Stability Issue

Linear Spring Constant

F = k \Delta

Rotational Spring

M = k \Delta \theta

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Column Buckling

Not Rigid

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\begin{align*}
\Delta \leq M_{\text{hinge}} &= 0 \\
P \sin \Delta \theta \left( \frac{L}{2} \right) - M_{\text{spring}} &= 0 \\
\frac{P}{2} L \sin \Delta \theta &= M_{\text{spring}} \\
\frac{P}{2} L \sin \Delta \theta &= \kappa_{\text{rot}} \Delta \theta
\end{align*}
For Small Angles

\[
\frac{PL}{2} \sin \Delta \theta = K_{\text{rot}} \Delta \theta
\]

\[
\Delta \theta \approx \sin \Delta \theta = \tan \Delta \theta
\]

\[
\frac{PL}{2} \Delta \theta = K_{\text{rot}} \Delta \theta
\]

\[
P = \frac{4K_{\text{rot}}}{L}
\]

Assumes Equilibrium
Stable  
Neutral  
Unstable
E quilibri u m  
E quilibrium  
E quilibrium

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Column Buckling

\[ D \leq M_{cut} = 0 \]
\[ M + P_y = 0 \]
\[ M = -P_y \]

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\[ EI \frac{d^2 y}{dx^2} = M \]
\[ EI \frac{d^2 y}{dx^2} = -Py \]
\[ \frac{d^2 y}{dx^2} + \left( \frac{Py}{EI} \right) = 0 \]

Capital \[ P \] - load

\[ Z = \frac{P}{EI} \]

Little \[ P \]

\[ \frac{d^2 y}{dx^2} + P \cdot y = 0 \]

Linear, homogeneous, 2nd order differential equation with constant coefficients

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General Solution

\[ y = A \sin px + B \cos px \]

1. \( x = 0 \), \( y = 0 \)
   
   \[ 0 = A \sin 0 + B \cos 0 \]
   
   \[ B = 0 \]

2. \( x = L \), \( y = 0 \)
   
   \[ 0 = A \sin pL \]
   
   \[ \sin pL = 0 \]
   
   \[ pL = n \pi \]
   
   \[ pL = n \pi \quad n = 1, 2, 3, \ldots \]
Column Buckling

\[ P L = \pi^2 \left( \frac{EI}{L^2} \right) \]

1, 2, 3, ... 

\[ P = \frac{(n\pi)^2 EI}{L^2} \]

\[ P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \]

Euler's Buckling Equation