1. **Summer Day**

\[ S = 21.0 \text{ mJ/m}^2\text{-day} \]
\[ S_r = 0.08 \cdot 21.0 = 1.7 \text{ mJ/m}^2\text{-day} \]

\[ L = 0 \cdot T_{air}^4 \cdot E_{am} = 0 \cdot T_{air}^4 \left(0.740 + 0.0049e\right) \quad T_{air} = 80^\circ F = 26.7^\circ C = 293.8 K \]
\[ e_g = 35.1 \text{ mb at } T = 26.7^\circ C \quad \text{(from Table K2.1, H of H)} \]
\[ e = 0.05 \cdot 35.1 = 22.8 \text{ mb} \]

\[ L = (4.903 \cdot 10^{-9}) \cdot (299.8)^4 \cdot (0.740 + 0.0049 (22.8)) = 33.7 \text{ mJ/m}^2\text{-day} \]
\[ L_r = 0.08 \cdot 33.7 = 1.0 \text{ mJ/m}^2\text{-day} \]

\[ M = 0 \cdot T_{in}^4 \cdot E_o \quad T_{in} = 75^\circ F = 23.9^\circ C = 297.0 K \]
\[ m = (4.903 \cdot 10^{-9}) \cdot (297.0)^4 \cdot (0.97) = 37.0 \text{ mJ/m}^2\text{-day} \]

\[ Q_{in} = C_p \left( u \cdot V_{in} \cdot T_{in} \right) \quad T_{in} = 68^\circ F = 20.0^\circ C = 293.2 K \]
\[ Q_{in} = (4.184 \cdot 10^{-3})(1000)(20)(293.2) = 92,068 \text{ mJ} \]

\[ Q_{out} = C_p \left( u \cdot V_{out} \cdot T_{out} \right) = (4.184 \cdot 10^{-3})(1000)(125)(293.2) \]
\[ Q_{out} = 155,435 \text{ mJ} \]

\[ \Delta Q = C_p \left( u \cdot V_{in} \cdot T_{in} - u \cdot V_{out} \cdot T_{out} \right) \]

Assume \( T_{end} = T_{start} = T_{in} \) \; \( S_{end} - S_{start} = \Delta S = V_{in} - V_{out} \)
\[ \Delta Q = (4.184 \cdot 10^{-3})(1000)(293.2) \left[20 - 125\right] = -62174 \text{ mJ} \]
1. (continued)

\[ \lambda = 2.501 - 0.002361 \times (23.9) = 2.44 \text{ m}^2/\text{kg} \]

\[ B = 0.411 \frac{P}{1000} \frac{T_{\text{w}} - T_{\text{air}}}{e_s(T_{\text{w}}) - e_{\text{air}}} \]

\[ e_s(T_{\text{w}} = 23.9^\circ\text{C}) = 2.968 \text{ kPa} = 29.68 \text{ mb} \]

\[ B = 0.411 \frac{0.95}{1000} \frac{(23.9^\circ\text{C}) - (26.7^\circ\text{C})}{(29.68 \text{ mb}) - (22.8 \text{ mb})} = -0.2487 \]

\[ E = \frac{(21.0 - 1.7) + (33.7 - 1.0) - (37.0) + \frac{92068 - 155435 + 62174}{2250 \cdot (1 \text{ day})}}{(2.44) \cdot (1000) (1 + 0.2487)} \]

\[ E = 7.89 \times 10^{-3} \text{ m/day} = 7.9 \text{ mm/day} = 0.31 \text{ in/day} \]

2. Winter Day

\[ S = 13.0 \text{ m}^3/\text{m}^2 \cdot \text{day} \]

\[ S_r = 0.08 \times 13.0 = 1.0 \text{ m}^3/\text{m}^2 \cdot \text{day} \]

\[ L \triangleq e_s(T = 50^\circ\text{F} = 10^\circ\text{C}) = 12.28 \text{ mb} \quad e = 12.28 \times 0.75 = 9.21 \text{ mb} \]

\[ L = (4.903 \times 10^{-9}) (283.2 \text{ K})^4 (0.740 + 0.0049 (9.21)) = 24.8 \text{ m}^2/\text{m}^2 \cdot \text{day} \]

\[ L_r = 0.03 \times 24.8 = 0.7 \text{ m}^2/\text{m}^2 \cdot \text{day} \]

\[ M \triangleq T_{\text{w}} = 61^\circ\text{F} = 16.1^\circ\text{C} = 283.5 \text{ K} \]

\[ M = (4.903 \times 10^{-9}) (283.5 \text{ K})^4 (0.97) = 33.3 \text{ m}^2/\text{m}^2 \cdot \text{day} \]
\[ Q_{in} = C_p \rho w (\frac{\Delta T}{10}) \Rightarrow T_{in} = 55^\circ F = 12.8^\circ C = 285.9 K \]
\[ Q_{in} = (4.1868 \times 10^{-3})(1000)(285.9)(132) = 158005 \text{ MJ} \]
\[ Q_{out} = (4.1868 \times 10^{-3})(1000)(289.3)(52) = 62935 \text{ MJ} \]
\[ \Delta Q = C_p \rho w T \Delta S = (4.1868 \times 10^{-3})(1000)(289.3)(132 - 52) = 96899 \text{ MJ} \]
\[ \lambda = 2.501 - 0.002361(14.1) = 2.46 \text{ mJ/kg} \]
\[ B \downarrow c_s \left( T_o = 14.1^\circ C \right) = 1.331 \text{ kPa} = 19.31 \text{ mb} \]
\[ B = 0.611 \frac{(1000)}{1000} \frac{(16.1 - 10)}{(18.31 - 9.21)} = 0.4096 \]
\[ E = \frac{(13.0 - 1.0) + (24.8 - 0.9) - (33.3) + \frac{158005 - 62935 - 96899}{(2250)(1 \text{ day})}}{(2.46)(1000)(1 + 0.4096)} \]
\[ E = 5.67 \times 10^{-4} \text{ m/day} = 0.57 \text{ mm/day} = 0.02 \text{ in/day} \]
3. **Meyer - Summer Day**

From calculations in #1: 

\[ e_s(T_w) = 29.68 \text{ mb} = 0.879 \text{ in Hg} \]

\[ e = 22.8 \text{ mb} = 0.675 \text{ in Hg} \]

Assume \( W_2 = W_2 \)

\[ E = (0.36) \left( 0.879 - 0.675 \right) \left( 1 + \frac{8}{10} \right) = 0.13 \text{ in/day} \]

**Meyer - Winter Day**

\[ e_s(T_w) = 18.31 \text{ mb} = 0.542 \text{ in Hg} \]

\[ e = 9.21 \text{ mb} = 0.273 \text{ in Hg} \]

\[ E = (0.36) \left( 0.542 - 0.273 \right) \left( 1 + \frac{12}{10} \right) = 0.21 \text{ in/day} \]

Compare winter to summer here and to Energy Budget estimates!

4. **Penman - Summer**

\[ W_2 = 8 \text{ m/hr} = 3.58 \text{ m/s} \]

For \( T_{air} = 26.7 \degree C \), \( \Delta = 0.206 \text{ kbp/k} \), \( \gamma = 0.0672 \text{ kbp/k} \)

All other quantities were calculated earlier:

\[ E = \frac{\Delta}{\Delta + \gamma} \left( R_n + \frac{Q_{in} - Q_{out} - \Delta Q}{Aw + t} \right) + \frac{\gamma}{\Delta + \gamma} \frac{(6.43 \cdot 10^{-4}) (1 + 0.536 W_2)(e_e - e)}{\lambda} \]

\[ E = \frac{0.206}{0.206 + 0.0672} \left[ \frac{21.1 - 0.7 + 83.7 - 1.0}{2.258} + \frac{92068 - 155435 - 62174}{2258 \cdot 1} \right] + \]

\[ \frac{0.0672}{0.206 + 0.0672} \left[ \frac{6.43 \cdot 10^{-4} \cdot (1 + 0.536 (3.58))(29.68 - 22.8)}{2.44} \right] = \]
\[ E = 1.72 \times 10^{-2} \text{ m/day} = 17.2 \text{ mm/day} = 0.68 \text{ in/day} \]

**Penman – Winter**

\[ E = \frac{0.082}{0.082 + 0.0661} \left[ \left( 13.0 - 1.0 + 24.8 - 0.9 \right) + \frac{1578.05 - 622.85 - 96.389}{2250 \text{ (1/day)}} \right] + \]

\[ \frac{0.0661}{0.082 + 0.0661} \left[ \frac{(6.48 \times 10^{-4})(1 + 0.53)(5.47)(18.31 - 5.21)}{2.46} \right] \]

\[ E = 1.21 \times 10^{-2} \text{ m/day} = 12.1 \text{ mm/day} = 0.48 \text{ in/day} \]
5. Pan A, surrounded by dry, bare area "Case B"
   upwind fetch ≈ 250m, RH = 55%, Wind = 11 mi/hr = 4.9 m/s
   Correction factor = 0.59

Pan B, Case B, upwind fetch ≈ 90m, Corr. factor = 0.61

Pan C, surrounded by green, watered crop, "Case A"
   upwind fetch ≈ 200m, Screened, so increase by about 10-20% (use 15%)
   corr. factor = 0.76 \times 1.15 = 0.87

Pan D, Case A, upwind fetch ≈ 150m, Screened
   corr. factor = 0.75 \times 1.15 = 0.86

Pan E, Goats have been drinking water; Disregard.

<table>
<thead>
<tr>
<th>Pan</th>
<th>Measurement (in)</th>
<th>Corrected Estimate (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.55</td>
<td>0.32</td>
</tr>
<tr>
<td>B</td>
<td>0.47</td>
<td>0.29</td>
</tr>
<tr>
<td>C</td>
<td>0.35</td>
<td>0.30</td>
</tr>
<tr>
<td>D</td>
<td>0.37</td>
<td>0.32</td>
</tr>
<tr>
<td>E</td>
<td>8.62</td>
<td></td>
</tr>
</tbody>
</table>
6. \( \phi = 0.435 \), \( \psi = 21.3 \text{ cm} \), \( K_{\text{sat}} = 0.14 \text{ cm/hr} \), \( \Theta_{\text{init}} = 0.253 \),
\( i = 0.35 \text{ in/hr} = 0.89 \text{ cm/hr} \)

\[
F_p = \frac{(\phi - \Theta_{\text{init}}) \psi}{\left( \frac{i}{K_{\text{sat}}} - 1 \right)} = \frac{(0.435 - 0.253) (21.3 \text{ cm})}{\left( \frac{0.89 \text{ cm/hr}}{0.14 \text{ cm/hr}} - 1 \right)} = 0.72 \text{ cm}
\]

\[t_p = \frac{F_p}{i} = \frac{0.72 \text{ cm}}{0.89 \text{ cm/hr}} = 0.81 \text{ hrs} \quad \text{Time that runoff begins}
\]

Two ways possible to find runoff depth:

1. Analytical solution of integrated infiltration equation (preferred method):

   Solve for \( t_p' \) as time to infiltrate \( F_p \) under initially ponded conditions:

\[
t_p' = \frac{1}{(0.14 \text{ cm/hr})} \cdot \left\{ (0.72 \text{ cm}) - (21.3 \text{ cm})(0.435 - 0.253) \cdot \ln \left[ 1 + \frac{0.72 \text{ cm}}{(0.435 - 0.253)(21.3 \text{ cm})} \right] \right\}
\]

\[t_p' = 0.43 \text{ hrs}
\]

Then for \( t = 10 \text{ hrs} \), find \( F \) from:

\[
K_{\text{sat}} (t - t_p + t_p') = F - \psi (\phi - \Theta_{\text{init}}) \cdot \ln \left[ 1 + \frac{F}{(\phi - \Theta_{\text{init}}) \psi} \right]
\]

\[
(0.14)(10 - 0.81 + 0.43) = F - (21.3)(0.435 - 0.253) \cdot \ln \left[ 1 + \frac{F}{(0.435 - 0.253)(21.3 \text{ cm})} \right]
\]

\[1.3468 = F - 3.877 \cdot \ln \left[ 1 + \frac{F}{3.877} \right]
\]
6. (continued)

Solve for \( F \) using programmable calculator, Excel Solver, Matlab, Scienc with Isaac Newton's ghost, etc:

\[ F = 4.19 \text{ cm} = 1.65 \text{ in} \]

For 10 hr storm at \( i = 0.35 \text{ in/hr} \), \( P = 3.50 \text{ in} \)

\[ \text{Runoff} = P - F = 3.50 \text{ in} - 1.65 \text{ in} = 1.85 \text{ in} \]

(2) Recursive solution computing \( f \) and \( F \) and time intervals. Set up table of \( f \) and \( F \) versus time. At each time step, using \( F \) calculate \( f \) at that time and assume it is constant until next time step. Find amount of infiltration in time period and add to previous \( F \).

\[
\begin{array}{ccc}
 t (\text{hrs}) & F (\text{cm}) & f (\text{cm/hr}) \\
 0.81 & 0.72 & 0.89 \\
 1.0 & 0.89 & 0.75 \\
 2.0 & 1.45 & 0.47 \\
 3.0 & 2.11 & 0.40 \\
 4.0 & 2.51 & 0.36 \\
 5.0 & 2.87 & 0.33 \\
 6.0 & 3.20 & 0.31 \\
 7.0 & 3.51 & 0.29 \\
 8.0 & 3.80 & 0.28 \\
 9.0 & 4.08 & 0.27 \\
 10.0 & 4.35 & \\
\end{array}
\]

\[
F(t=10\text{hrs}) = 4.35\text{ cm} = 1.71\text{ in}
\]

\[
\text{Runoff} = 3.50 - 1.71 = 1.79\text{ in}
\]

Note that error is introduced here as \( \Delta t \) becomes larger. This method will overestimate \( F \).
7. Horton model, \( f_o = 22.5 \text{ cm/hr} = 8.86 \text{ in/hr} \), \( f_c = 0.7 \text{ cm/hr} = 0.28 \text{ in/hr} \), \( \beta = 2.05 \text{ hr}^{-1} \)

\[
f = f_c + (f_o - f_c) e^{-\beta t}
\]

<table>
<thead>
<tr>
<th>( t ) (hrs)</th>
<th>( f ) (in/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.86</td>
</tr>
<tr>
<td>0.5</td>
<td>3.36</td>
</tr>
<tr>
<td>1.0</td>
<td>1.38</td>
</tr>
<tr>
<td>1.5</td>
<td>0.68</td>
</tr>
<tr>
<td>2.0</td>
<td>0.42</td>
</tr>
<tr>
<td>2.5</td>
<td>0.33</td>
</tr>
<tr>
<td>3.0</td>
<td>0.30</td>
</tr>
</tbody>
</table>

We need to balance the infiltration curve so that at time \( t_p \) when \( i = f \), cumulative \( F \) and \( P \) to that point are equal.

To balance the \( f \) curve, we start it at time \( t_{f_0} \). At time \( t_p \), which is \( t_{f_0} \) after \( t_{f_0} \), ponding occurs and \( f = i \).

\[
f = i = f_c + (f_o - f_c) e^{-\beta t_f'}
\]

\[
t_f' = -\frac{1}{\beta} \ln \left[ \frac{i - f_c}{f_o - f_c} \right] = -\frac{1}{2.05} \ln \left( \frac{0.85 - 0.28}{8.86 - 0.28} \right) = 2.35 \text{ hrs}
\]

\[
F_p = f_c t_f' + \left( \frac{f_o - f_c}{\beta} \right) (1 - e^{-\beta t_f'}) = (0.28)(2.35) + \left( \frac{8.86 - 0.28}{2.05} \right)(1 - e^{-2.05(2.35)}) = 4.81 \text{ in}
\]
7. (continued)

Next, we want to find when cumulative $P$ would equal $F_p$.

$$P = i \cdot t \implies \text{for ponding, } P_p = i \cdot t_p \; ; \text{Thus, } t_p = \frac{P_p}{i} = \frac{F_p}{i}$$

$$t_p = \frac{4.81 \text{ in}}{0.35 \text{ in/hr}} = 13.74 \text{ hrs}$$

and the beginning of the balanced infiltration curve, $t_{p0}$, would be

$$t_{p0} = t_p - F_p' = 13.74 - 2.35 = 11.39 \text{ hrs}.$$

If the storm lasts only 10 hrs, then ponding would not occur before the end of the storm. Thus, runoff is zero under these conditions.
8. Interception, \( I = a + bP^n \)  

For pine, \( a = 0.05 \), \( b = 0.20 \), \( c = 0.5 \)

From hydrograph, total \( P = 3.275 \text{ in} \sim 3.28 \text{ in} \)

\[ I = 0.05 + 0.20(3.28)^{0.5} = 0.46 \text{ in} \]

At time \( 1.16 \text{ hrs} \), \( P = 0.46 \text{ in} \), so interception takes all precip up to that time.

Infiltration Curve - Horton, \( f_0 = 2.0 \text{ in/hr} \), \( f_i = 0.1 \text{ in/hr} \), \( B = 1.86 \text{ hr}^{-1} \)

<table>
<thead>
<tr>
<th>( t ) (hr)</th>
<th>( f ) (in/hr)</th>
<th>( F ) (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>0.85</td>
<td>0.47</td>
</tr>
<tr>
<td>1.0</td>
<td>0.40</td>
<td>0.96</td>
</tr>
<tr>
<td>1.5</td>
<td>0.22</td>
<td>1.18</td>
</tr>
<tr>
<td>2.0</td>
<td>0.15</td>
<td>1.20</td>
</tr>
<tr>
<td>2.5</td>
<td>0.12</td>
<td>1.34</td>
</tr>
<tr>
<td>3.0</td>
<td>0.11</td>
<td>1.32</td>
</tr>
<tr>
<td>3.5</td>
<td>0.10</td>
<td>1.37</td>
</tr>
</tbody>
</table>

\( t_0 \) is not constant, a simple analytical solution is difficult. It is probably easier to iterate on \( t_0 \) to find acceptable location for \( t_0 \). The goal of the iteration is to find \( t_0 \) such that infiltration before ponding and post-interception precipitation before ponding are equal. Graphically:

![Graphical representation of infiltration curve]
3. (continued)

\[
\ln \left( \frac{i - f_c}{f_0 - f_c} \right) \quad \text{Here, use } t_f' = \frac{-\ln \left( \frac{i - f_c}{f_0 - f_c} \right)}{-\rho} \quad \text{as in last problem}
\]

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$t_f$ (hr)</th>
<th>$t_p$ (hr)</th>
<th>$F_p$ (in)</th>
<th>$P_p$ (in)</th>
<th>post-interception</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.16</td>
<td>1.70</td>
<td>0.70</td>
<td>0.40</td>
<td>$F_p &gt; P_p$, so increase $t_f$</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>2.06</td>
<td>0.11</td>
<td>0.22</td>
<td>$F_p &lt; P_p$, so decrease $t_f$</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>2.0</td>
<td>0.67</td>
<td>0.64</td>
<td>close enough!</td>
</tr>
</tbody>
</table>

So, $t_p$ is at 2.0 hrs. Runoff begins.

Infiltration rate $f$ reaches a constant value of 0.10 in/hr ($f_c$) at 3.5 hours after $t_f$, which would be $3.5 + 1.5 = 5.0$ hrs.

Reinfall rate decreases to below 0.10 in/hr at time 5.5 hrs, so runoff would end then.

Total infiltration during storm would be sum of $F(t_f' = 4.0$ hrs) to Co + last 0.5 hrs of reinfall (when $i < f_c$).

\[
F(t_f' = 4.0$ hrs) = (0.1 \text{ in/hr})(4.0$ hrs) + \frac{(2.0 - 0.1 \text{ in/hr})}{(1.86 \text{ hr}^{-1})} (1 - e^{-0.26}(4)) = 1.42 \text{ in}
\]

Last 0.5 hrs of $P = (0.05 \text{ in/hr})(0.5$ hr) = 0.025 in $\sim 0.03$ in

Total Infiltration = 1.45 in

Losses = Interception $+$ Infiltration $+$ Depression Storage $= 0.46 \text{ in} + 1.45 \text{ in} + 0.05 \text{ in} = 1.96 \text{ in}$

Runoff = $P - \text{Losses} = 4.28 \text{ in} - 1.96 \text{ in} = 2.32 \text{ in}$

See next page for diagram of losses on hyetograph.
Problem 8 Hyetograph

Blue hatched area is $P_p$; Orange hatched area is $F_p$. In order to balance infiltration curve, $t_{fo}$ and $t_p$ must be found so that $P_p = F_p$. 
2. Given below is a storm hyetograph. If this storm falls on a watershed whose soil is a sandy clay, is covered by grass 10 inches high, and has depression storage capacity of 0.10 in, what will be the depth of runoff for this storm on this watershed? You should use the Green-Ampt model for your infiltration calculations and assume initial volumetric soil moisture content to be 0.250. (50 points)

\[- P = 1.5 \times 4 + 3 \times 4 + 1 \times 4 = 22 \text{in} \quad - 4 \text{pts}\]

\[\text{Interception: } a = 0.005 \times \frac{10}{12} = 0.00417 \quad ; \quad b = 0.08 \times \frac{10}{12} = 0.067 \quad , \quad n = 1.0 \quad ; \quad I = 0.00417 \times (8 + 0.067(22)) \quad - 6 \text{pts}\]

\[\text{I} = 1.48 \text{in} \quad \text{[From rainfall-hydrometric data]} \quad - 6 \text{pts}\]

\[\text{Infiltration: } \phi = 0.430 \quad ; \quad \gamma = 23.90 \text{cm} = 9.41 \text{in} \quad \text{[from ref. table]} \quad ; \quad \Theta \text{init} = 0.250 \quad - 4 \text{pts}\]

\[\text{Assume ponding occurs while } i = 1.5 \text{in/hr} \]

\[\frac{F_p}{(1.5 \text{in/hr} \quad - 1)} = 0.055 \text{in} \quad ; \quad t_p = \frac{0.055 \text{in}}{1.5 \text{in/hr}} = 0.037 \text{hr} \quad - 3 \text{pts}\]
(Work space for #2)

\[ t_p = \frac{1}{(0.047 \text{ in/hr})} \left\{ 0.055 \text{ in} - (9.41 \text{ in})(0.430 - 0.250) \right\} \ln\left[ 1 + \frac{0.055}{(9.41)(0.430 - 0.250)} \right] \]

\[ t_p = 0.019 \text{ hr} \]

4 pts

End of storm is \( t = 12 - \frac{1.5 \text{ in}}{1.5 \text{ in/hr}} = 11.01 \text{ hrs} \)

4 pts

Solve for \( F \):

\[ K_{sat}(t - t_p + t_p') = F - \frac{F}{\gamma (\phi - \Theta_{init})} \ln\left[ 1 + \frac{F}{\gamma (\phi - \Theta_{init})} \right] \]

\[ (0.047 \text{ in/hr})(11.01 - 0.037 + 0.019) \text{ hrs} = F - (9.41 \text{ in})(0.430 - 0.250) \ln\left[ 1 + \frac{F}{(9.41)(0.43 - 0.25)} \right] \]

\[ F = 1.69 \text{ in} \]

8 pts

Depr. Stor. = 0.10 \text{ in} \]

2 pts

\[ \Sigma \text{ Losses} = I + F + DS = 1.48 + 1.69 + 0.10 = 3.27 \text{ in} \]

5 pts

\[ \delta = P - \Sigma \text{ Losses} = 22 \text{ in} - 3.27 \text{ in} = 18.73 \text{ in} \]

6 pts