CVEN 627 – HW3 Solutions

(1) Dingman # 7-9

PET from Penman eqn (7-55): $\Delta = \frac{2508.3}{(T+237.3)^2} \cdot \exp\left(\frac{17.3\,T}{T+237.3}\right) = \frac{2508.3}{(20.7+237.3)^2} \cdot \exp\left[\frac{17.3(20.7)}{20.7+237.3}\right]$

$\Delta = 0.151 \frac{kPa}{K}$

$\rho_a = 1.220\ kg/m^3$ (constant, p. 273)

$c_a = 10^{-3} \frac{MJ}{kg \cdot K}$ (constant, p. 274)

(eqn 7-49) $C_{at} = \frac{u}{6.25 \cdot \left[\ln \frac{z_m - z_d}{z_0}\right]^2} = \frac{(2.48 \frac{m}{s})}{6.25 \cdot \left[\ln \frac{3.66 - (0.7)(0.50)\ m}{(0.1)(0.50)\ m}\right]^2} = 2.26 \cdot 10^{-2} \frac{m}{s}$

(eqn 7-4) $e_a^* = 0.611 \cdot \exp\left(\frac{17.3 \cdot T_a}{T_a + 237.3}\right) = 0.611 \cdot \exp\left(\frac{17.3(20.7)}{(20.7)+237.3}\right) = 2.45\ kPa$

(eqn 7-8) $\lambda_v = 2.50 - 2.36 \cdot 10^{-3} \cdot T = 2.50 - 2.36 \cdot 10^{-3}(20.7) = 2.45 \frac{MJ}{kg}$

(eqn 7-13) $\gamma = \frac{c_a \cdot P}{0.622\,\lambda_v} = \frac{10^{-3}(101.3)}{0.622(2.45)} = 0.0665 \frac{kPa}{K}$

eqn 7-55: $E = \frac{\Delta \cdot (K+L) + \rho_a\, c_a\, C_{at}\, e_a^*\,(1-W_a)}{\rho_w\, \lambda_v\, (\Delta + \gamma)}$

$$E = \frac{\left(0.151 \frac{kPa}{K}\right)\left(13.9 \frac{MJ}{m^2 \cdot d}\right) + \left(1.220 \frac{kg}{m^3}\right)\left(10^{-3} \frac{MJ}{kg \cdot K}\right)\left(2.26 \cdot 10^{-2} \frac{m}{s}\right)\left(2.45\ kPa\right)\left(1 - 0.49\right)\left(86400 \frac{s}{d}\right)}{\left(10^3 \frac{kg}{m^3}\right)\left(2.45 \frac{MJ}{kg}\right)\left(0.151 + 0.0665 \frac{kPa}{K}\right)}$$

$E = 9.52 \cdot 10^{-3} \frac{m}{d} = 9.52 \frac{mm}{d} = 0.375 \frac{in}{d}$

Using Penman-Monteith eqn (7-56): need to include $C_{leaf} = C^*_{leaf} \cdot f_K \cdot f_\rho \cdot f_T \cdot f_\theta$

- assume that K is such that $f_K(K) = 0.8$ (we can't really know K from the data given; Fig 7-13 is also based on instantaneous radiation).
- assume soil moisture deficit $\Delta\theta \approx 0$, so that $f_\theta(\Delta\theta) \approx 1.0$, which is consistent with finding "potential" ET
- for $T_a = 20.7°C$, Fig 7-13 gives $f_T(T_a) \cong 1.0$
- $\Delta e_a = e^*_a(1 - W_a) = (2.45)(1 - 0.49) = 1.25 \text{ kPa}$

From eqn D-8c: $\Delta \rho_v = \dfrac{2.17 \, \Delta e_a}{T_a} = \dfrac{2.17(1.25)}{(20.7 + 273.15)} = 0.00923 \dfrac{kg}{m^3} = 9.23 \dfrac{g}{m^3}$

$f_\rho(\Delta\rho_v) = 0.3$

$C_{leaf} = (0.0050)(0.8)(0.3)(1.0)(1.0) = 0.0012 \frac{m}{s}$

$C_{can} = f_s \cdot LAI \cdot C_{leaf} = (0.5) \cdot (4.5) \cdot 0.0012 = 0.0027 \frac{m}{s}$

(0.5: see p. 298)

(eqn 7-56)

$$ET = \frac{\Delta \cdot (K + L) + \rho_a \cdot c_a \cdot C_{at} \cdot e^*_a(1 - W_a)}{\rho_w \cdot \lambda_v \cdot \left[\Delta + \gamma\left(1 + \frac{C_{at}}{C_{can}}\right)\right]} = 2.67 \cdot 10^{-3} \frac{m}{d} = 2.67 \frac{mm}{d} = 0.105 \frac{in}{d}$$

Hamon eqn (7-63): $PET = 29.8 \cdot D \cdot \dfrac{e_a^*}{T_a + 273.2}$

$$PET = 29.8(14.8) \cdot \frac{(2.45)}{(20.7)+273.2}$$

$$PET = 3.68\ \frac{mm}{d} = 0.145\ \frac{in}{d}$$

Malmstrom eqn (7-64): $PET = 40.9 \cdot e_a^* = 40.9 \cdot (2.45) = 100.2\ \frac{mm}{month}$

$$= 3.34\ \frac{mm}{d} = 0.132\ \frac{in}{d}$$

Priestly-Taylor eqn (7-65): $PET = \dfrac{\alpha \cdot \Delta \cdot (K+L)}{\rho_w \cdot \lambda_v \cdot (\Delta + \gamma)}$

$$PET = \frac{1.26 \cdot (0.151)(13.9)}{(1000)(2.45)(0.151+0.0665)} = 4.96\ \frac{mm}{d} = 0.195\ \frac{in}{d}$$

(2) Soil: Clay Loam, Table 6-1: $\phi =$ ~~0.482~~ 0.476, $\psi_{ae} = -63.0\,cm$, $b = 8.52$

$$\theta_{fc} = (0.476)\left(\frac{|-63.0|}{340}\right)^{1/8.52} = 0.391 \quad \text{, from eqn 6-19}$$

$$\theta_{wp} = (0.476)\left(\frac{|-63.0|}{15000}\right)^{1/8.52} = 0.250 \quad \text{, from eqn 6-20}$$

Transition from stage 1 to 2 evap: $\theta_{trans} = \theta_{wp} + \frac{1}{4}(\theta_{fc} - \theta_{wp})$

$$= (0.250) + \frac{1}{4}(0.391 - 0.250) = 0.285$$

Over a 5 cm thick soil layer, this means that total evap. at transition will be:

$$E_{trans} = (\theta_{fc} - \theta_{trans})\,5\,cm = (0.391 - 0.285)\,5\,cm$$

$$= 0.529\ cm = F_1$$

Potential evap. rate from Eqn 7-33 should match what was computed earlier from eqn 7-55 (both are Penman eqn), but let's use eqn 7-33 here just to be sure:

$$E = \frac{\Delta\cdot(K+L) + \gamma\cdot K_E\cdot\rho_w\cdot\lambda_v\cdot u\cdot e_a^*\,(1-W_a)}{\rho_w\cdot\lambda_v\cdot(\Delta+\gamma)}$$

where
$$K_E = \frac{0.622\cdot\rho_a}{P\cdot\rho_w}\cdot\frac{1}{6.25\cdot\left[\ln\frac{z_m - z_d}{z_0}\right]^2} = \frac{0.622\cdot(1.220)}{(101.3)(10^3)}\cdot\frac{1}{6.25\cdot\left[\ln\frac{3.66 - 0.7\cdot0.50}{0.1\cdot0.50}\right]^2}$$

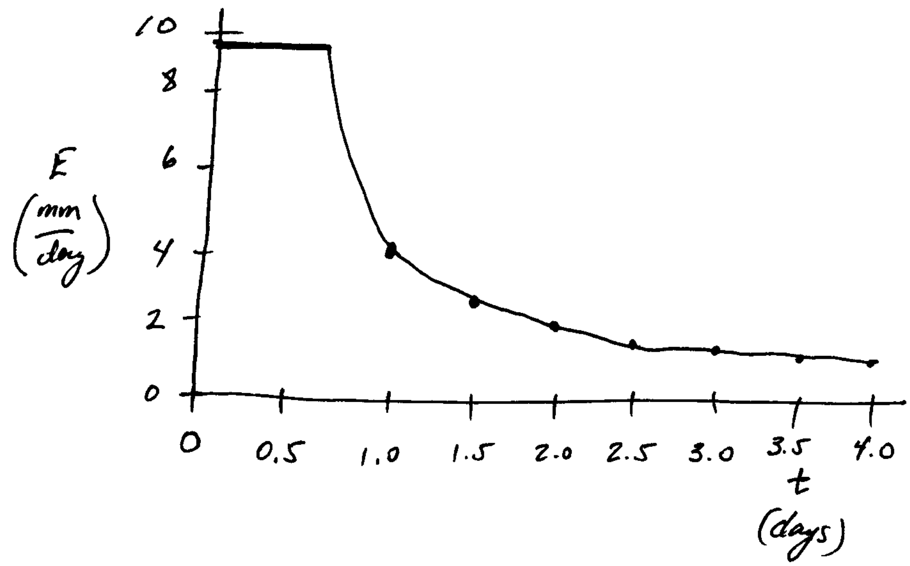
$$K_E = 6.818\cdot10^{-8}\ kPa^{-1}$$

$$E = \frac{(0.151)\cdot(13.9) + (0.0665)(6.818\cdot10^{-8})(10^3)(2.45)(2.48)(2.45)(1-0.49)(86400)}{(10^3)(2.45)(0.151+0.0665)} = 9.51\,\frac{mm}{day}\ \checkmark$$

Transition occurs at time t_1. From eqn 7-43: $t_1 = \frac{F_1}{\overline{E}_1} = \frac{5.29\ mm}{9.57\ \frac{mm}{day}} = 0.56\ day$

Stage 2 evap. from eqn 7-45: $E_2(t) = \frac{8}{\pi^2} \cdot \overline{E}_1 \cdot \frac{t_1}{t} = 0.811\ \frac{F_1}{t}$

t (days)	E_2 (mm/day)
1.0	4.29
1.5	2.86
2.0	2.15
2.5	1.72
3.0	1.43
3.5	1.23
4.0	1.07



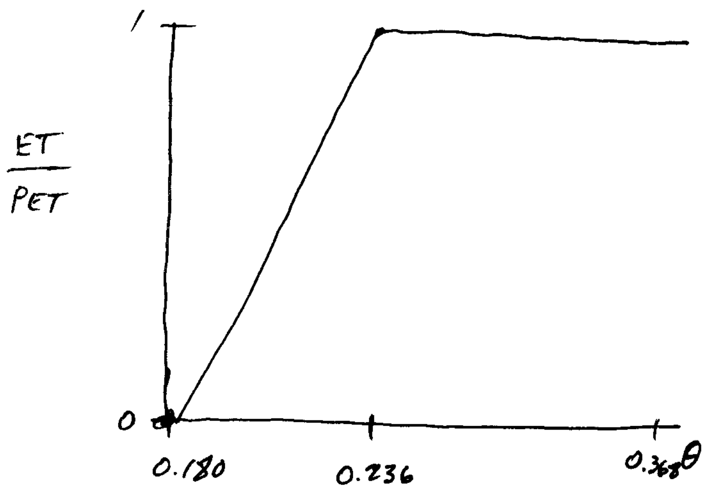
(3) Silt-loam soil, Table 6-1: $\phi = 0.485$, $\psi_{ae} = -78.6\,cm$, $b = 5.30$

$$\theta_{fc} = 0.485 \cdot \left(\frac{78.6}{340}\right)^{1/5.30} = 0.368$$

$$\theta_{wp} = 0.485 \left(\frac{78.6}{15000}\right)^{1/5.30} = 0.180$$

$$\theta_{rel} = \frac{\theta - \theta_{wp}}{\theta_{fc} - \theta_{wp}} \rightsquigarrow \theta = \theta_{wp} + \theta_{rel}(\theta_{fc} - \theta_{wp})$$

Transition at $\theta_{rel} = 0.3$: $\theta = (0.180) + (0.3)(0.368 - 0.180) = 0.236$



FOREST-DAY 1	SNOWMELT COMPUTATIONS					SnowMelt.xls
	Ripe snowpack, ground heat input neglected.					S.L. Dingman
		Input Values				*Physical Hydrology*, 2nd Ed.
Site:	Measurement height, *za* =	2.00	m			
	Roughness height, *zo* =	0.0020	m			
	Forest cover, *F* =	0.80				
	Albedo, *a* =	0.50	From Figure 5-23 with ripe snow			
	Snow density, ρ*s* =	400	kg/m^3			
Weather:	Clear-sky solar radiation, *Kcs* =	16.80	MJ/(m^2 day) - See Appendix E; SolarRad.xls			
	Cloud cover, *C* =	0.0	0 <= *C* <= 1			
	Air temp, *Ta* =	4.0	oC			
	Relative humidity, *Wa* =	0.50	0 <= *Wa* <= 1			
	Wind speed in open, *vao* =	5.00	m/s			
	Rain, *r* =	0	mm/day			
	Atmospheric Pressure, *P* =	100.0	kPa			
		Computed Values				
		(Positive = heat input)		**% Energy**	**% Water**	**Equation used:**
	Net solar radiation, *K* =	0.381	MJ/(m^2 day)	120.8%		5-27, 5-30, 5-33
	Vapor pressure, *ea* =	0.407	kPa			D-7
	Atmospheric emissivity, ε*at* =	0.935				5-40
	Net long-wave radiation, *L* =	-0.236	MJ/(m^2 day)	-74.8%		5-42
	Net radiation, *K* + *L* =	0.145	MJ/(m^2 day)	46.0%		
	Adjusted wind speed, *va* =	1.800	m/s			5-44
	Air density, ρ*a* =	1.253	kg/m^3			D-5
	Richardson No., *Ri* =	0.088				D-55
	Stability factor, Φ*M* =	1.842				Table D-5
	Stability factors, Φ*V*, Φ*H* =	1.842				Table D-5
	Sensible-heat transfer coefficient, *KH* =	0.365	(MJ s)/(m^3 K day)			D-50
	Latent-heat transfer coefficient, *KLE* =	5.575	(MJ s)/(m^3 kPa day)			D-43
	Sensible-heat transfer rate, *H* =	0.774	MJ/(m^2 day)	245.3%		D-52
	Latent-heat transfer rate, *LE* =	-0.603	MJ/(m^2 day)	-191.3%		D-45
	Evaporation =	0.244	mm/day		0.0%	D-45/(latent heat)
	Heat input from rain, *R* =	0.000	MJ/(m^2 day)	0.0%	0.0%	5-47a
	Total heat input rate, *S* =	0.315	MJ/(m^2 day)	100.0%		5-26
Energy Bal:	Total melt =	0.9	mm/day		100.0%	5-22
	Total ablation =	1.2	mm/day			melt + evap
	Total water output =	0.9	mm/day		100.0%	melt + rain + cond
Hybrid:	Total melt =	8.4	mm/day			5-61
	Total water output =	8.4	mm/day			melt + rain
Temp Index:	Total melt =	20.2	mm/day			5-60 (weighted)
	Total water output =	20.2	mm/day			melt + rain

FOREST-DAY 2	SNOWMELT COMPUTATIONS					SnowMelt.xls
	Ripe snowpack, ground heat input neglected.					S.L. Dingman
		Input Values				*Physical Hydrology*, 2nd Ed.
Site:	Measurement height, *za* =	2.00	m			
	Roughness height, *zo* =	0.0020	m			
	Forest cover, *F* =	0.80				
	Albedo, *a* =	0.50	From Figure 5-23 with ripe snow			
	Snow density, ρ*s* =	400	kg/m^3			
Weather:	Clear-sky solar radiation, *Kcs* =	16.80	MJ/(m^2 day) - See Appendix E; SolarRad.xls			
	Cloud cover, *C* =	1.0	0 <= *C* <= 1			
	Air temp, *Ta* =	4.0	oC			
	Relative humidity, *Wa* =	1.00	0 <= *Wa* <= 1			
	Wind speed in open, *vao* =	5.00	m/s			
	Rain, *r* =	10	mm/day			
	Atmospheric Pressure, *P* =	100.0	kPa			
		Computed Values				
		(Positive = heat input)		**% Energy**	**% Water**	**Equation used:**
	Net solar radiation, *K* =	0.131	MJ/(m^2 day)	4.7%		5-27, 5-30, 5-33
	Vapor pressure, *ea* =	0.814	kPa			D-7
	Atmospheric emissivity, ε*at* =	0.982				5-40
	Net long-wave radiation, *L* =	1.124	MJ/(m^2 day)	40.2%		5-42
	Net radiation, *K* + *L* =	1.255	MJ/(m^2 day)	44.9%		
	Adjusted wind speed, *va* =	1.800	m/s			5-44
	Air density, ρ*a* =	1.253	kg/m^3			D-5
	Richardson No., *Ri* =	0.088				D-55
	Stability factor, Φ*M* =	1.842				Table D-5
	Stability factors, Φ*V*, Φ*H* =	1.842				Table D-5
	Sensible-heat transfer coefficient, *KH* =	0.365	(MJ s)/(m^3 K day)			D-50
	Latent-heat transfer coefficient, *KLE* =	5.575	(MJ s)/(m^3 kPa day)			D-43
	Sensible-heat transfer rate, *H* =	0.774	MJ/(m^2 day)	27.7%		D-52
	Latent-heat transfer rate, *LE* =	0.600	MJ/(m^2 day)	21.5%		D-45
	Condensation =	0.243	mm/day		1.3%	D-45/(latent heat)
	Heat input from rain, *R* =	0.167	MJ/(m^2 day)	6.0%	53.7%	5-47a
	Total heat input rate, *S* =	2.796	MJ/(m^2 day)	100.0%		5-26
Energy Bal:	Total melt =	8.4	mm/day		45.0%	5-22
	Total ablation =	8.4	mm/day			melt + evap
	Total water output =	18.6	mm/day		100.0%	melt + rain + cond
Hybrid:	Total melt =	11.8	mm/day			5-61
	Total water output =	21.8	mm/day			melt + rain
Temp Index:	Total melt =	20.2	mm/day			5-60 (weighted)
	Total water output =	30.2	mm/day			melt + rain

FIELD-DAY 1		SNOWMELT COMPUTATIONS			SnowMelt.xls
		Ripe snowpack, ground heat input neglected.			S.L. Dingman
		Input Values			*Physical Hydrology*, 2nd Ed.
Site:	Measurement height, *za* =	2.00	m		
	Roughness height, *zo* =	0.0020	m		
	Forest cover, *F* =	0.00			
	Albedo, *a* =	0.50	From Figure 5-23 with ripe snow		
	Snow density, ρ*s* =	400	kg/m^3		
Weather:	Clear-sky solar radiation, *Kcs* =	16.80	MJ/(m^2 day) - See Appendix E; SolarRad.xls		
	Cloud cover, *C* =	0.0	0 <= *C* <= 1		
	Air temp, *Ta* =	4.0	oC		
	Relative humidity, *Wa* =	0.50	0 <= *Wa* <= 1		
	Wind speed in open, *vao* =	5.00	m/s		
	Rain, *r* =	0	mm/day		
	Atmospheric Pressure, *P* =	100.0	kPa		

		Computed Values				
		(Positive = heat input)		**% Energy**	**% Water**	**Equation used:**
	Net solar radiation, *K* =	8.694	MJ/(m^2 day)	360.6%		5-27, 5-30, 5-33
	Vapor pressure, *ea* =	0.407	kPa			D-7
	Atmospheric emissivity, ε*at* =	0.677				5-40
	Net long-wave radiation, *L* =	-7.705	MJ/(m^2 day)	-319.6%		5-42
	Net radiation, *K* + *L* =	0.989	MJ/(m^2 day)	41.0%		
	Adjusted wind speed, *va* =	5.000	m/s			5-44
	Air density, ρ*a* =	1.253	kg/m^3			D-5
	Richardson No., *Ri* =	0.011				D-55
	Stability factor, *ΦM* =	1.063				Table D-5
	Stability factors, *ΦV*, *ΦH* =	1.063				Table D-5
	Sensible-heat transfer coefficient, *KH* =	0.365	(MJ s)/(m^3 K day)			D-50
	Latent-heat transfer coefficient, *KLE* =	5.575	(MJ s)/(m^3 kPa day)			D-43
	Sensible-heat transfer rate, *H* =	6.455	MJ/(m^2 day)	267.8%		D-52
	Latent-heat transfer rate, *LE* =	-5.034	MJ/(m^2 day)	-208.8%		D-45
	Evaporation =	2.038	mm/day		0.0%	D-45/(latent heat)
	Heat input from rain, *R* =	0.000	MJ/(m^2 day)	0.0%	0.0%	5-47a
	Total heat input rate, *S* =	2.411	MJ/(m^2 day)	100.0%		5-26
Energy Bal:	Total melt =	7.2	mm/day		100.0%	5-22
	Total ablation =	9.3	mm/day			melt + evap
	Total water output =	7.2	mm/day		100.0%	melt + rain + cond
Hybrid:	Total melt =	11.0	mm/day			5-61
	Total water output =	11.0	mm/day			melt + rain
Temp Index:	Total melt =	13.8	mm/day			5-60 (weighted)
	Total water output =	13.8	mm/day			melt + rain

FOREST-DAY 2		SNOWMELT COMPUTATIONS			SnowMelt.xls
		Ripe snowpack, ground heat input neglected.			S.L. Dingman
		Input Values			*Physical Hydrology*, 2nd Ed.

Site:	Measurement height, *za* =	2.00	m			
	Roughness height, *zo* =	0.0020	m			
	Forest cover, *F* =	0.00				
	Albedo, *a* =	0.50	From Figure 5-23 with ripe snow			
	Snow density, ρ*s* =	400	kg/m^3			
Weather:	Clear-sky solar radiation, *Kcs* =	16.80	MJ/(m^2 day) - See Appendix E; SolarRad.xls			
	Cloud cover, *C* =	1.0	0 <= *C* <= 1			
	Air temp, *Ta* =	4.0	oC			
	Relative humidity, *Wa* =	1.00	0 <= *Wa* <= 1			
	Wind speed in open, *vao* =	5.00	m/s			
	Rain, *r* =	10	mm/day			
	Atmospheric Pressure, *P* =	100.0	kPa			
		Computed Values				
		(Positive = heat input)		**% Energy**	**% Water**	**Equation used:**
	Net solar radiation, *K* =	2.982	MJ/(m^2 day)	21.8%		5-27, 5-30, 5-33
	Vapor pressure, *ea* =	0.814	kPa			D-7
	Atmospheric emissivity, ε*at* =	0.912				5-40
	Net long-wave radiation, *L* =	-0.905	MJ/(m^2 day)	-6.6%		5-42
	Net radiation, *K* + *L* =	2.077	MJ/(m^2 day)	15.2%		
	Adjusted wind speed, *va* =	5.000	m/s			5-44
	Air density, ρ*a* =	1.253	kg/m^3			D-5
	Richardson No., *Ri* =	0.011				D-55
	Stability factor, *ΦM* =	1.063				Table D-5
	Stability factors, *ΦV*, *ΦH* =	1.063				Table D-5
	Sensible-heat transfer coefficient, *KH* =	0.365	(MJ s)/(m^3 K day)			D-50
	Latent-heat transfer coefficient, *KLE* =	5.575	(MJ s)/(m^3 kPa day)			D-43
	Sensible-heat transfer rate, *H* =	6.455	MJ/(m^2 day)	47.1%		D-52
	Latent-heat transfer rate, *LE* =	5.006	MJ/(m^2 day)	36.5%		D-45
	Condensation =	2.027	mm/day		3.8%	D-45/(latent heat)
	Heat input from rain, *R* =	0.167	MJ/(m^2 day)	1.2%	18.8%	5-47a
	Total heat input rate, *S* =	13.706	MJ/(m^2 day)	100.0%		5-26
Energy Bal:	Total melt =	41.0	mm/day		77.3%	5-22
	Total ablation =	41.0	mm/day			melt + evap
	Total water output =	53.1	mm/day		100.0%	melt + rain + cond
Hybrid:	Total melt =	14.2	mm/day			5-61
	Total water output =	24.2	mm/day			melt + rain
Temp Index:	Total melt =	13.8	mm/day			5-60 (weighted)
	Total water output =	23.8	mm/day			melt + rain

(5) I'll use Total Water Output as calculated by Energy Balance.

From Eqn 5-51 : $K_h^* = 0.0602 \cdot \exp(-0.00957 \cdot (400)) = 1.31 \cdot 10^{-3}$ m/s

From Fig 5-20 : $\phi = 0.58$, $\theta_{ret} = 0.013$

Rate of travel down through the snowpack from Eqn 5-52:

$$U_z = \frac{c}{\phi - \theta_{ret}} \cdot {K_h^*}^{1/c} \cdot {V_z'}^{(c-1)/c}$$

We'll assume that snow depth is 0.725m as in Example 5-10, since no other info. is given.

Lag time $t_{LAG} = \dfrac{h_s}{U_z}$

Case	Melt rate (mm/day), V_z'	$U_z\left(\frac{m}{s}\right)$	U_z $\frac{m}{day}$	t_{LAG} (day)
Forest - Day 1	0.9	$2.76 \cdot 10^{-6}$	0.239	3.04
Forest - Day 2	18.6	$2.08 \cdot 10^{-5}$	1.80	0.40
Field - Day 1	7.2	$1.10 \cdot 10^{-5}$	0.954	0.76
Field - Day 2	53.1	$4.18 \cdot 10^{-5}$	3.62	0.20