$$\left[\begin{array}{c|c} n\,\underline{A}_{11} & \underline{A}_{12} \\ \hline A_{21} & \underline{0} \end{array}\right] \times \left[\begin{array}{c} \underline{\Delta Q} \\ \hline \underline{\Delta H} \end{array}\right] = \left[\begin{array}{c} -\underline{dE} \\ \hline -\underline{dq} \end{array}\right]$$

For our example (assume D-W):

$$\left[\begin{array}{ccccccc|ccccc}
2K_1|Q_1| & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\
0 & 2K_2|Q_2| & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\
0 & 0 & 2K_3|Q_3| & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2K_4|Q_4| & 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\
0 & 0 & 0 & 0 & 2K_5|Q_5| & 0 & 0 & 0 & 0 & -1 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & 2K_6|Q_6| & 0 & 0 & 0 & 0 & -1 & +1 \\
0 & 0 & 0 & 0 & 0 & 0 & 2K_7|Q_7| & -1 & 0 & 0 & +1 & 0 \\
\hline
+1 & 0 & +1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & & & & 0 \\
0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & & & & 0 \\
0 & 0 & 0 & 0 & +1 & -1 & +1 & 0 & & & & 0 \\
0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \times \left[\begin{array}{c}
\Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \\ \Delta Q_6 \\ \Delta Q_7 \\ \hline \Delta H_C \\ \Delta H_D \\ \Delta H_E \\ \Delta H_F \\ \Delta H_G
\end{array}\right] =$$

$$\left[\begin{array}{c}
-(-H_A + H_C + K_1 Q_1|Q_1|) \\
-(-H_B + H_D + K_2 Q_2|Q_2|) \\
-(-H_D + H_C + K_3 Q_3|Q_3|) \\
-(-H_D + H_E + K_4 Q_4|Q_4|) \\
-(-H_E + H_F + K_5 Q_5|Q_5|) \\
-(-H_F + H_G + K_6 Q_6|Q_6|) \\
-(-H_C + H_F + K_7 Q_7|Q_7|) \\
\hline
-(Q_1 + Q_3 - Q_7 - D_C) \\
-(Q_2 - Q_3 - Q_4 - D_D) \\
-(Q_4 - Q_5 - D_E) \\
-(Q_5 - Q_6 + Q_7 - D_F) \\
-(Q_6 - D_G)
\end{array}\right]$$

In each iteration, find deltas from:

$$\begin{bmatrix} \underline{\Delta Q} \\ \hline \underline{\Delta H} \end{bmatrix} = \left[\begin{array}{c|c} n\,\underline{A}_{11} & \underline{A}_{12} \\ \hline \underline{A}_{21} & \underline{0} \end{array}\right]^{-1} \cdot \begin{bmatrix} -\underline{dE} \\ \hline -\underline{dg} \end{bmatrix}$$

Then,

$$\begin{bmatrix} {}_{m+1}\underline{Q} \\ \hline {}_{m+1}\underline{H} \end{bmatrix} = \begin{bmatrix} {}_{m}\underline{Q} \\ \hline {}_{m}\underline{H} \end{bmatrix} + \begin{bmatrix} \underline{\Delta Q} \\ \hline \underline{\Delta H} \end{bmatrix}$$

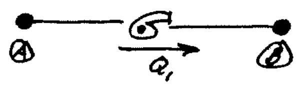
Procedure:
1. From network topology, form $\underline{A}_{21}$ and $\underline{A}_{12}$ matrices.
2. From pipe chars. and choice of head loss formula, form $\underline{A}_{11}$ matrix.
3. Guess initial values of all Q and H values (e.g. "1" for Q and some fgn value for H).
4. Calculate $\left[\begin{array}{c|c} n\,\underline{A}_{11} & \underline{A}_{12} \\ \hline \underline{A}_{21} & \underline{0} \end{array}\right]^{-1}$ based on this iteration's Q values.
5. Calculate $\begin{bmatrix} -\underline{dE} \\ \hline -\underline{dg} \end{bmatrix}$ based on this iteration's Q and H values.
6. Calculate $\begin{bmatrix} \underline{\Delta Q} \\ \hline \underline{\Delta H} \end{bmatrix}$ and then $\begin{bmatrix} {}_{m+1}\underline{Q} \\ \hline {}_{m+1}\underline{H} \end{bmatrix}$.
7. If $\begin{bmatrix} \Delta Q \\ \hline \Delta H \end{bmatrix}$ satisfies convergence criterion, stop.

   Else, iterate ④ → ⑦.

→ Look at Excel file with coded example.

Pumps in Gradient Method

The pipe equation for



becomes:

$$H_A + \left(AQ_1^2 + BQ + H_c\right) - H_B - K_1 Q_1 |Q_1|^{n-1} = 0$$

~ or ~

$$-H_A + H_B + K_1 Q_1 |Q_1|^{n-1} - \left(AQ_1^2 + BQ_1 + H_c\right) = 0 = F_{P1}$$

Which alters Newton method implementation:

$$\frac{\partial F_{P1}}{\partial Q_1} = n K_1 |Q_1|^{n-1} - 2AQ_1 - B$$

and the element in the $\underline{A}_{11}$ matrix will become

$$\begin{bmatrix} K_1 |Q_1|^{n-1} + \frac{-2AQ_1 - B}{n} & 0 & \cdots & \\ 0 & K_2 |Q_2|^{n-1} & 0 & \cdots \\ \vdots & 0 & \ddots & \\ \vdots & \vdots & & \end{bmatrix}$$

since $\underline{A}_{11}$ gets multiplied by n.