"Aggies do not lie, cheat, or steal, nor do they tolerate those who do." — Aggie Code of Honor

By my signature below I pledge that my conduct on this exam is consistent in every way with the Aggie Code of Honor:

Signature: __________________________________________

1. This exam consists of Four (4) problems that are equally weighted.
2. Some problems have multiple parts.
3. Be sure to carefully read and properly analyze each question that is asked. Do not jump to unfounded conclusions, but also do not overlook or oversimplify problems either.
4. Be sure to show all work, including sketches, Free Body Diagrams, and calculations, and organize your solution procedure as clearly and systematically as possible.
5. Work problems in the space provided on the exam sheets.
6. Work efficiently, neatly, and use pencil.
7. Clearly indicate final answers by enclosing in a "box" or place answer in box if box is provided. Include any and all appropriate units.

Problem 1: ______ / 10
Problem 2: ______ / 10
Problem 3: ______ / 10
Problem 4: ______ / 10

TOTAL: ______ / 40

Allowed
1 sheet of notes 8½x11 in
1 side.

calculator

Not Allowed
Book &
Class notes

WLB
Problem 1. Use the stress transformation equations as discussed in class to determine the normal, $\sigma_{xx}$, and shear, $\tau_{xy}$, stresses on plane $AA$ which is located $70^\circ$ CW from the horizontal plane as shown. Enter your results in the blanks provided and show your results on the sketch provided. Show all work to receive full credit.

\[
\begin{align*}
\sigma_x &= +6000 \\
\sigma_y &= -3000 \\
\tau_{xy} &= -2000
\end{align*}
\]

\[
\sigma_0 = \frac{\sigma_x + \sigma_y}{2} + \tau_{xy} \cos 2\theta + \tau_{xy} \sin 2\theta
\]

\[
\begin{align*}
\sigma_{20^\circ} &= \frac{6000 + (-3000)}{2} + \frac{6000 - (-3000)}{2} \cos 40^\circ - 2000 \sin 40^\circ \\
\sigma_{20^\circ} &= 1500 + 4500 (1.766) - 2000 (1.643) \\
\sigma_{20^\circ} &= +3661
\end{align*}
\]

\[
\begin{align*}
\tau_0 &= -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta
\end{align*}
\]

\[
\begin{align*}
\tau_{20^\circ} &= -\left(\frac{6000 - (-3000)}{2}\right) \sin 40^\circ + (-2000) \cos 40^\circ \\
\tau_{20^\circ} &= -(4500)(1.643) - 2000 (1.766) \\
\tau_{20^\circ} &= -4425.5
\end{align*}
\]

\[
\sigma_{xx} = +3661 \quad \text{and} \quad \tau_{xy} = -4425.5
\]
Problem 2. A cylindrical pressure vessel with an inside diameter of 20 in. and a ¼ in. wall thickness is subject to an axial compressive load of 80,000 lbs, a torque of 40,000 ft-lb, and an internal pressure of 250 psi as shown. Use Mohr’s circle to determine the maximum in-plane principal stress, \( \sigma_{max} \), the minimum in-plane principal stress, \( \sigma_{min} \), the maximum in-plane shearing stress, \( \tau_{max} \), and the absolute maximum shearing stress, \( \tau_{abs} \), on the outer cylindrical surface. Show all work to receive full credit.

\[
A = \pi \left( \frac{10.25^2}{10^2} \right) \\
A = \frac{40,000 \text{ lbs}}{15,904 \text{ in}^2} = 2.50 \text{ in}^2
\]

\[
J = \frac{\pi}{2} \left( \frac{10.25^4}{10^4} \right) \\
J = 1630.7 \text{ in}^4
\]

**Compressive Load**

\[
\frac{P}{A} = 80,000 \text{ lbs} = 5030.2 \frac{\text{lb}}{\text{in}^2}
\]

**Torsion**

\[
T = \frac{T}{J} = \frac{40,000 \times 12}{1630.7} = 3017.1 \frac{\text{lb}}{\text{in}^2}
\]

**Pressure Vessel**

\[
\frac{P}{A} = \frac{250}{10} = 10,000 \frac{\text{lb}}{\text{in}^2}
\]

\[
\frac{P}{J} = \frac{250}{1630.7} = 0.15 \frac{\text{lb}}{\text{in}^2}
\]

\[
\sigma_x = 10,000 \frac{\text{lb}}{\text{in}^2} \quad \sigma_y = -30.2 \frac{\text{lb}}{\text{in}^2} \quad \tau_{xy} = 3017.1 \text{ in}^2
\]

All are in \( \frac{\text{lb}}{\text{in}^2} \).
\( T_{\text{cont}} = \frac{N_x + V_y}{2} = \frac{10,000 - 50.2}{2} = 4984.9 \ \frac{\text{lb}}{\text{in}} \)

\( R_d = \sqrt{\left(\frac{N_x - V_y}{2}\right)^2 + N_{xy}^2} = \sqrt{\left(\frac{10,000 - (-30.2)}{2}\right)^2 + (30.17.1)^2} = 5852.7 \ \frac{\text{lb}}{\text{in}} \)

**In-Plane**

\( T_{\text{max}} = T_{\text{cont}} + R_d = 4984.9 + 5852.7 = 10837.6 \ \frac{\text{lb}}{\text{in}} \)

\( T_{\text{min}} = T_{\text{cont}} - R_d = 4984.9 - 5852.7 = -867.8 \ \frac{\text{lb}}{\text{in}} \)

\( T_{\text{max}} = 5852.7 \ \frac{\text{lb}}{\text{in}} \)

**Absolute Maximum Shear Stress**
Problem 3. Use singularity functions as discussed in class to write complete equations for the slope and deflection of the beam loaded as shown. Assume that \( E = 29 \times 10^6 \) psi and that \( I = 100 \) in. Solve for the constants of integration and determine the maximum deflection, \( \delta_c \), at point C on the beam and enter this value in the blank provided. Show all work to receive full credit.

\[
\begin{align*}
\delta_c & = 0 \\
M_A & = 0 \\
\pm F_x & = 0 \\
A_x & = 0
\end{align*}
\]

\[
\begin{align*}
\pm F_y & = 0 \\
A_y & = 200 - 200 = 0 \\
A_y & = 400 \text{ lb}
\end{align*}
\]

\[
\begin{align*}
\delta_c & = 0.17 \text{ in}
\end{align*}
\]

\[
\begin{align*}
E \frac{dy}{dx} & = -25x^2 + 25(x-4)^2 + 400x - 1700 \\
E \frac{d^2y}{dx^2} & = -8.333x^3 + 8.333(x-4)^3 + 200x^2 - 1700x + C_1 \\
E \frac{dy}{dx} & = -2.083x^4 + 2.083(x-4)^4 + 66.67x^3 - 850x^2 + C_1 + C_2 \\
\end{align*}
\]

\[
\begin{align*}
& @ x = 0, \frac{dy}{dx} = 0 \implies C_1 = 0 \\
& @ x = 0, y = 0 \implies C_2 = 0
\end{align*}
\]
\[ y_c = \frac{1}{E_1} \left[ -2.083 (b)^4 + 2.083 (4)^4 + 66.67 (b)^3 - 850 (b)^2 \right] \]
\[ y_c = \frac{1}{E_1} \left[ -8532 + 533 + 34135 - 54400 \right] \]
\[ y_c = -\frac{28264}{E_1} \]
\[ y_c = -\frac{(28264)(1728)}{(29 \times 10^6)(100)} \]
\[ y_c = -0.17 \text{ in} \]
Problem 4. Find the external reactions for the beam loaded as shown. Use the method of superposition. Assume that EI is constant. Summarize your results on a FBD of the beam. Show all work to receive full credit.

\[ \begin{align*}
S_B &= -\frac{50}{24EI} \\
&= -\frac{50}{24EI} \left( x^4 - 4Lx^3 + 6L^2x^2 \right) \\
&= -\frac{50}{24EI} \left( 4 - 4(2)(4) + 6(4)(4)^2 \right) \\
&= -9.067
\end{align*} \]

\[ \begin{align*}
S_B &= -\frac{50}{24EI} \left( 2.56 - 2.048 + 61.44 \right) \\
S_B &= -9.067
\end{align*} \]

\[ \begin{align*}
S_B' &= -\frac{PL^3}{3EI} \\
&= -\frac{PL^3}{3EI} (4)^3 \\
S_B'' &= -\frac{-By}{3EI} \\
S_B &= 21.33 \frac{By}{EI}
\end{align*} \]

\[ \begin{align*}
S_B' + S_B'' &= 0 \\
&= -\frac{50}{24EI} + 21.33 \frac{By}{EI} = 0 \\
By &= 425.1 \text{ lb}
\end{align*} \]

\[ \begin{align*}
M_A &= 1000 \text{ lb}\cdot\text{ft} \\
M_A &= 50 \text{ lb}\cdot\text{ft} + 425.1 \text{ lb}\cdot\text{ft} \\
\Rightarrow M_A &= 500 \text{ lb}\cdot\text{ft} + 425.1 \text{ lb}\cdot\text{ft} \\
A_y &= -400 + 425.1 = 0 \\
A_y &= 25.1 \\
M_A &= 100A 5\text{ ft} - 16 \\
M_A &= 500 + 16
\end{align*} \]
### Appendix D. Beam Deflections and Slopes

<table>
<thead>
<tr>
<th>Beam and Loading</th>
<th>Elastic Curve</th>
<th>Maximum Deflection</th>
<th>Slope at End</th>
<th>Equation of Elastic Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image 1" /></td>
<td>$y = \frac{PL^3}{3EI}$</td>
<td>$\theta_a = \frac{PL^2}{2EI}$</td>
<td>$y = \frac{P}{6EJ} \left( x^3 - 3Lx \right)$</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Image 2" /></td>
<td>$y = \frac{ML^2}{2EI}$</td>
<td>$\theta_a = \frac{ML}{6EI}$</td>
<td></td>
<td>$y = \frac{-M}{2EI}x^3$</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image 3" /></td>
<td>$y = \frac{wL^4}{8EI}$</td>
<td>$\theta_a = \frac{wL^3}{6EI}$</td>
<td>$y = -\frac{w}{24EI} \left( x^4 - 4Lx^2 + 6L^2 \right)$</td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Image 4" /></td>
<td>$y = \frac{-ML^2}{2EI}$</td>
<td>$\theta_a = \frac{-ML}{5EI}$</td>
<td></td>
<td>For $x \leq \frac{L}{2}$: $y = \frac{P}{48EI} \left( 6x^2 - 3L^2x \right)$</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image 5" /></td>
<td>$y = \frac{PL^2}{16EI}$</td>
<td>$\theta_a = \frac{PL^2}{6EI}$</td>
<td>For $x &lt; a$: $\theta_a = \frac{PB \left( x^2 - b^2 \right)}{6EI}$</td>
<td>For $x = a$: $\theta_a = \frac{-PBa^2}{6EI}$</td>
</tr>
<tr>
<td><img src="image6.png" alt="Image 6" /></td>
<td>$y = \frac{-wL^3}{384EI}$</td>
<td>$\theta_a = \frac{wL^3}{24EI}$</td>
<td>$y = -\frac{w}{24EI} \left( x^4 - 2Lx^2 + L^2 \right)$</td>
<td></td>
</tr>
<tr>
<td><img src="image7.png" alt="Image 7" /></td>
<td>$y = \frac{-ML^2}{5EI}$</td>
<td>$\theta_a = \frac{ML}{6EI}$</td>
<td>$\theta_b = \frac{-ML}{5EI}$</td>
<td>$y = \frac{-M}{6EI} \left( x^3 - Lx \right)$</td>
</tr>
</tbody>
</table>