A solid steel shaft is fixed at both ends as shown. The diameter of section AB is 2 in. and the diameter of section BC is 3 in. Calculate the support reactions and the maximum shearing stress in the shaft. Assume that $G = 11 \times 10^6 \frac{in^2}{lb}$. 

Equilibrium Relationships

\[ \sum T = 0 \]

\[ T_A + T_C - 10,000 \text{ in-lb} = 0 \]

\[ T_A + T_C = 10,000 \text{ in-lb} \]
<table>
<thead>
<tr>
<th>Examine Section</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA</td>
<td>T_{AB}</td>
</tr>
</tbody>
</table>

\[ T_{PAB} = \frac{\pi}{2} \left( r_2^4 - r_1^4 \right) \]  
\[ T_{PAB} = \frac{\pi}{2} \left( \left( \frac{1}{2} \right)^4 - 0^4 \right) \]  
\[ T_{PAB} = 1.571 \text{ in}^4 \]  

\[ \sigma = \tau = 0 \]

\[ TA - T_{AB} = 0 \]

\[ T_{AB} = TA \]

\[ T_{AB} = \frac{T_{AB} \cdot r_{AB}}{I_{PAB}} \]

\[ \phi_{AB} = \frac{T_{AB} \cdot L_{AB}}{E \cdot A_{AB}} \]

\[ T_{AB} = \frac{TA}{1.571 \text{ in}^4} \]

\[ \phi_{AB} = \frac{7.638 \cdot TA}{57.2 \text{ in} \cdot \text{lb}} \]
**Examined Section BC**

\[ T_A - 10,000 - T_{BC} = 0 \]
\[ T_{BC} = T_A - 10,000 \]

\[ \tau_{BC} = \frac{T_{BC}}{I_{BC}} \]
\[ \phi_{BC} = \frac{T_{BC} L_{BC}}{G_{BC}} \]

\[ T_{BC} = \frac{(T_A - 10,000)(1.75)}{7.952} \]
\[ \phi_{BC} = 1.258 \frac{T_A}{G_{st+1}} - 12,580 \frac{1}{G_{st+1}} \]
Compatibility

\[ \phi_{AB} + \phi_{BC} = \phi_{AC} = 0 \quad (25) \]

\[ 7.638 \frac{TA}{641} + 1.258 \frac{TB}{85} - \frac{12580}{257} = 0 \]

\[ 8.816 TA = 12580 \quad (27) \]

\[ TA = 1414.1 \text{ in}-\text{lb} \quad (28) \]

Then plugging back into equation (2)

\[ TA + TC = 10000 \text{ in}-\text{lb} \quad (29) \]

\[ 1414.1 + TC = 10000 \quad (30) \]

\[ TC = 8585.9 \text{ in}-\text{lb} \quad (31) \]

Check stresses

From equation (21)

\[ TB_C = 220 TA = 220 \text{ in}-\text{lb} \quad (32) \]

\[ TB_C = 220 (1414.1) - 220 \text{ in}-\text{lb} \quad (33) \]
\[ T_{BC} = -1890 \text{ in-lb} \]

From equation (10):
\[ T_{AB} = 1.6365T_A \]
\[ T_{AB} = (1.6365)(14/1,1) \]
\[ T_{AB} = 900 \text{ psi} \]

Materials do not care whether shearing stresses are + or -.
All that matters is magnitude.

because \[ |-1890| > 900 \]

The maximum shearing stress occurs in section BC and is equal to 1890 psi.