A copper bar has an initial length of 30 in and is supported as shown. The left end is fixed and there is a gap of 0.009 in. at the right end. Calculate the compressive stress in the member if the temperature increases by 50°F. Assume \( E = 16 \times 10^6 \) psi; and \( \alpha = 9.6 \times 10^{-6} / \circ F \).

\[ A_0 = 2 \text{ in}^2 \]

First remove the constraint at B and allow the bar to elongate freely. In this case the elongation of the bar, \( \delta_{AB} \), will be given as follows

\[ \delta_{AB} = L \Delta T \]
\[ \Delta_{AB} = \left(9.6 \times 10^{-6} \text{ } ^\circ \text{F}^{-1}\right) (50 \text{ } ^\circ \text{F}) (30 \text{ in.}) \]

\[ \Delta_{AB} = 0.144 \text{ in.} \]

Thus, the bar elongates to a length of 30.0144 in. as shown.

However, the constraint at B prevents the length of the bar from exceeding 30.009 in.

Thus, it can be seen that the reaction at B must make up the difference by causing a compressive deformation of

\[ 0.144 - 0.09 = 0.054 \text{ in.} \]
as shown below:

\[ A_x \]

\[ B_x \]

\[ 30.09 \]  
\[ 30.0194 \] 

The deformation of the bar due to the compressive force \( B_x \) is

\[ \delta_{AB}'' = 0.0054 \text{ in} = \frac{B_x L_{AB}}{A_{AB} E_{AB}} \quad (4) \]

\[ 0.0054 \text{ in} = \left( \frac{30 \text{ in}}{2 \text{ in}^2} \right) \left( 10 \times 10^6 \text{ lb/in}^2 \right) \quad (5) \]

\[ B_x = 5760 \text{ lb} \quad (6) \]

Then by equilibrium

\[ \sum F_x = 0 \]

\[ A_x - B_x = 0 \quad (7) \]

\[ A_x = B_x = 5760 \text{ lb} \quad (8) \]
So a F.B.D. of the bar is given as follows

The internal force in section AB is found using a F.B.D. as follows

\[ 5760 \rightarrow \rightarrow P_{AB} \]
\[ \Rightarrow \Sigma F_x = 0 \]
\[ 5760 + P_{AB} = 0 \]
\[ P_{AB} = -5760 \text{ lb} \quad (10) \]

or

5760 lb compression

The corresponding compressive stress is

\[ \sigma_{AB} = \frac{P_{AB}}{A_{AB}} \quad (11) \]
\[ \overrightarrow{AB} = \frac{-5700 \text{ lbs}}{2 \text{ in}^2} \]

\[ \overrightarrow{AB} = \frac{-2880 \text{ lb}}{\text{ in}^2} \]