5.5 Bending Strength of Compact Shapes

**Example 5.4 pg 203c continued (Lb = 20')**

b. \( L_b = 20 \) ft and \( C_b = 1.0 \). First, determine \( L_p \) and \( L_r \)

\[
L_p = 1.76r_s \sqrt{\frac{E}{F_y}} = 1.76(2.46) \sqrt{\frac{29,000}{50}} = 104.3 \text{ in.} = 8.692 \text{ ft}
\]

The following terms will be needed in the computation of \( L_r \):

\[
r_s = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{121(5380)}}{103} = 7.833 \text{ in.}^2
\]

\[
r_s = \sqrt{7.833} = 2.799 \text{ in.}
\]

\( r_s \) can also be found in the dimensions and properties tables. For a W14 x 68, it is given as 2.80 in.

\[
h_o = d - t_f = 14.0 - 0.720 = 13.28 \text{ in.}
\]

\( h_o \) can also be found in the dimensions and properties tables. For a W14 x 68, it is given as 13.3 in.

For a doubly-symmetric I-shape, \( c = 1.0 \). From AISC Equation F2-6

\[
L_r = 1.95r_s \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \frac{1 + 6.76 \left( \frac{0.7F_y S_x h_o}{E J_c} \right)^2}{1 + 6.76 \left( \frac{0.7(50)(103)(13.28)}{29,000(3.01)(1.0)} \right)^2}} \]

\[
= 1.95(2.799) \frac{29,000}{0.7(50)} \frac{3.01(1.0)}{103(13.28)} \sqrt{1 + \frac{1 + 6.76 \left( \frac{0.7(50)(103)(13.28)}{29,000(3.01)(1.0)} \right)^2}{1 + 6.76 \left( \frac{0.7(50)(103)(13.28)}{29,000(3.01)(1.0)} \right)^2}}
\]

\[
= 351.3 \text{ in.} = 29.28 \text{ ft}
\]

Since \( L_p < L_g < L_r \)

\[
M_n = C_b \left[ M_p - \left( M_p - \frac{0.7F_y S_x}{L_r - L_p} \right) \left( L_b - L_p \right) \right] \leq M_p
\]

\[
= 1.0 \left[ 5.750 - \left( 5.750 - \frac{0.7 \times 50 \times 103}{12} \right) \left( \frac{20 - 8.692}{29.28 - 8.692} \right) \right]
\]

\[
= 4572 \text{ in.-kips} = 381.0 \text{ ft-kips} < M_p = 479.2 \text{ ft-kips}
\]

**LRFD Solution** The design strength is

\[
M_n \leq \phi_b M_n = 0.90(381.0) = 343 \text{ ft-kips}
\]
Table 3-10 (continued)

W-Shapes

Available Moment vs. Unbraced Length
c. $L_b = 30\text{ ft}$ and $C_b = 1.0$

$L_b > L_r = 29.28\text{ ft}$, so elastic lateral-torsional buckling controls. Begin from AISC Equation F2-4,

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_t)^2} \sqrt{1 + 0.078 \frac{J_c}{S_s h_o \left(\frac{L_b}{r_t}\right)^2}}$$

Variables found on $[1-25](203\text{ h})$

From AISC Equation F2-3,

$$M_n = F_{cr} S_s = 33.90(103) = 3492\text{ in.-kips} = 291.0\text{ ft-kips} < M_p = 479.2\text{ ft-kips}$$

**LRFD Solution** $M_{u} < \phi_{M_n} = 0.90(291.0) = 262\text{ ft-kips}$

If the moment within the unbraced length $L_b$ is uniform (constant), there is no moment gradient and $C_b = 1.0$. If there is a moment gradient, the value of $C_b$ is given by

$$C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

(AISC Equation F1-1)

(See next page)
Why Absolute Value? Both beams shown have a flange in compression wanting to L.T.B.

WORST CASE

\[ C_b = 1.0 \]

The purpose of \( C_b \) is to account for the fact that you did not stress every fiber on the compressive side of the beam between the points of lateral support] to the same extreme values as assumed by Eq. 5.5 pg. 204 Segui (Timoshenko)

Eq. 2-6 [16, 1-47] LRFD

Study two ranges: AB & BC A B L B L B L B 12' 16' 12' 12'
If the moment within the unbraced length $L_b$ is uniform (constant), there is no moment gradient and $C_b = 1.0$. If there is a moment gradient, the value of $C_b$ is given by

$$C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

(AISC Equation F1-1)

where

$M_{\text{max}} =$

$M_A =$

$M_B =$

$M_C =$

AISC Equation F1-1 is valid for doubly-symmetric members and for singly-symmetric members in single curvature.

When the bending moment is uniform, the value of $C_b$ is

$$C_b = \frac{12.5 M}{2.5 M + 3M + 4M + 3M} = 1.0 \text{ WORST CASE}$$
EXAMPLE 5.5

SOLUTION  Because of symmetry, the maximum moment is at midspan, so

\[ M_{\text{max}} = M_B = \frac{1}{8} wL^2 = M_{\frac{L}{2}} = M_{\text{m A X}} \]

Also because of symmetry, the moment at the quarter point equals the moment at the three-quarter point. From Figure 5.14,

\[ M_{\frac{L}{4}} = M_{\frac{3L}{4}} = M_A = M_C = \frac{wL}{2} \left( \frac{L}{4} \right) - \frac{wL}{4} \left( \frac{L}{8} \right) = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3}{32} wL^2. \]

FIGURE 5.14

Since this is a W shape (doubly symmetric), AISC Equation (F1-1) is applicable.

\[ C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} = \frac{12.5 \left( \frac{1}{8} \right)}{2.5 \left( \frac{1}{8} \right) + 3 \left( \frac{3}{32} \right) + 4 \left( \frac{1}{8} \right) + 3 \left( \frac{3}{32} \right)} = 1.14. \]

ANSWER  \( C_b = 1.14. \) Also see [3-18] (210b)
4 beam floor system

$L_b = 40'$

Top View (Plan View)

Laterally Braced at quarter-point between beams
TOP VIEW

L_b = 10'
Figure 5.15 shows the value of $C_b$ for several common cases of loading and lateral support. Values of $C_b$ for other cases can be found in Part 3 of the Manual, "Design of Flexural Members."

For unbraced cantilever beams, AISC specifies a value of $C_b$ of 1.0. A value of 1.0 is always conservative, regardless of beam configuration or loading, but in some cases it may be excessively conservative.

The effect of $C_b$ on the nominal strength is illustrated in Figure 5.16. Although the strength is directly proportional to $C_b$, this graph clearly shows the importance of observing the upper limit of $M_p$, regardless of which equation is used for $M_n$.

Part 3 of the Steel Construction Manual, "Design of Flexural Members," contains several useful tables and charts for the analysis and design of beams. For example, Table 3-2, "W Shapes, Selection by $Z_x$," (hereafter referred to as the "$Z_x$ table"), lists shapes commonly used as beams, arranged in order of available flexural strength—both $\phi_b M_{px}$ and $M_{px}/\Omega_b$. Other useful constants that are tabulated include $L_p$ and $L_r$ (which is particularly tedious to compute). These two constants can
Figure 5.15 shows the value of $C_b$ for several common cases of loading and support. Values of $C_b$ for other cases can be found in Part 3 of the Manual, "Design of Flexural Members."

**Figure 5.15**

- (a) $L_b = L$, $C_b = 1.14$
- (b) $L_b = L/2$, $C_b = 1.30$
- (c) $L_b = L$, $C_b = 1.32$
- (d) $L_b = L/2$
- (e) $L_b = L$, $C_b = 2.27$
- (f) $AB$ and $CD$:
  - $C_b = 1.67$
  - $BC$:
    - $C_b = 1.00$

**Notes:**
- $L_b = L$
- $C_b = 1.67$
- Don't study this region; study each region individually.
Table 3-1
Values for $C_b$ for Simply Supported Beams

<table>
<thead>
<tr>
<th>Load</th>
<th>Lateral Bracing Along Span</th>
<th>$C_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{L}{2}$</td>
<td>None</td>
<td>$\frac{L}{2}$, 1.32</td>
</tr>
<tr>
<td></td>
<td>Load at midpoint</td>
<td></td>
</tr>
<tr>
<td>$\frac{L}{3}$</td>
<td>None</td>
<td>$\frac{L}{3}$, 1.14</td>
</tr>
<tr>
<td></td>
<td>Loads at third points</td>
<td></td>
</tr>
</tbody>
</table>

**At third points**

<table>
<thead>
<tr>
<th></th>
<th>1.01</th>
<th>1.02</th>
<th>1.03</th>
<th>1.04</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>At quarter points</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At fifth points</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Lateral bracing must always be provided at points of support per AISC Specification Chapter F.
Table 3-2 (continued)  
W-Shapes  
Selection by $Z_x$

| Shape | $Z_x$ | $M_{SLO}$ | $\phi_0 M_{SB}$ | $M_{260G}$ | $\phi_0 M_{260}$ | $\phi_0 BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{SB}$ | $\phi_0 V_{SB}$ | Kips
|-------|--------|------------|----------------|------------|----------------|-------------|------|------|------|---------|----------------|------|
| W21x55 | 126 | 314 | 473 | 92 | 289 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W14x74 | 126 | 314 | 473 | 96 | 294 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W18x60 | 123 | 307 | 461 | 96 | 284 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W12x79 | 119 | 297 | 446 | 87 | 281 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W14x68 | 115 | 287 | 431 | 80 | 270 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W13x88 | 113 | 282 | 424 | 72 | 259 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W19x55 | 112 | 279 | 420 | 72 | 258 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234
| W21x60 | 110 | 274 | 413 | 65 | 248 | 16.6 | 16.3 | 6.11 | 17.4 | 1140 | 458 | 234

ASD | LRFD

1 Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

$\Omega_0 = 1.67, \phi_0 = 0.90$

$\Omega_y = 1.50, \phi_y = 1.00$

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
Study span AB only

Study spans BC & CD only

\[ C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_a + 4M_b + 3M_c} \]

FIGURE 1.67 (pg 210a)

Cb uses Mmax within length of current interest between support points.
\[ C_b = \frac{12.5 \cdot M_{\text{max}}}{2.5 \cdot M_{\text{max}} + 0 + 0 + 0} = 5 \]

Often you will be unable to use \( C_b = 5 \).

\[ [3-25] \]
\[ (210 \text{ba}) \]
\[ 473 = \phi \cdot M_p \]
\[ \text{KFT} \]
\[ 270 = \phi \cdot M_R \]
\[ \text{KFT} \]

\[ W \cdot 21 \times 5.5 \]
\[ [3-25] \]
\[ (210 \text{ba}) \]

\[ 5 \times 231 \text{KFT} = 1155 \text{KFT} > \phi \cdot M_p = 473 \text{ KFT} \]
Table 3-10 (continued)

W Shapes

<table>
<thead>
<tr>
<th>Available Moment vs. Unbraced Length</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ASD</strong></td>
</tr>
<tr>
<td>160</td>
</tr>
<tr>
<td>234</td>
</tr>
<tr>
<td>228</td>
</tr>
</tbody>
</table>

**Available Moment, \( M_{y}/\Omega \) (1 kip-ft increments)**

<table>
<thead>
<tr>
<th><strong>kip-ft</strong></th>
<th><strong>kip-ft</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>210</td>
<td>204</td>
</tr>
<tr>
<td>198</td>
<td>192</td>
</tr>
<tr>
<td>186</td>
<td>180</td>
</tr>
<tr>
<td>160</td>
<td>154</td>
</tr>
</tbody>
</table>

**Unbraced Length (0.5-ft increments)**

18  20  22  24  26  28  30  32  34

**American Institute of Steel Construction, Inc.**
FIGURE 5.16

The diagrams illustrate the bending strength of noncompact shapes. The curves show the moment capacity of beams with compact and noncompact sections, as indicated by the compact shapes notation.

Also be found in several other tables in Part 3 of the Manual. We cover additional design aids in other sections of this chapter.

5.6 BENDING STRENGTH OF NONCOMPACT SHAPES

\[(199^1/2)[16.1-49] \quad M_n = M_p - (M_p - 0.7F_sS_s) \left( \frac{\lambda_e - \lambda_p}{\lambda_v - \lambda_p} \right) \]

(AISC Equation F3-1)

where

\[\lambda_e = \frac{b_f}{2f_f}\]

\[\lambda_p = 0.38 \sqrt{\frac{E}{F_y}}\]

\[\lambda_v = 1.0 \sqrt{\frac{E}{F_y}}\]

\[\frac{(M_p - 0.7F_sS_s)(L_b - L_p)}{L_r - L_p)}\]

\[L_b \text{ not a factor.}\]

\[F3-1 \approx F2-2[16.1-47]\]

(199^1/2)

except no \(C_b\) since no \(L_b\)

and \(\lambda_f\) replaces \(L_b\)

The webs of all hot-rolled shapes in the Manual are compact, so the noncompact shapes are subject only to the limit states of lateral-torsional buckling and flange local...
EXAMPLE 5.6

A simply supported beam with a span length of 45 feet is laterally supported at its ends and is subjected to the following service loads:

- Dead load = 400 lb/ft (including the weight of the beam)
- Live load = 1000 lb/ft

If $F_y = 50$ ksi, is a W14 × 90 adequate?

SOLUTION

Determine whether the shape is compact, noncompact, or slender:

\[
\begin{bmatrix}
1 - 2.5 \\
12 \bar{b}
\end{bmatrix} = \begin{bmatrix}
16.1 - 17 \\
(200 \bar{a}a)
\end{bmatrix}
\]

\[
0.38 \sqrt{\frac{29,000}{50}} = 9.15
\]

\[
0 \sqrt{\frac{29,000}{50}} = 24.1
\]

The shape is noncompact. Check the capacity based on the limit of buckling:

\[
\text{1-kips} \begin{bmatrix}
1 - 2.5 \\
212 \bar{b}
\end{bmatrix} \begin{bmatrix}
16.1 - 49 \\
(199 \bar{a}a)
\end{bmatrix}
\]

\[
:143 \left( \frac{10.2 - 9.15}{24.1 - 9.15} \right)
\]

\[
\text{mit state of lateral-torsional buckling. From the}
\]

\[
\begin{bmatrix}
3 - 24 \\
(212 \bar{c})
\end{bmatrix}
\]

\[
\text{is by } \text{elastic LTB.}
\]
From Part 1 of the Manual,

\[
4.101 \text{ in. (14th ed.)} \left\{ 1.25 \right\} (2126) \]

\[1.66\]

ded, simply supported beam with lateral support at the ends,

(Fig. 5.15a) \[3 \left[ 18 \right] (2126)\]

metric I shape, \(c = 1.0\). AISC Equation F2-4 gives \(\left[ 16.1 - 4 \right] (1998)\)

\[
\sqrt{1 + 0.078 \frac{J_c}{S_x h_0} \left( \frac{L_b}{r_{fe}} \right)^2} \left[ 9,000 \right] \left[ \frac{4,06(1.0)}{143(13.3)} \left( \frac{45 \times 12}{4.11} \right) \right]^{2} = 37.20 \text{ ksi} \]

on F2-3, \(7.20(143) = 5320 \text{ in.-kips} < M_p = 7850 \text{ in.-kips} \)

the nominal strength based on flange local buckling, so lateral-controls.

is

320\) = 4788 \text{ in.-kips} = 399 \text{ ft-kips}

nd moment are

\[1.6w_L = 1.2(0.400) + 1.6(1.000) = 2.080 \text{ kips/ft} \]

\[= \frac{1}{8}(2.080)(45)^2 = 527 \text{ ft-kips} > 399 \text{ ft-kips} \quad (N.G.) \]

he beam does not have adequate moment strength.