Section: 501  Name: KEY
Student Number:

"Aggies do not lie, cheat, or steal, nor do they tolerate those who do." – Aggie Code of Honor

By my signature below I pledge that my conduct on this exam is consistent in every way with the Aggie Code of Honor:

Signature:

1. This exam consists of Four (4) problems that are equally weighted
2. Some problems have multiple parts.
3. Be sure to carefully read and properly analyze each question that is asked. Do not jump to unfounded conclusions, but also do not overlook or oversimplify problems either.
4. Be sure to show all work, including sketches, Free Body Diagrams, and calculations, and organize your solution procedure as clearly and systematically as possible.
5. Work problems in the space provided on the exam sheets.
6. Work efficiently, neatly, and use pencil.
7. Clearly indicate final answers by enclosing in a "box" or place answer in box if box is provided. Include any and all appropriate units.

Problem 1: _______ / 10
Problem 2: _______ / 10
Problem 3: _______ / 10
Problem 4: _______ / 10

TOTAL: _______ / 40
Problem 1 (25 points). Use the stress transformation equations as discussed in class and presented on the internet to calculate the maximum and minimum normal stresses (principal stresses) acting at the point shown. Show these stresses on a properly oriented block as was done in class and on the internet. Show all work to receive full credit.

\[ \sigma_x = 12,000 \quad \tau_{xy} = -5,000 \]

\[ \tan 2\theta_p = \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \]

\[ \tan 2\theta_p = \frac{2 (-5,000)}{12,000 - (-3,000)} \]

\[ \tan 2\theta_p = 0.667 \]

\[ 2\theta_p = -33.69^\circ \]

or

\[ 2\theta_p = 33.69^\circ + 180^\circ = 146.31^\circ \]

\[ \sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \]

\[ \sigma_\theta = \frac{12,000 + (-3,000)}{2} + \frac{12,000 - (-3,000)}{2} \cos 2\theta + (-5,000) \sin 2\theta \]

\[ \sigma_\theta = 4,500 + 7,500 \cos 2\theta - 5,000 \sin 2\theta \]

\[ \tau_{xy} = -33.69^\circ \]

\[ \sigma_{45^\circ} = 4,500 + 7,500 \cos 33.69^\circ - 5,000 \sin 33.69^\circ \]

\[ \sigma_{16.8^\circ} = 4,500 + 6,240.4 + 2,773.5 \]

\[ \sigma_{16.8^\circ} = 13,514 \]

\[ \sigma_{73.16^\circ} = 4,500 + 7,500 \cos 146.31^\circ - 5,000 \sin 146.31^\circ \]

\[ \sigma_{73.16^\circ} = 4,500 - 6,240.4 - 2,773.5 \]

\[ \sigma_{73.16^\circ} = -4,514 \]
Problem 2 (25 points). Use the Mohr's circle approach as discussed in class and presented on the internet to calculate the normal and shearing stresses acting on plane $AA$ of the element shown. Show these stresses on a properly oriented block as was done in class and on the internet. Show all work to receive full credit.

\[ \sqrt{x} = 10,000, \quad \sqrt{y} = 5,000, \quad \sqrt{xy} = 2,000 \]

\[ \sqrt{r} = \frac{\sqrt{x} + \sqrt{y}}{z} \]

\[ \sqrt{r} = \frac{10,000 + 5,000}{z} = 7,500 \]

\[ \sqrt{y} = \sqrt{\frac{x - y}{z}} + \sqrt{\frac{x + y}{z}} \]

\[ \sqrt{y} = \sqrt{\frac{10,000 - 5,000}{z}} + \sqrt{5,000} \]

\[ R_{ad} = 320.16 \]

\[ \sin \theta = \frac{3,200}{320.16} \]

\[ 2\theta = 38.66^\circ \]

\[ \sqrt{2\theta} = \sqrt{38.66^\circ} \]

\[ \sqrt{AA} = 7,500 - \cos 35.08^\circ (320.16) \]

\[ \sqrt{AA} = 4880 \]

\[ \sqrt{AA} = - \sin 35.08^\circ (320.16) \]

\[ \sqrt{AA} = - 1840 \]
Problem 3 (25 points). The vertical, right circular cylinder is subjected to an internal pressure of 300 psi, an axial load of 150,000 lb, and a torque of 800,000 in-lb as shown. The inner diameter of the cylinder is 20 in. and the outer diameter of the cylinder is 21 in. Calculate the combined state of stress acting on the surface of the cylinder at point A as shown. Show your results on the sketch at the bottom of the page. Show all work to receive full credit.

\[ A = \pi \left( r_{\text{outer}}^2 - r_{\text{inner}}^2 \right) = \pi \left( 10.5^2 - 10^2 \right) = 32.2 \text{ in}^2 \]

\[ T = \frac{P r_{\text{inner}}}{Z} = \frac{150,000 \times 10}{32.2 \times 10^2} = 4658.4 \text{ psi} \]

Combined stresses as follows:

- Axial Load:
  \[ V = \frac{P}{A} = \frac{150,000 \times 10}{32.2 \times 10^2} = 4658.4 \text{ psi} \] (Compression)

- Torque Load:
  \[ T = \frac{P r}{J} = \frac{150,000 \times 10}{3385.2} = 4581.4 \text{ psi} \]

- Pressure Vessel:
  \[ \sigma_{\text{hoop}} = \frac{P r}{t} = 1500 \text{ psi} \]
  \[ \sigma_{\text{long}} = \frac{P r}{2t} = 750 \text{ psi} \]
Problem 4 (25 points). Use the double integration method and singularity functions as discussed in class and presented on the internet to calculate the equations for the slope and deflection of the beam loaded as shown. Work the problem all the way through determination of the constants of integration. Show all work to receive full credit.

\[ \mathcal{E} F_y = 0 \]

\[ A_y - 3000 \times -3.333 \times 1 \text{lb} = 0 \]

\[ A_y = 6.500 \text{ lb} \]

\[ \mathcal{E} M_A = 0 \]

\[ M_A = -3000(3) - 3.333(9.5) + 833.3 \times 1 \text{ lb} \]

\[ M_A = 41.417 \text{ ft} \times -1 \text{ lb} \]

\[ M_c = 0 \]

\[ M_{CD} + 500 \times \frac{z}{2} + 3000 \times \frac{z}{x} \]

\[ + 41.417 - 6500 \times x = 0 \]

\[ M_{CD} = -250 \times \frac{z}{x} - 3000 \times \frac{z}{x^3} + 6500 \times x - 41.417 \]

\[ EI \frac{d^2y}{dz^2} = -250 \times \frac{z}{x^2} - 3000 \times \frac{z}{x^3} + 6500 \times x - 41.417 + C_1 \]

\[ EI \frac{dy}{dx} = -250 \times \frac{z}{x^2} + \frac{3000}{x} \times \frac{z}{x^3} + \frac{6500}{x} \times x - 41.417 + C_1 \times x \]

\[ EI y = -250 \times \frac{z}{x^2} - \frac{3000}{x} \times \frac{z}{x^3} + \frac{6500}{x} \times x - 41.417 \times \frac{x}{2} \times C_1 \times x \]

@ \( x = 0 \), \( \frac{dy}{dx} = 0 \) \:
\[ 0 = - \frac{1}{2} (0 - 0 + 0 - 0 + 0 + C_1) \]

\[ C_1 = 0 \]

@ \( x = 0 \), \( y = 0 \) \:
\[ 0 = -0 - 0 + 0 + 0 + 0 + C_2 \]

\[ C_2 = 0 \]

\[ \frac{dy}{dx} = \frac{1}{EI} \left[ \theta + \frac{3}{3} \times \frac{z}{x^3} - 1500 \times \frac{z}{x^3} + 3250x^2 - 41.417x \right] \]

\[ y = \frac{1}{EI} \left[ -150 \times \frac{z}{x^3} + 3250x + 10.83 \times x^3 - 20708.5x^2 \right] \]