.S.C.E. Manual of Engineering Practice*, No. 25, A.S.C.E., 1942.
[e]From J.H. Keenan and F.G. Keyes, *Thermodynamic Properties of Steam*, John Wiley & Sons, 1936.
[f]Compiled from many sources including those indicated: *Handbook of Chemistry and Physics*, 54th ed., The CRC Press, 1973, and *Handbook of Tables for Applied Engineering Science*, The Chemical Rubber Co., 1970.
Vennard, J.K. and Robert L. Street, *Elementary Fluid Mechanics*, John Wiley & Sons, Inc., 1954.

PROPERTIES OF WATER ENGLISH UNITS

Temperature (°F)	Specific Weight γ (lb/ft³)	Mass Density ρ (lb • sec²/ft⁴)	Absolute Viscosity μ (× 10⁻⁵ lb • sec/ft²)	Kinematic Viscosity ν (× 10⁻⁵ ft²/sec)	Vapor Pressure p_v (psi)
32	62.42	1.940	3.746	1.931	0.09
40	62.43	1. 940	3.229	1.664	0.12
50	62.41	1.940	2.735	1.410	0.18
60	62.37	1.938	2.359	1.217	0.26
70	62.30	1.936	2.050	1.059	0.36
80	62.22	1.934	1.799	0.930	0.51
90	62.11	1.931	1.595	0.826	0.70
100	62.00	1.927	1.424	0.739	0.95
110	61.86	1.923	1.284	0.667	1.24
120	61.71	1.918	1.168	0.609	1.69
130	61.55	1.913	1.069	0.558	2.22
140	61.38	1.908	0.981	0.514	2.89
150	61.20	1.902	0.905	0.476	3.72
160	61.00	1.896	0.838	0.442	4.74
170	60.80	1.890	0.780	0.413	5.99
180	60.58	1.883	0.726	0.385	7.51
190	60.36	1.876	0.678	0.362	9.34
200	60.12	1.868	0.637	0.341	11.52
212	59.83	1.860	0.593	0.319	14.7

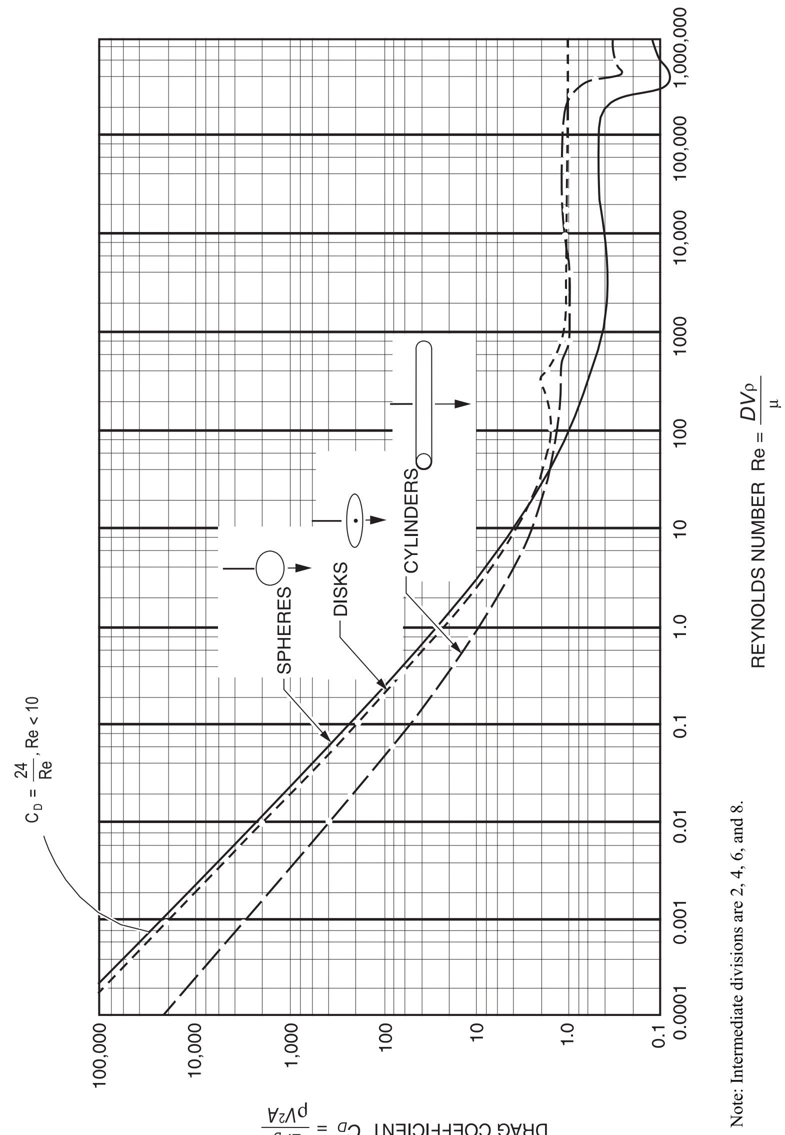
MOODY (STANTON) DIAGRAM

	e, (ft)	e, (mm)
Riveted steel	l0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015



From ASHRAE (The American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.)

DRAG COEFFICIENTS FOR SPHERES, DISKS, AND CYLINDERS



Note: Intermediate divisions are 2, 4, 6, and 8.