9. Mixing in Lakes and Reservoirs

Mixing in lakes and reservoirs is largely controlled by stratification. Stratification reduces vertical exchange and can drive horizontal exchange by enforcing a preferred vertical structure. Stratification arises from temperature, dissolved solids and suspended particulate variations. The latter two components (chemical and suspended particulate stratification) are generally negligible in the bulk fluid of inland lakes and reservoirs. Thus, stratification is dominantly dependent on temperature variation, which in turn is a function of the overall energy balance and the internal mixing processes of a lake.

This chapter begins by discussing the development of stratification followed by a description of the energy inputs to a lake or reservoir. From the annual energy input variation a typical annual cycle for a temperate lake is introduced: lakes are stratified in winter and summer and have complete mixing events, called turnover, that occur in the spring and fall. The chapter closes with a brief introduction to the dominant mixing processes that influence the stratification structure. These mixing processes are also the dominant processes affecting the fate and transport of contaminants in lakes and reservoirs.

Reservoir dynamics is an active and important area of research in environmental fluid mechanics. Many of the concepts introduced here are general. They relate to most lakes and help understand the important characteristics of stratification. The detailed mixing processes, however, can be significant and quite complicated. For instance, exchange between sediments and the open water can be controlled by seiches, groundwater exchange, redox chemistry, and microturbulence, among other factors. The interested reader is encouraged to get more details in Imboden & Wüst (1995), Lorke et al. (2003), and Fischer et al. (1979) and their bibliography sections.

9.1 Concepts and definitions

Mixing in inland lakes and reservoirs belongs to the field of physical limnology (the study of the physical properties of bodies of fresh water). The study of lake and reservoir mixing is an active and important area of research in environmental fluid mechanics. Many of the concepts that will be introduced in this chapter are general. They relate to most lakes and help understand the important characteristics of stratification. The detailed mixing processes, however, can be significant to transport and quite complicated. For instance, exchange between sediments and the open water can be controlled by a combination of seich motion, groundwater exchange, redox chemistry and kinetics, microturbulence, and stratification, among other factors. Many good and
comprehensive texts are available. The reader is pointed to the classic text by Wetzel (1983) and books more closely related to fluid mechanics by Fischer et al. (1979) and by Imboden & Wüest (1995). Much of the material in this chapter is taken from the latter two sources.

In general, lakes and reservoirs are stratified systems. Nearly every water body is stratified to some degree, but the stratification in lakes and reservoirs is strong enough to play a dominant role in mixing.

Because inland lakes are primarily made up of fresh water, the dominant stratification component is thermal stratification. Locally, at boundaries or river inflows, dissolved solids or suspended particulate stratification can be important. In estuaries, where saline seawater meets inland freshwater, strong concentration gradients can dominate the stratification. Here we restrict our attention to inland water bodies. Thus, the vertical stratification structure that we consider in this chapter is controlled by thermal stratification.

The development of stratification in lakes and reservoirs is due to three major factors (Imboden & Wüest 1995). First, because lakes and reservoirs are comparatively stagnant, they have low flow velocities, often laminar in nature. As a result, mixing is controlled by molecular diffusion, which allows stable gradients in temperature and dissolved solids concentrations to persist. Second, lakes and reservoirs have long residence times. This is important since heating, cooling, and chemical processes in lakes are slow. If the flow through the lake is too fast, there is not adequate time for stratification to develop and turbulent mixing is too rigorous. The limit of this case would be a river, where the residence time in a given section is short and stratification is negligible. Third, lakes and reservoirs form in (sometimes deep) depressions which naturally reduce the interaction between surface and bottom water. This topographic effect supports the layered nature of stratification. Hence, vertical stratification in lakes and reservoirs becomes increasingly important as the lake becomes more laminar, has longer residence times and deeper bottom water. Conversely, processes that increase turbulence, decrease residence time and increase interaction with bottom water reduce stratification and enhance mixing.

9.2 Energy budget

From a fundamental viewpoint the total energy in a lake is the sum of the thermal and mechanical energy. Mechanical energy is further subdivided into potential energy, $E_P$, and kinetic energy, $E_K$. Thermal energy, $E_T$, is the energy stored in the random velocity distribution of atoms and molecules (Imboden & Wüest 1995). Thus, the energy budget for a lake can be written as

$$\frac{dE_{tot}}{dt} = \frac{dE_P}{dt} + \frac{dE_K}{dt} + \frac{dE_T}{dt} \quad (9.1)$$

where $E_{tot}$ is the total energy of the lake. The potential energy is usually defined in a lake as the difference between the current lake state and the potential energy of the lake if it were completely mixed (unstratified). Since the potential energy of a mixed system is higher than for a stratified system, this difference is always negative. The kinetic energy describes the fluid motion, and the thermal energy is the heat content of the lake.

Figure 9.1 shows a schematic of all the major energy inputs to a typical lake. Some of these in-
puts, such as solar radiation, directly heat the water and, thereby, enhance stratification. Others of these components create water motion that results in mixing, thereby reducing stratification. Some inputs cause both effects; thus, there is interaction among all these energy inputs, which we will discuss in more detail in the section on mixing. Here we briefly discuss the effect of each input on stratification.

### 9.2.1 Radiation

Two major types of radiation play a role in lake and reservoir stratification: short-wave ultraviolet radiation and long-wave infrared radiation.

Short-wave radiation originates from the sun. Some of the solar radiation is reflected at the water surface and the remainder penetrates into the lake. Most of the radiation that penetrates is absorbed in the water column and converted to heat. Different wavelengths are absorbed at different depths. Because blue travels the farthest, it heats the deepest layers and can also travel full circle and escape from the lake, thereby, giving the lake its blue appearance.
Long-wave radiation originates from black-body radiation. Both the lake and the atmosphere emit black-body radiation. Long-wave atmospheric radiation is partially reflected at the lake surface, and the penetrating radiation is absorbed, causing heating. Black-body radiation emitted by the lake results in a loss of thermal energy and cooling of the lake water.

9.2.2 Evaporation and condensation

Evaporation is the conversion of liquid water to water vapor; condensation is the conversion of water vapor to liquid water. Both of these reactions are accompanied by a flux of heat. Evaporation from the lake surface extracts heat from the lake and results in cooling of the water surface. We experience this ourselves in the summer when a cool breeze evaporates our sweat, thereby, cooling our skin. Condensation extracts heat from the atmosphere and adds it to the water surface, resulting in heating at the water surface. We experience this in a sauna when we pore water over the hot coals. The steam produced condenses on our skin and the heat we receive from this process gives us the impression that the sauna becomes much hotter. Thus, evaporation cools the lake surface, and condensation heats the lake surface.

Evaporation and condensation are also accompanied by a flux of water; thus, they affect the total water budget of the lake.

9.2.3 Wind

Wind is the major external input responsible for mixing. Wind adds kinetic energy to the lake and reorganizes the potential energy. Wind affects the lake through the shear it imparts on the water surface. This shear drags the water in the downwind direction, adding kinetic energy and causing surface currents, surface waves, and a so-called surface set up: the mean lake surface downwind is tilted upward compared to the upwind side of the lake. This set up results in a basin-scale circulation: bottom-water return currents compliment the surface-water motion. Although the surface set up may only be a few millimeters or centimeters, this results in a much greater (sometimes order meters) tilting of the internal isopcnals, lines of constant density. Thus, the set up results in a re-organization of the potential energy in the lake: downwind regions have increased potential energy. When the wind stops, this potential energy is released, causing basin-scale waves called seiches.

9.2.4 Direct inflows and outflows

Direct inflows to lakes and reservoirs can include surface inflow (rivers and streams), groundwater inflow, and precipitation. Outflows from the lake can be surface or groundwater outflow.

Each of these flows is accompanied by a flux of heat; these flows may also add or remove kinetic energy. Among the inflows and outflows, surface flows have the greatest potential for kinetic energy input to a lake. However, the impact of rain drops should not be neglected as an input of kinetic energy.

When inflows and outflows result in unstable density gradients, as in cold rain water falling on a warm lake surface, density currents may also be induced, creating changes in kinetic and
potential energy. Hence, each inflow or outflow must be considered separately in a given system to determine its effect on the total energy budget and the stratification.

9.3 Seasonal lake cycle

The seasonal cycle of lake stratification is a popular topic in classical physical limnology. It results from the seasonal variation in solar radiation and controls the long-term temperature and distribution of chemicals (in particular oxygen) in the lake. Figure 9.2 shows a schematic of this seasonal cycle for a temperate lake.

In the summer, the high solar radiation input and warm air temperatures contribute to a strong thermal stratification of the lake. Surface water is warmer than bottom water. Winds tend to keep the surface water mixed, and this upper mixed region of the lake is called the epilimnion. Below the mixing action of the wind and the penetration depth of the solar radiation, a strong temperature and accompanying density gradient develops. This region of strong gradients is called the thermocline, or sometimes picnocline. Below the thermocline a weaker temperature
gradient is observed and the water is cool and comparatively quiescent. The bottom region of the lake is called the hypolimnion. In very deep lakes, where solar radiation and thermal conduction cannot penetrate to the lake bottom, the bottom water will have a temperature near 4 °C, the temperature of maximum density of water. The strong density gradients in the thermocline inhibit exchange between the epilimnion and the hypolimnion. Thus, bottom water in a stratified lake does not actively interact with the atmosphere and can easily become deprived of important dissolved gases.

As the air temperature gets cooler and the solar radiation input decreases in the fall, the surface water begins to cool. Eventually, the surface water and thermocline cool down to the temperature of the hypolimnion and the lake is no-longer stratified. In this state the lake can easily be mixed, even by a light wind; thus, the lake is expected to completely mix, or in other words experience a turnover event. This gives the bottom water an opportunity to aerate with the atmosphere.

If the air temperature in the winter goes below 4 °C so that the surface water can cool below this temperature, then the surface water becomes lighter than the bottom water and a so-called winter inverse stratification develops. The term inverse refers to the fact that the surface water is colder than the bottom water; however, the surface water remains less dense. When ice forms at the water surface, wind mixing is not possible and the winter density profile may not exhibit a well-defined thermocline.

As the surface water heats up again in the spring, the lake will again reach a state of thermal homogeneity and in the presence of a light wind will turn over. As the surface waters become warmer, stratification sets in and we return to the summer stratification state.

Lakes that experience this full cycle of stratification states are called dimictic because they stratify twice. Lakes that only stratify in the summer are called monomictic. These classifications are important to limnologists because they affect which species of plants and aquatic life will populate the lake. These classifications are also helpful for environmental fluid mechanics because they suggest the expected base-state of the stratification in each season. However, the actual state of a lake and the mixing mechanisms active on a given day are dependent on many other factors discussed in the following section.

9.4 Transport and mixing mechanisms

In general, the active mixing processes in a lake or reservoir are driven by diurnal (daily) forcing, such as by wind, inflows and outflows, radiation exchange, and chemistry. These processes affect the local mixing characteristics and also influence the global lake stratification. The stratification forces a preferred horizontal motion and inhibits vertical exchange. Figure 9.3, taken directly from Imboden & Wüest (1995), illustrates the common types of mixing mechanisms in lakes and reservoirs. These processes are discussed briefly in the following.
9.4.1 Wind

As already illustrated in the section on the energy budget, wind is the dominant external energy input responsible for mixing. Wind generates local currents that, as a result of the lake boundaries, also induce basin-scale motions.

Near the lake surface wind is responsible for surface waves and surface currents. Larger surface waves generate Stokes drift, an induced current in the wave propagation direction. Wave breaking is an important mixing mechanism and can greatly enhance gas exchange with the atmosphere. Strong winds (above about 3 m/s) also induce Langmuir circulations, large-scale counter-rotating helical vortices. The alternating convergence and divergence zones of the Langmuir circulation are visible by the appearance of streaks on the water surface in the direction of the wind. Langmuir circulation is largely responsible for mixing of the surface layer and deepening of the epilimnion.

The wind-induced basin-scale currents at the water surface are responsible for horizontal mixing and result in a basin-scale circulation, as the bottom water must flow upwind to satisfy mass conservation. The bottom recirculation currents generate boundary mixing events, discussed separately in the section below.

Also, as introduced above, the presence of the wind generates the lake set up. When the wind stops, basin-scale internal waves, called seiches, and other internal waves develop which also result in boundary currents and boundary mixing. In essence, wind is the stirring rod that keeps the lake in motion.

9.4.2 Boundary mixing

Boundaries are important locations for mixing because of the shear that develops between the ambient flow and the no-slip condition at the boundary. When a region of fluid near the boundary
becomes mixed, it must intrude at a level of neutral buoyancy, forming a series of finger-line layers of mixed fluid. In regions of strong topographic changes, boundary mixing is locally more intense.

**9.4.3 Inflows and outflows**

Inflows and outflows create mixing through their own kinetic energy and through an input of buoyancy.

River inflows can add a large input of both kinetic energy and buoyancy. River inflows that are more dense than the surface water plunge to a level of neutral buoyancy and form internal currents, called intrusions. In the presence of appropriate bottom topography, these internal flows may have internal hydraulic jumps, which have associated regions of strong mixing. Shear flows in density stratification also can form Kelvin-Helmholz instabilities (sometimes called billows and their formation is sometimes called billowing), which mix across density gradients.

When river inputs contain high suspended solids or stir-up the bottom sediments, turbidity currents may develop. Turbidity currents are density currents formed by suspended particulate flows. These flows entrain ambient water and mix at the boundaries.

Man-made water withdrawals and waste inputs also create regions of enhanced mixing. Withdrawals create regions of shear, and inputs create jets and plumes. In stratified lakes, jets and plumes form intrusions at their level of neutral buoyancy.

**9.4.4 Radiation**

Radiation, both long- and short-wave, causes mixing when an unstable density gradient is generated. For example, when suspended particles, such as algae, in the water column concentrate the solar radiative heating in a certain water layer, this region may become warmer than the water above. This warmer bottom water must rise, forming a thermal convection current. Another common example is surface cooling at night due to black-body back-radiation from the lake surface. The surface water cools, becoming more dense than the underlying water, and forming a downward convecting thermal.

**9.4.5 Chemistry**

Phase-change reactions and dissolution can create density gradients that generate buoyancy-driven flows. Mineralization at the lake bed, for instance, can generate small density currents. Evaporation at the water surface cools the water surface and may cause convective mixing. Double diffusion is another possible mixing mechanism when gradients of concentration and temperature create transport in opposing directions.

**9.5 Lake mixing regimes**

As is clear from the previous section, many processes are responsible for mixing in lakes and reservoirs. This mixing is controlled by stratification, both in the form of convective currents
in the presence of locally unstable stratification and in intrusion flows in stably stratified environments. A non-dimensional number that expresses the mixing potential of a shear flow in a stably stratified ambient is the Richardson number, \( Ri \), defined in Chapter 8 as

\[
Ri = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2.
\]  

(9.2)

Other similar numbers can be developed to describe the mixing potential of other mixing mechanisms.

For a strongly stratified reservoir acted on by wind mixing, we can use a modified version of \( Ri \) to classify the mixing regime. The velocity gradient in the surface water is proportional to the shear velocity due to the wind, given by

\[
u_s = \sqrt{\frac{0.013 \rho_a U_{10}^2}{\rho_w}}
\]  

(9.3)

where \( \rho_a \) is the density of the air, \( \rho_w \) is the density of the water and \( U_{10} \) is the wind speed at an elevation of 10 m above the water. The term \( (g\Delta \rho)/\rho \) can be approximated as the reduced gravity between the epilimnion and the hypolimnion, namely

\[
g' = g \frac{\rho_h - \rho_e}{\bar{\rho}}
\]  

(9.4)

where \( \rho_h \) is the hypolimnion density, \( \rho_e \) is the epilimnion density, and \( \bar{\rho} \) is the average lake density. Finally, the density differences occur over a depth about equal to the depth of mean density, or

\[
\bar{h} = z(\bar{\rho})
\]  

(9.5)

where \( \bar{h} \) is the depth at which the density is equal to \( \bar{\rho} \). Combining these approximations, the Richardson number is

\[
Ri_* = \frac{g' \bar{h}}{u_s^2}.
\]  

(9.6)

This form of \( Ri \) gives the ratio of the stability due to the stratification compared to the instability caused by the wind stirring.

Fischer et al. (1979) suggest four mixing regimes for different ranges of \( Ri_* \). The values of \( Ri_* \) are compared to the lake mean depth, \( h \), and the length of the wind fetch across the lake \( L \). For the derivation of these critical regimes refer to Fischer et al. (1979). The regimes are are defined in Fischer et al. (1979) as follows:

1. **Regime A:** \( Ri_* > L^2/(2h)^2 \). The deepening process proceeds very slowly by turbulent erosion. The interface setup is small and internal waves persist for long times. Any longitudinal gradients are quickly obliterated by longitudinal gravitationally driven mixing. Mixing and internal motions are completely uncoupled and the interface remains sharp.

2. **Regime B:** \( L/(2h) < Ri_* < L^2/(2h)^2 \). Internal waves are the predominant feature of this regime. Entrainment and billowing have minor effects on the internal waves, yet the wave amplitude can be quite severe. Erosion keeps the interface sharp and it is energized almost exclusively by surface stirring. Billowing should be observable with the interface becoming
Wea k stratification
severe storm

Strong stratification
weak winds

D
C

Fig. 9.4. Schematic of the mixing sequences for the different regimes of mixed layer deepening taken directly from Fischer et al. (1979).

less defined the closer $Ri_*$ is to $L/(2h)$. At this boundary the thermocline surfaces at one end, becomes smeared and essentially deepens to the bottom at the downwind end all in much the same time. Furthermore, at the boundary $Ri_* = L/(2h)$ the mean kinetic energy being utilized by the billowing process is approximately equal to that used for deepening. The billowing will be rapid, but the presence of the turbulence in the upper layer may hide the organized structure associated with the billowing. Internal waves are heavily damped.

3. Regime C: $1 < Ri_* < L/(2h)$. Throughout this regime the thermocline will be diffuse and steeply inclined. The process deepens the epilimnion rapidly to the bottom, the mixing being predominantly energized by shear production. The final state will be a longitudinal temperature gradient which will be mixed horizontally.

4. Regime D: $Ri_* < 1$. Deepening is now so rapid and chaotic that the interface will not be well defined. The flow will appear a little like the reverse of a lock exchange flow.

These mixing regimes and their development as the wind persists over time are illustrated in Figure 9.4.

Similar exercises and modified $Ri$ numbers can be used to evaluate the strength of mixing for the other mechanisms described in the previous section. Interested readers are pointed to a series of lectures by Jörg Imberger available over the web\(^1\).

\(^1\) http://www.cwr.uwa.edu.au/Presentations/
Summary

This chapter introduced the role of stratification and discussed the dominant energy inputs and mixing mechanisms in inland lakes and reservoirs. Energy inputs are in the form of mechanical (kinetic and potential) and thermal energy. Because chemical and suspended particulate stratification is often negligible in inland water bodies, temperature is the dominant stratification mechanism. Heat inputs to the lake are, thus, the dominant factors creating the stratification. Mechanical energy is the dominant input that erodes the stratification. Because of the seasonal variation in solar radiation, lakes undergo an annual cycle of mixing and stratification. On a daily basis, diurnal forcing is responsible for mixing. Since wind is often the dominant mixing mechanism, a lake mixing classification scheme was introduced as a function of the Richardson number in the lake.

Exercises

9.4 Atmospheric heat flux. Table 9.1 gives two profiles of temperature measured 47 days apart during the summer. What is the mean atmospheric heat flux into the lake during this time assuming that no other energy inputs are responsible for heat changes in the lake? Recall that the heat content of water, \( C_H \), in kJ/m\(^3\) is given by

\[
C_H = c_p \rho_w T \tag{9.7}
\]

where \( c_p = 4.2 \text{ kJ/(kg-K)} \) is the specific heat of water, \( \rho_w \) is the density of the water in kg/m\(^3\), and \( T \) is the water temperature in K.

9.4 Wind mixing. On August 1 the wind on the lake in problem 1 was blowing at a velocity of \( U_0 = 4.5 \text{ m/s} \) for 6 hours. Estimate the modified Richardson number, \( R_i^* \), and predict the mixing state of the lake.

9.4 Vertical diffusion coefficients. From the temperature data in Table 9.1 compute the buoyancy frequency as a function of depth. Where do you expect the lowest values of the vertical turbulent diffusion coefficient and why?
### Table 9.1. Lake temperature data for use in Exercises 1 and 3. Taken from Nepf (1995).

<table>
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<tr>
<th>Depth [m]</th>
<th>June 15 Temperature [°C]</th>
<th>August 1 Temperature [°C]</th>
<th>Cross-sectional Area [m²]</th>
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<td>15.0</td>
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</tr>
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<td>8.4</td>
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</tr>
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References


ASCE & WPCF (1992), ‘Design and construction of urban stormwater management systems’.


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