of equal forces acting at each flange. As an approximation, each flange can be considered to resist each of these forces independently. Consequently, the problem is reduced to a case of bending of two shapes, each one loaded through its shear center. In each of the two situations depicted in Figure 5.49, only about half the cross section is considered to be effective with respect to its y axis; therefore, when considering the strength of a single flange, use half the tabulated value of $Z_y$ for the shape.
Design of Roof Purlins

Roof purlins that are part of a sloping roof system can be subjected to biaxial bending of the type just described. For the roof purlin shown in Figure 5.50, the load is vertical, but the axes of bending are inclined. This condition corresponds to the loading of Figure 5.49a. The component of load normal to the roof will cause bending about the x-axis, and the parallel component bends the beam about its y-axis. If the purlins are simply supported at the trusses (or rigid frame rafters), the maximum bending moment about each axis is \( \frac{wL^2}{8} \), where \( w \) is the appropriate component of load. If sag rods are used, they will provide lateral support with respect to x-axis bending and will act as transverse supports for y-axis bending, requiring that the purlin be treated as a continuous beam. For uniform sag-rod spacings, the formulas for continuous beams in Part 3 of the Manual can be used.
Example 5.19

A roof system consists of trusses of the type shown in Figure 5.51 spaced 15 feet apart. Purlins are to be placed at the joints and at the midpoint of each top-chord member. Sag rods will be located at the center of each purlin. The total gravity load, including an estimated purlin weight, is 42 psf of roof surface, with a live-load-to-dead-load ratio of 1.0. Assuming that this is the critical loading condition, use A36 steel and select a channel shape for the purlins.

LRFD Solution

For the given loading condition, dead load plus a roof live load with no wind or snow, load combination 3 will control:

\[ w_u = 1.2w_D + 1.6L_r = 1.2(21) + 1.6(21) = 58.80 \text{ psf} \]

The width of roof surface tributary to each purlin is

\[ \sqrt{\frac{15}{10}} = 7.906 \text{ ft} = \sqrt{\left(\frac{60}{8}\right)^2 + \left(\frac{10}{4}\right)^2} \]

Then

Purlin load = 58.80(7.906) = 464.9 lb/ft

Normal component = \( \frac{3}{\sqrt{10}} (464.9) = 441.0 \text{ lb/ft} \) \((x-x\text{axis})\)

Parallel component = \( \frac{1}{\sqrt{10}} (464.9) = 147.0 \text{ lb/ft} \) \((y-y\text{axis})\)

and

\[ M_{ux} = \frac{1}{8} (0.4410)(15)^2 = 12.40 \text{ ft-kips, } X-X \text{ request} \]

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, Continuous Beams, \( M_{ux} = \frac{1}{8} (0.4410)(15)^2 \), the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by

\[ M = 0.125wl^2 \]

where

\( w = \text{uniform load intensity} \)
\( l = \text{span length} \)
### Table 3-22c

**Continuous Beams**

**Moments and Shear Coefficients – Equal Spans, Equally Loaded**

#### Uniform Load

<table>
<thead>
<tr>
<th>Moment in terms of $w_i$</th>
<th>Shear in terms of $w_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+0.07$</td>
<td>$+0.07$</td>
</tr>
<tr>
<td>$+0.06$</td>
<td>$+0.05$</td>
</tr>
<tr>
<td>$+0.05$</td>
<td>$+0.06$</td>
</tr>
<tr>
<td>$+0.07$</td>
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<tr>
<td>$+0.08$</td>
<td>$+0.09$</td>
</tr>
<tr>
<td>$+0.09$</td>
<td>$+0.10$</td>
</tr>
<tr>
<td>$+0.10$</td>
<td>$+0.10$</td>
</tr>
</tbody>
</table>

#### Concentrated Loads

<table>
<thead>
<tr>
<th>Moment in terms of $P_i$</th>
<th>Shear in terms of $P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+1.65$</td>
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</tr>
<tr>
<td>$+1.66$</td>
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<tr>
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<td>$+1.71$</td>
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</table>

#### Concentrated Loads

<table>
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<th>Shear in terms of $P_i$</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td>$+1.39$</td>
<td>$+1.39$</td>
</tr>
</tbody>
</table>

### American Institute of Steel Construction, Inc.
The width of roof surface tributary to each purlin is
\[ \frac{15 \times \sqrt{10}}{2} \approx 7.906 \text{ ft} \]

Then
\[ \text{Purlin load} = 58.80(7.906) = 464.9 \text{ lb/ft} \]

Normal component \[= \frac{3}{\sqrt{10}} (464.9) = 441.0 \text{ lb/ft} \]

Parallel component \[= \frac{1}{\sqrt{10}} (464.9) = 147.0 \text{ lb/ft} \]

and
\[ M_{\text{max}} = \frac{1}{8} (0.4410)(15)^2 = 12.40 \text{ ft-kips} \]

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 5-27 of "Shears, Moments, and Deflections," in Part 3 of the Manual, the bending moment at the interior support with only one span loaded is
\[ M = \frac{0.125 w L^2}{8} \]

where
- \( w \) = uniform load intensity
- \( L \) = span length (two equal spans)

With both spans loaded, the moment can be obtained by superposition:
\[ M = M_{\text{max}} = \frac{1}{16} w L^2 \]
\[ \therefore M_{\text{ky}} = \frac{1}{8} (0.1470)(15/2)^2 = 1.034 \text{ ft-kips} \]

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of 15\(\frac{2}{3} \) ft, try a C10 × 15.3. For \( C_b = 1.0, \phi_b M_{\text{ax}} = 33.2 \text{ ft-kips} \). From Figure 5.15b, \( C_b \) is 1.30 for the load and lateral support conditions of this beam. Therefore,
Chapter 5  Beams

The maximum moment about the y axis is therefore

\[ M_{y} = 0.125(0.1470)(15.2)^2 = 1.034 \text{ ft-kips} \]

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of \( \frac{15}{2} = 7.5 \) ft, try a C10 \( \times \) 15.3.

For \( C_b = 1.0 \), \( \phi_b M_{nx} = 33.0 \) ft-kips. From Figure 5.15b, \( C_b \) is 1.30 for the load and lateral support conditions of this beam. Therefore,

\[ \phi_b M_{nx} = 1.30(33.0) = 42.90 \text{ ft-kips} \]

From the uniform load table for C shapes,

\[ \phi_b M_{py} = 43.0 \text{ ft-kips} > 42.90 \text{ ft-kips} \]

\[ \Phi_{M_p} = \frac{0.9 (15.1 \text{ in}^3)}{36 \text{ in}} \]

\[ \frac{1}{1.15} = 0.86 \]

\[ \phi_M M_{py} = \phi_b (1.6 F_y S_y) = 0.90(1.6)(36)(1.15) \]

\[ = 59.62 \text{ in.-kips} = 4.968 \text{ ft-kips} \]

Because the load is applied to the top flange, use only half this capacity to account for the torsional effects. From Equation 5.22,

\[ \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{12.40}{42.9} + \frac{1.034}{4.968/2} = 0.705 < 1.0 \text{ (OK)} \]

The shear is

\[ V = \frac{0.4410(15)}{2} = 3.31 \text{ kips} \]

From the uniform load tables,

\[ \phi V_n = 46.7 \text{ kips} > 3.31 \text{ kips} \text{ (OK)} \]

Answer  Use a C10 \( \times \) 15.3.

**ASD Solution**  For dead load plus a roof live load, load combination 3 will control:

\[ q_d = q_D + q_{LR} = 42 \text{ psf} \]
# Table 3-1

## Values of $C_b$ for Simply Supported Beams

<table>
<thead>
<tr>
<th>Load</th>
<th>Lateral Bracing Along Span</th>
<th>$C_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Load at midpoint" /></td>
<td>None</td>
<td><img src="image2" alt="Value1" /></td>
</tr>
<tr>
<td><img src="image3" alt="Load at third points" /></td>
<td>None</td>
<td><img src="image4" alt="Value2" /></td>
</tr>
<tr>
<td><img src="image5" alt="Load at quarter points" /></td>
<td>None</td>
<td><img src="image6" alt="Value3" /></td>
</tr>
<tr>
<td><img src="image7" alt="Load at fifth points" /></td>
<td>None</td>
<td><img src="image8" alt="Value4" /></td>
</tr>
</tbody>
</table>

**Note:** Lateral bracing must always be provided at points of support per AISC Specification Chapter F.

**LATERALLY BRACED**

**AMERICAN INSTITUTE OF STEEL CONSTRUCTION**
### Table 1-5

#### C Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area, $A$</th>
<th>Depth, $d$</th>
<th>Web Thickness, $t_w$</th>
<th>Flange Thickness, $t_f$</th>
<th>Distance</th>
<th>Workable Gage</th>
<th>$t_w$</th>
<th>$h_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15×50</td>
<td>14.7</td>
<td>15.0</td>
<td>0.716</td>
<td>3/16</td>
<td>3/8</td>
<td>3/2</td>
<td>3/4</td>
<td>0.650</td>
</tr>
<tr>
<td>×40</td>
<td>11.8</td>
<td>15.0</td>
<td>0.520</td>
<td>1/4</td>
<td>3/2</td>
<td>3/4</td>
<td>0.650</td>
<td>9/16</td>
</tr>
<tr>
<td>×33.9</td>
<td>10.0</td>
<td>15.0</td>
<td>0.400</td>
<td>3/16</td>
<td>3/8</td>
<td>3/4</td>
<td>0.650</td>
<td>9/16</td>
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<tr>
<td>C12×30</td>
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<td>3/4</td>
<td>0.501</td>
<td>1/2</td>
</tr>
<tr>
<td>×25</td>
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<td>12.0</td>
<td>0.387</td>
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<td>3/8</td>
<td>3/4</td>
<td>0.501</td>
<td>1/2</td>
</tr>
<tr>
<td>×20.7</td>
<td>6.08</td>
<td>12.0</td>
<td>0.282</td>
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<td>3/8</td>
<td>3/4</td>
<td>0.501</td>
<td>1/2</td>
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<tr>
<td>C10×30</td>
<td>8.81</td>
<td>10.0</td>
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<td>3/8</td>
<td>3/8</td>
<td>0.436</td>
<td>1/8</td>
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<tr>
<td>×25</td>
<td>7.34</td>
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<td>3/8</td>
<td>3/8</td>
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<td>1/8</td>
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<tr>
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<td>1/8</td>
<td>3/8</td>
<td>3/8</td>
<td>0.436</td>
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<tr>
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<td>10.0</td>
<td>0.240</td>
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<td>3/8</td>
<td>3/8</td>
<td>0.436</td>
<td>1/8</td>
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<tr>
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<td>3/8</td>
<td>2/8</td>
<td>2/4</td>
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<tr>
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<td>3/8</td>
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<td>0.413</td>
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<tr>
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<td>0.233</td>
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<td>3/8</td>
<td>2/8</td>
<td>2/4</td>
<td>0.413</td>
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<tr>
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<td>3/8</td>
<td>2/8</td>
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<tr>
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<td>0.220</td>
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<td>3/8</td>
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<td>0.419</td>
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<td>3/8</td>
<td>2/8</td>
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<td>3/8</td>
<td>2/8</td>
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<tr>
<td>×9.8</td>
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<td>0.210</td>
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<td>3/8</td>
<td>2/8</td>
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<td>2/8</td>
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<td>1/16</td>
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<td>1/16</td>
<td>1/16</td>
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<td>1/16</td>
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<td>1/16</td>
<td>1/16</td>
<td>0.273</td>
<td>1/4</td>
</tr>
</tbody>
</table>

---

The actual size, combination, and orientation of fastener components should be compared with the geometry of the cross-section to ensure compatibility.

Flange is too narrow to establish a workable gage.
<table>
<thead>
<tr>
<th>Nominal Wt.</th>
<th>Shear Ctr.</th>
<th>Axis X-X</th>
<th>Axis Y-Y</th>
<th>Torsional Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_o$</td>
<td>$l$</td>
<td>$S$</td>
<td>$r$</td>
</tr>
<tr>
<td>lb/ft</td>
<td>in.</td>
<td>in.$^2$</td>
<td>in.$^3$</td>
<td>in.$^3$</td>
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<td>5.45</td>
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</tr>
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<td>4.29</td>
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<td>4.61</td>
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<tr>
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<td>13.5</td>
<td>3.87</td>
</tr>
</tbody>
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### Table 3-8 (continued)

**Maximum Total Uniform Load, kips**

**Shapes**

<table>
<thead>
<tr>
<th>Design</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15.3</th>
<th>20</th>
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</thead>
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<tr>
<td>2</td>
<td>174</td>
<td>262</td>
<td>136</td>
<td>205</td>
<td>98.0</td>
</tr>
<tr>
<td>3</td>
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**Beam Properties**

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</table>

**Note:** Beams must be laterally supported if Table 3-8 is used. Available strength tabulated above heavy line is limited by available shear strength.
Design of Roof Purlins

Roof purlins that are part of a sloping roof system can be subjected to biaxial bending of the type just described. For the roof purlin shown in Figure 5.50, the load is vertical, but the axes of bending are inclined. This condition corresponds to the loading of Figure 5.49a. The component of load normal to the roof will cause bending about the $x$ axis, and the parallel component bends the beam about its $y$ axis. If the purlins are simply supported at the trusses (or rigid frame rafters), the maximum bending moment about each axis is $wL^2/8$, where $w$ is the appropriate component of load. If sag rods are used, they will provide lateral support with respect to $x$-axis bending and will act as transverse supports for $y$-axis bending, requiring that the purlin be treated as a continuous beam. For uniform sag-rod spacings, the formulas for continuous beams in Part 3 of the Manual can be used.

![Figure 5.50](image)

EXAMPLE 5.19

A roof system consists of trusses of the type shown in Figure 5.51 spaced 15 feet apart. Purlins are to be placed at the joints and at the midpoint of each top-chord member. Sag rods will be located at the center of each purlin. The total gravity load, including an estimated purlin weight, is 42 psf of roof surface, with a live-load-to-dead-load ratio of 1.0. Assuming that this is the critical loading condition, use A36 steel and select a channel shape for the purlins.

![Figure 5.51](image)
For the given loading condition, dead load plus a roof live load with no wind or snow, load combination 3 will control:

\[ w_u = 1.2w_D + 1.6L_r = 1.2(21) + 1.6(21) = 58.80 \text{ psf} \]

The width of roof surface tributary to each purlin is

\[ \frac{15 \sqrt{10}}{3} = 7.906 \text{ ft} \]

Then

Purlin load = 58.80(7.906) = 464.9 \text{ lb/ft}

Normal component = \(\frac{3}{\sqrt{10}}\)(464.9) = 441.0 \text{ lb/ft}

Parallel component = \(\frac{1}{\sqrt{10}}\)(464.9) = 147.0 \text{ lb/ft}

and

\[ M_{ux} = \frac{1}{8}(0.4410)(15)^2 = 12.40 \text{ ft-kips} \]

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, "Continuous Beams," the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by

\[ M = 0.125w\ell^2 \]

where

- \( w \) = uniform load intensity
- \( \ell \) = span length

The maximum moment about the \( y \) axis is therefore

\[ M_{yy} = 0.125(0.1470)(15/2)^2 = 1.034 \text{ ft-kips} \]

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of \( 15/2 = 7.5 \text{ ft} \), try a C10 \( \times \) 15.3.

For \( C_b = 1.0 \), \( \phi_b M_{nx} = 33.0 \text{ ft-kips} \). From Figure 5.15b, \( C_b \) is 1.30 for the load and lateral support conditions of this beam. Therefore,

\[ \phi_b M_{nx} = 1.30(33.0) = 42.90 \text{ ft-kips} \]

From the uniform load table for C shapes,

\[ \phi_b M_{px} = 42.9 \text{ ft-kips} \quad \therefore \text{ Use } \phi_b M_{nx} = 42.9 \text{ ft-kips.} \]
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This shape is compact (no footnote in the uniform load tables), so

\[ \phi_b M_{ny} = \phi_b M_{py} = \phi_b F_y Z_y = 0.90(36)(2.34) = 75.82 \text{ in.-kips} = 6.318 \text{ ft-kips} \]

But

\[ \frac{Z_y}{S_y} = \frac{2.34}{1.15} = 2.03 > 1.6 \]

\[ \therefore \phi_b M_{ny} = \phi_b (1.6F_yS_y) = 0.90(1.6)(36)(1.15) \]

\[ = 59.62 \text{ in.-kips} = 4.968 \text{ ft-kips} \]

Because the load is applied to the top flange, use only half this capacity to account for the torsional effects. From Equation 5.22,

\[ \frac{M_{nx}}{\phi_b M_{nx}} + \frac{M_{ny}}{\phi_b M_{ny}} = \frac{12.40}{42.9} + \frac{1.034}{4.968/2} = 0.705 < 1.0 \quad \text{(OK)} \]

The shear is

\[ V_n = \frac{0.4410(15)}{2} = 3.31 \text{ kips} \]

From the uniform load tables,

\[ \phi_V V_n = 46.7 \text{ kips} > 3.31 \text{ kips} \quad \text{(OK)} \]

**ANSWER**

Use a C10 \times 15.3.

For dead load plus a roof live load, load combination 3 will control:

\[ q_a = q_D + q_{Lr} = 42 \text{ psf} \]

The width of roof surface tributary to each purlin is

\[ \frac{15 \sqrt{10}}{2 \frac{3}{2}} = 7.906 \text{ ft} \]

Then

Purlin load = 42(7.906) = 332.1 lb/ft

Normal component = \[ \frac{3}{\sqrt{10}} (332.1) = 315.1 \text{ lb/ft} \]

Parallel component = \[ \frac{1}{\sqrt{10}} (332.1) = 105.0 \text{ lb/ft} \]
and

$$M_{ax} = \frac{1}{8} wL^2 = \frac{1}{8} (0.3151)(15)^2 = 8.862 \text{ ft-kips}$$

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, "Continuous Beams," the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by

$$M = 0.125w\ell^2$$

where

- $w =$ uniform load intensity
- $\ell =$ span length

The maximum moment about the y axis is therefore

$$M_{oy} = 0.125(0.1050)(15/2)^2 = 0.7383 \text{ ft-kips}$$

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of $15/2 = 7.5 \text{ ft}$, try a $C10 \times 15.3$.

For $C_b = 1.0, M_{nx}/\Omega_b = 22.0 \text{ ft-kips}$. From Figure 5.15b, $C_b = 1.30$ for the load and lateral support conditions of this beam. Therefore,

$$M_{nx}/\Omega_b = 1.30(22.0) = 28.60 \text{ ft-kips}$$

From the uniform load tables,

$$M_{px}/\Omega_b = 28.6 \text{ ft-kips}$$

$\therefore$ Use $M_{nx}/\Omega_b = 28.6 \text{ ft-kips}.$

This shape is compact (no footnote in the uniform load tables), so

$$M_{ny}/\Omega_b = M_{py}/\Omega_b = F_yZ_y/\Omega_b = 36(2.34)/1.67 = 50.44 \text{ in.-kips} = 4.203 \text{ ft-kips}$$

But

$$\frac{Z_y}{S_y} = \frac{2.34}{1.15} = 2.03 > 1.6$$

$\therefore$ $M_{ny}/\Omega_b = 1.6F_yS_y/\Omega_b = 1.6(36)(1.15)/1.67 = 39.66 \text{ in.-kips} = 3.300 \text{ ft-kips}$

Because the load is applied to the top flange, use only half of this capacity to account for the torsional effects. From Equation 5.23,

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{oy}}{M_{ny}/\Omega_b} = \frac{8.862}{28.6} + \frac{0.7383}{3.300/2} = 0.757 < 1.0 \quad (OK)$$
The maximum shear is
\[ V_n = \frac{0.3151(15)}{2} = 2.36 \text{ kips} \]

From the uniform load tables,
\[ \frac{V_n}{\Omega_v} = 31.0 \text{ kips} > 2.36 \text{ kips} \quad \text{(OK)} \]

**ANSWER** Use a C10 \( \times \) 15.3.

### 5.16 BENDING STRENGTH OF VARIOUS SHAPES

W, S, M, and C shapes are the most commonly used hot-rolled shapes for beams, and their bending strength has been covered in the preceding sections. Other shapes are sometimes used as flexural members, however, and this section provides a summary of some of the relevant AISC provisions. All equations are from Chapter F of the Specification. (The width-to-thickness limits are from Chapter B.) Nominal strength is given for compact and noncompact hot-rolled shapes, but not for slender shapes or shapes built up from plate elements. No numerical examples are given in this section, but Example 6.11 includes the computation of the flexural strength of a structural tee-shape.

I. **Square and Rectangular HSS and Box-Shaped Members (AISC F7):**

   a. Width-to-thickness parameters (see Figure 5.52):

      i. Flange:

         \[ \lambda = \frac{b}{t} \quad \lambda_p = 1.12 \sqrt{\frac{E}{F_y}} \quad \lambda_r = 1.40 \sqrt{\frac{E}{F_y}} \]

      ii. Web:

         \[ \lambda = \frac{h}{t} \quad \lambda_p = 2.42 \sqrt{\frac{E}{F_y}} \quad \lambda_r = 5.70 \sqrt{\frac{E}{F_y}} \]

   If the actual dimensions \( b \) and \( h \) are not known, they may be estimated as the total width or depth minus three times the thickness. The *design* thickness, which is 0.93 times the nominal thickness, should be used. (\( b/t \) and \( h/t \) ratios for HSS are given in the Manual in Part 1, "Dimensions and Properties.")
b. Bending about the major axis (loaded in the plane of symmetry):

i. Compact shapes: For compact shapes, the strength will be based on the limit state of yielding.

\[ M_n = M_p = F_y Z \quad \text{(AISC Equation F7-1)} \]

(Because of the high torsional resistance of closed cross-sectional shapes, lateral-torsional buckling of HSS need not be considered, even for rectangular shapes bent about the strong axis.)

ii. Noncompact shapes: The nominal strength \( M_n \) will be the smaller value computed from the limit states of flange local buckling (FLB) and web local buckling (WLB). For FLB,

\[ M_n = M_p - (M_p - F_y S) \left( 3.57 \frac{b}{t_f} \sqrt{\frac{F_y}{E}} - 4.0 \right) \leq M_p \quad \text{(AISC Equation F7-2)} \]

For WLB,

\[ M_n = M_p - (M_p - F_y S_w) \left( 0.305 \frac{h}{t_w} \sqrt{\frac{F_y}{E}} - 0.738 \right) \leq M_p \quad \text{(AISC Equation F7-5)} \]

c. Bending about the minor axis: The provisions are the same as for bending about the major axis.

II. Round HSS (AISC F8):

a. Width-to-thickness parameters:

\[ \lambda = \frac{D}{t} \quad \lambda_p = \frac{0.07E}{F_y} \quad \lambda_r = \frac{0.31E}{F_y} \]

where \( D \) is the outer diameter.
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b. Nominal bending strength: There is no LTB limit state for circular (or square) shapes. The strength is limited by local buckling.

Compact shapes:

\[ M_n = M_p = F_y Z \]  \hspace{1cm} (AISC Equation F8-1)

Noncompact shapes:

\[ M_n = \left( \frac{0.021E}{D/t} + F_y \right) S \]  \hspace{1cm} (AISC Equation F8-2)

III. Tees Loaded in the Plane of Symmetry (AISC F9):

a. Width-to-thickness parameters:

i. Flange:

\[ \lambda = \frac{b_f}{2t_f} \quad \lambda_p = 0.38 \frac{E}{F_y} \quad \lambda_r = 1.0 \frac{E}{F_y} \]

ii. Web (stem):

\[ \lambda = \frac{d}{t_w} \quad \lambda_p = 0.84 \frac{E}{F_y} \quad \lambda_r = 1.03 \frac{E}{F_y} \]

b. Compact shapes: The strength will be smaller values for the limit states of yielding and LTB. For yielding,

\[ M_n = M_p \]  \hspace{1cm} (AISC Equation F9-1)

where

\[ M_p = F_y Z_x \leq 1.6 M_y \] for stems in tension  \hspace{1cm} (AISC Equation F9-2)

\[ M_p = F_y Z_x \leq M_y \] for stems in compression  \hspace{1cm} (AISC Equation F9-3)

where \( M_y \) = yield moment = \( F_y S \).

For LTB,

\[ M_n = M_{cr} = \frac{\pi \sqrt{EI_y GJ}}{L_b} \left[ B + \sqrt{1 + B^2} \right] \]  \hspace{1cm} (AISC Equation F9-4)

where

\[ B = \pm 2.3 \left( \frac{d}{L_b} \right) \sqrt{\frac{I_y}{J}} \]  \hspace{1cm} (AISC Equation F9-5)

The positive sign is used for \( B \) when the stem is in tension, and the negative sign is used when the stem is in compression anywhere along the unbraced length.
c. Noncompact shapes: The strength will be the lower value obtained for flange local buckling, web local buckling, and LTB. For FLB,

\[ M_n = M_p - (M_p - 0.7F_y S_{xc}) \left( \frac{\lambda_c - \lambda_{pc}}{\lambda_r - \lambda_p} \right) \leq 1.6M_y \] (AISC Equation F9-6)

where

\[ S_{xc} = \text{elastic section modulus referred to the compression flange} \]

For WLB,

\[ M_n = F_{cr}S_x \] (AISC Equation F9-8)

where

\[ F_{cr} = F_y \quad \text{when} \quad \frac{d}{t_w} \leq 0.84 \sqrt{\frac{E}{F_y}} \] (AISC Equation F9-9)

\[ F_{cr} = \left[ 2.55 - 1.84 \frac{d}{t_w} \sqrt{\frac{F_y}{E}} \right] F_y \quad \text{when} \quad 0.84 \sqrt{\frac{E}{F_y}} < \frac{d}{t_w} \leq 1.03 \sqrt{\frac{E}{F_y}} \] (AISC Equation F9-10)

For LTB, see AISC Equations F9-4 and F9-5 given previously.

IV. Double Angles Loaded in the Plane of Symmetry (AISC F9 and F10):

a. Width-to-thickness parameters: Use the limiting ratios for single angles from Table B4.1b:

\[ \lambda = \frac{b}{t} \quad \lambda_p = 0.54 \sqrt{\frac{E}{F_y}} \quad \lambda_r = 0.91 \sqrt{\frac{E}{F_y}} \]

where \( b \) is the leg length and \( t \) is the thickness.

b. Compact shapes: The strength will be the smaller value for the limit states of yielding and LTB. This is the same as for compact tees.

c. Noncompact shapes: The strength will be the smaller value for the limit states of LTB (same as for tees) and local buckling of the angle leg that is in compression. For local buckling (see user notes in AISC F9),

\[ M_n = F_y S_c \left( 2.43 - 1.72 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}} \right) \leq 1.5M_y \] (AISC Equation F10-7)

V. Solid Rectangular Bars (AISC F11): The applicable limit states are yielding and LTB for major axis bending; local buckling is not a limit state for either major or minor axis bending.
a. Bending about the major axis:

For \( \frac{L_b d}{t^2} \leq \frac{0.08E}{F_y} \),

\[ M_n = M_p = F_y Z \leq 1.6M_y \]  

where \( M_y \) = yield moment = \( F_y S \).  

For \( \frac{0.08E}{F_y} < \frac{L_b d}{t^2} \leq \frac{1.9E}{F_y} \),

\[ M_n = C_b \left[ 1.52 - 0.274 \left( \frac{L_b d}{t^2} \right) \frac{F_y}{E} \right] M_y \leq M_p \]  

(AISC Equation F11-2)

For \( \frac{L_b d}{t^2} > \frac{1.9E}{F_y} \),

\[ M_n = F_{cr} S \leq M_p \]  

(AISC Equation F11-3)

where

\[ F_{cr} = \frac{1.9EC_b}{L_b d/t^2} \]  

(AISC Equation F11-4)

\( t \) = width of bar (dimension parallel to axis of bending)  
\( d \) = depth of bar

b. Bending about the minor axis:

\[ M_n = M_p = F_y Z \leq 1.6M_y \]  

(AISC Equation F11-1)

where \( M_y \) = yield moment = \( F_y S \).

VI. Solid Circular Bars (AISC F11):

\[ M_n = M_p = F_y Z \leq 1.6M_y \]  

(AISC Equation F11-1)

(For a circle, \( Z/S = 1.7 > 1.6 \), so the upper limit always controls.)

For flexural members not covered in this summary (single angles, slender shapes, unsymmetrical shapes, and shapes built up from plate elements), refer to Chapter F of the AISC Specification. (Shapes built up from plate elements are also covered in Chapter 10 of this book.)
6.1 DEFINITION

While many structural members can be treated as axially loaded columns or as beams with only flexural loading, most beams and columns are subjected to some degree of both bending and axial load. This is especially true of statically indeterminate structures. Even the roller support of a simple beam can experience friction that restrains the beam longitudinally, inducing axial tension when transverse loads are applied. In this particular case, however, the secondary effects are usually small and can be neglected. Many columns can be treated as pure compression members with negligible error. If the column is a one-story member and can be treated as pinned at both ends, the only bending will result from minor accidental eccentricity of the load. For many structural members, however, there will be a significant amount of both effects, and such members are called beam–columns. Consider the rigid frame in Figure 6.1. For the given loading condition, the horizontal member \( AB \) must not only support the vertical uniform load but must also assist the vertical members in resisting the concentrated lateral load \( P \). Member \( CD \) is a more critical case, because it must resist the load \( P_1 + P_2 \) without any assistance from the vertical members. The reason is that the \( x \)-bracing, indicated by dashed lines, prevents sidesway in the lower story. For the direction of \( P_2 \) shown, member \( ED \) will be in tension and member \( CF \) will be slack, provided that the bracing elements have been designed to resist only tension. For this condition to occur, however, member \( CD \) must transmit the load \( P_1 + P_2 \) from \( C \) to \( D \).

The vertical members of this frame must also be treated as beam–columns. In the upper story, members \( AC \) and \( BD \) will bend under the influence of \( P_1 \). In addition, at \( A \) and \( B \), bending moments are transmitted from the horizontal member through the rigid joints. This transmission of moments also takes place at \( C \) and \( D \) and is true in any rigid frame, although these moments are usually smaller than those resulting from lateral loads. Most columns in rigid frames are actually beam–columns, and the effects of bending should not be ignored. However, many isolated one-story columns can be realistically treated as axially loaded compression members.
Another example of beam-columns can sometimes be found in roof trusses. Although the top chord is normally treated as an axially loaded compression member, if purlins are placed between the joints, their reactions will cause bending, which must be accounted for. We discuss methods for handling this problem later in this chapter.

6.2 INTERACTION FORMULAS

The relationship between required and available strengths may be expressed as

\[
\frac{\text{required strength}}{\text{available strength}} \leq 1.0
\]

(6.1)

For compression members, the strengths are axial forces. For example, for LRFD,

\[
\frac{P_u}{\phi_c P_n} \leq 1.0 \quad P_u \leq \phi_c P_n
\]

and for ASD,

\[
\frac{P_u}{P_n/\Omega_c} \leq 1.0
\]

These expressions can be written in the general form

\[
\frac{P_r}{P_c} \leq 1.0 \quad \frac{P_u}{\phi_c P_n} \leq 1.0
\]

where

- \( P_r \) = required axial strength
- \( P_c \) = available axial strength

If more than one type of resistance is involved, Equation 6.1 can be used to form the basis of an interaction formula. As we discussed in Chapter 5 in conjunction with biaxial bending, the sum of the load-to-resistance ratios must be limited to unity. For
example, if both bending and axial compression are acting, the interaction formula would be

$$ \frac{P}{P_c} + \frac{M_r}{M_c} \leq 1.0 \quad \frac{M_u}{\phi_b M_m} $$

where

- $M_r$ = required moment strength
  - $= M_u$ for LRFD
  - $= M_n$ for ASD

- $M_c$ = available moment strength
  - $= \phi_b M_n$ for LRFD
  - $= \frac{M_n}{\Omega_b}$ for ASD

For biaxial bending, there will be two moment ratios:

$$ \frac{P}{P_c} + \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad \frac{P_u}{\phi_c P_m} \quad \frac{\phi_b M_{mx}}{\phi_b M_{my}} \leq 1.0 \quad (6.2) $$

where the $x$ and $y$ subscripts refer to bending about the $x$ and $y$ axes.

Equation 6.2 is the basis for the AISC formulas for members subject to bending plus axial compressive load. Two formulas are given in the Specification: one for small axial load and one for large axial load. If the axial load is small, the axial load term is reduced. For large axial load, the bending term is slightly reduced. The AISC requirements are given in Chapter H, “Design of Members for Combined Forces and Torsion,” and are summarized as follows:

Heavily loaded in compression:

For $\frac{P}{P_c} \geq 0.2,$

$$ \frac{P}{\phi_c P_m} \geq 0.2 \quad \frac{P_u}{\phi_c P_m} \leq \left( \frac{M_{ux}}{\phi_b M_{mx}} + \frac{M_{uy}}{\phi_b M_{my}} \right) \leq 1.0 $$

(AISC Equation H1-1a)

Lightly loaded in compression:

For $\frac{P}{P_c} < 0.2,$

$$ \frac{P}{2\phi_c P_m} \leq 0.2 \quad \frac{P_u}{2\phi_c P_m} \leq \left( \frac{M_{ux}}{\phi_b M_{mx}} + \frac{M_{uy}}{\phi_b M_{my}} \right) \leq 1.0 $$

(AISC Equation H1-1b)

These requirements may be expressed in either LRFD or ASD form.
LRFD Interaction Equations

For \( \frac{P_u}{\phi_c P_n} \geq 0.2, \)

\[
\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{xy}}{\phi_b M_{xy}} \right) \leq 1.0
\]  

(6.3)

For \( \frac{P_u}{\phi_c P_n} < 0.2, \)

\[
\frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{xy}}{\phi_b M_{xy}} \right) \leq 1.0
\]  

(6.4)

ASD Interaction Equations

For \( \frac{P_a}{P_n/\Omega_c} \geq 0.2, \)

\[
\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left( \frac{M_{ax}}{M_{ax}/\Omega_a} + \frac{M_{ay}}{M_{ay}/\Omega_b} \right) \leq 1.0
\]

(6.5)

For \( \frac{P_a}{P_n/\Omega_c} < 0.2, \)

\[
\frac{P_a}{2P_n/\Omega_c} + \left( \frac{M_{ax}}{M_{ax}/\Omega_a} + \frac{M_{ay}}{M_{ay}/\Omega_b} \right) \leq 1.0
\]

(6.6)

Example 6.1 illustrates the application of Equations 6.3–6.6.

**Example 6.1**

The beam–column shown in Figure 6.2 is pinned at both ends and is subjected to the loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC Specification interaction equation.

From the column load tables, the axial compressive design strength of a W10 \( \times \) 49 with \( F_y = 50 \) ksi and an effective length of \( K_c L = 1.0 \times 17 = 17 \) feet is

\[ \phi_c P_n = 405 \text{ kips} \]

Since bending is about the strong axis, the design moment, \( \phi_b M_{n3} \), for \( C_b = 1.0 \) can be obtained from the beam design charts in Part 3 of the Manual.
CHAPTER H
DESIGN OF MEMBERS FOR COMBINED FORCES AND TORSION

This chapter addresses members subject to axial force and flexure about one or both axes, with or without torsion, and members subject to torsion only.

The chapter is organized as follows:

H1. Doubly and Singly Symmetric Members Subject to Flexure and Axial Force
H2. Unsymmetric and Other Members Subject to Flexure and Axial Force
H3. Members Subject to Torsion and Combined Torsion, Flexure, Shear and/or Axial Force
H4. Rupture of Flanges with Holes Subject to Tension

User Note: For composite members, see Chapter I.

H1. DOUBLY AND SINGLY SYMMETRIC MEMBERS SUBJECT TO FLEXURE AND AXIAL FORCE

1. Doubly and Singly Symmetric Members Subject to Flexure and Compression

The interaction of flexure and compression in doubly symmetric members and singly symmetric members for which 0.1 ≤ (Iy/Ix) ≤ 0.9, constrained to bend about a geometric axis (x and/or y) shall be limited by Equations H1-1a and H1-1b, where Ix is the moment of inertia of the compression flange about the y-axis, in.⁴ (mm⁴).

User Note: Section H2 is permitted to be used in lieu of the provisions of this section.

(a) When \( \frac{P_r}{P_c} \geq 0.2 \)

\[
\frac{P_r}{P_c} + \frac{8}{9} \left( \frac{M_{cx}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0
\]

(H1-1a)

(b) When \( \frac{P_r}{P_c} < 0.2 \)

\[
\frac{P_r}{2P_c} + \left( \frac{M_{cx}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0
\]

(H1-1b)

where

\( P_r = \text{required axial strength using LRFD or ASD load combinations, kips (N)} \)
\( P_c = \text{available axial strength, kips (N)} \)

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\[ M_r = \text{required flexural strength using LRFD or ASD load combinations, kip-in.} \] (N-mm)
\[ M_c = \text{available flexural strength, kip-in. (N-mm)} \]
\[ x = \text{subscript relating symbol to strong axis bending} \]
\[ y = \text{subscript relating symbol to weak axis bending} \]

**For design according to Section B3.3 (LRFD):**
\[ P_r = \text{required axial strength using LRFD load combinations, kips (N)} \]
\[ P_c = \phi_c P_n = \text{design axial strength, determined in accordance with Chapter E, kips (N)} \]
\[ M_r = \text{required flexural strength using LRFD load combinations, kip-in. (N-mm)} \]
\[ M_c = \phi_b M_n = \text{design flexural strength determined in accordance with Chapter F, kip-in. (N-mm)} \]
\[ \phi_c = \text{resistance factor for compression} = 0.90 \]
\[ \phi_b = \text{resistance factor for flexure} = 0.90 \]

**For design according to Section B3.4 (ASD):**
\[ P_r = \text{required axial strength using ASD load combinations, kips (N)} \]
\[ P_c = P_n / \Omega_c = \text{allowable axial strength, determined in accordance with Chapter E, kips (N)} \]
\[ M_r = \text{required flexural strength using ASD load combinations, kip-in. (N-mm)} \]
\[ M_c = M_n / \Omega_b = \text{allowable flexural strength determined in accordance with Chapter F, kip-in. (N-mm)} \]
\[ \Omega_c = \text{safety factor for compression} = 1.67 \]
\[ \Omega_b = \text{safety factor for flexure} = 1.67 \]

2. **Doubly and Singly Symmetric Members Subject to Flexure and Tension**

The interaction of flexure and tension in doubly symmetric members and singly symmetric members constrained to bend about a geometric axis (x and/or y) shall be limited by Equations H1-1a and H1-1b

where

**For design according to Section B3.3 (LRFD):**
\[ P_r = \text{required axial strength using LRFD load combinations, kips (N)} \]
\[ P_c = \phi_t P_n = \text{design axial strength, determined in accordance with Section D2, kips (N)} \]
\[ M_r = \text{required flexural strength using LRFD load combinations, kip-in. (N-mm)} \]
\[ M_c = \phi_b M_n = \text{design flexural strength determined in accordance with Chapter F, kip-in. (N-mm)} \]
\[ \phi_t = \text{resistance factor for tension (see Section D2)} \]
\[ \phi_b = \text{resistance factor for flexure} = 0.90 \]

**For design according to Section B3.4 (ASD):**
\[ P_r = \text{required axial strength using ASD load combinations, kips (N)} \]
\[ P_c = P_n / \Omega_t = \text{allowable axial strength, determined in accordance with Section D2, kips (N)} \]
\( M_r = \text{required flexural strength using ASD load combinations, kip-in. (N-mm)} \)
\( M_c = M_n/\Omega_b = \text{allowable flexural strength determined in accordance with Chapter F, kip-in. (N-mm)} \)
\( \Omega_t = \text{Safety factor for tension (see Section D2)} \)
\( \Omega_b = \text{Safety factor for flexure} = 1.67 \)

For doubly symmetric members, \( C_b \) in Chapter F may be multiplied by \( \sqrt{1 + \frac{\alpha P_r}{P_{cy}}} \) for axial tension that acts concurrently with flexure.

where
\[
P_{cy} = \frac{\pi^2 EI_y}{L_b^2}
\]

and
\( \alpha = 1.0 \text{ (LRFD)}; \alpha = 1.6 \text{ (ASD)} \)

A more detailed analysis of the interaction of flexure and tension is permitted in lieu of Equations H1-1a and H1-1b.

3. **Doubly Symmetric Rolled Compact Members Subject to Single Axis Flexure and Compression**

For doubly symmetric rolled compact members with \((KL)_z \leq (KL)_y\) subjected to flexure and compression with moments primarily about their major axis, it is permissible to consider the two independent limit states, in-plane instability and out-of-plane buckling or lateral-torsional buckling, separately in lieu of the combined approach provided in Section H1.1.

For members with \( M_{ry}/M_{cy} \geq 0.05 \), the provisions of Section H1.1 shall be followed.

(a) For the limit state of in-plane instability, Equations H1-1 shall be used with \( P_c \), \( M_{rx} \) and \( M_{cx} \) determined in the plane of bending.

(b) For the limit state of out-of-plane buckling and lateral-torsional buckling:

\[
\frac{P_r}{P_{cy}} \left(1.5 - 0.5 \frac{P_r}{P_{cy}}\right) + \left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leq 1.0 \quad \text{(H1-2)}
\]

where
\( P_{cy} = \text{available compressive strength out of the plane of bending, kips (N)} \)
\( C_b = \text{lateral-torsional buckling modification factor determined from Section F1} \)
\( M_{cx} = \text{available lateral-torsional strength for strong axis flexure determined in accordance with Chapter F using } C_b = 1.0, \text{ kip-in. (N-mm)} \)

**User Note:** In Equation H1-2, \( C_b M_{cx} \) may be larger than \( \phi_b M_{px} \) in LRFD or \( M_{px}/\Omega_b \) in ASD. The yielding resistance of the beam-column is captured by Equations H1-1.
H2. UNSYMMETRIC AND OTHER MEMBERS SUBJECT TO FLEXURE AND AXIAL FORCE

This section addresses the interaction of flexure and axial stress for shapes not covered in Section H1. It is permitted to use the provisions of this Section for any shape in lieu of the provisions of Section H1.

\[
\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right| \leq 1.0
\]

(H2-1)

where

- \( f_{ra} \) = required axial stress at the point of consideration using LRFD or ASD load combinations, ksi (MPa)
- \( F_{ca} \) = available axial stress at the point of consideration, ksi (MPa)
- \( f_{rbw}, f_{rbz} \) = required flexural stress at the point of consideration using LRFD or ASD load combinations, ksi (MPa)
- \( F_{cbw}, F_{cbz} \) = available flexural stress at the point of consideration, ksi (MPa)
- \( w \) = subscript relating symbol to major principal axis bending
- \( z \) = subscript relating symbol to minor principal axis bending

For design according to Section B3.3 (LRFD):

- \( f_{ra} \) = required axial stress at the point of consideration using LRFD load combinations, ksi (MPa)
- \( F_{ca} = \phi_c F_{cr} \) = design axial stress, determined in accordance with Chapter E for compression or Section D2 for tension, ksi (MPa)
- \( f_{rbw}, f_{rbz} \) = required flexural stress at the point of consideration using LRFD or ASD load combinations, ksi (MPa)
- \( F_{cbw}, F_{cbz} = \frac{\phi_b M_n}{S} \) = design flexural stress determined in accordance with Chapter F, ksi (MPa). Use the section modulus for the specific location in the cross section and consider the sign of the stress.
- \( \phi_c \) = resistance factor for compression = 0.90
- \( \phi_t \) = resistance factor for tension (Section D2)
- \( \phi_b \) = resistance factor for flexure = 0.90

For design according to Section B3.4 (ASD):

- \( f_{ra} \) = required axial stress at the point of consideration using ASD load combinations, ksi (MPa)
- \( F_{ca} = \frac{F_{cr}}{\Omega_c} \) = allowable axial stress determined in accordance with Chapter E for compression, or Section D2 for tension, ksi (MPa)
- \( f_{rbw}, f_{rbz} \) = required flexural stress at the point of consideration using LRFD or ASD load combinations, ksi (MPa)
- \( F_{cbw}, F_{cbz} = \frac{M_n}{\Omega_b S} \) = allowable flexural stress determined in accordance with Chapter F, ksi (MPa). Use the section modulus for the specific location in the cross section and consider the sign of the stress.
- \( \Omega_c \) = safety factor for compression = 1.67
6.2 Interaction Formulas

For \( \frac{P_a}{\phi_nP_n} < 0.2, \)

\[
\frac{P_a}{2\phi_nP_n} + \left(\frac{M_{ax}}{\phi_xM_{xx}} + \frac{M_{ay}}{\phi_yM_{yy}}\right) \leq 1.0
\]  

(6.4)

ASD Interaction Equations

For \( \frac{P_a}{P_n/\Omega_c} \geq 0.2, \)

\[
\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left(\frac{M_{ax}}{M_{ax}/\Omega_b} + \frac{M_{ay}}{M_{ay}/\Omega_b}\right) \leq 1.0
\]  

(6.5)

For \( \frac{P_a}{P_n/\Omega_c} < 0.2, \)

\[
\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{ax}/\Omega_b} + \frac{M_{ay}}{M_{ay}/\Omega_b}\right) \leq 1.0
\]  

(6.6)

Example 6.1 illustrates the application of Equations 6.3–6.6.

---

**Example 6.1**

The beam–column shown in Figure 6.2 is pinned at both ends and is subjected to the loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC Specification interaction equation.

**Solution**

As we demonstrate in Section 6.3, the applied moments in AISC Equations H1-1a and b will sometimes be increased by moment amplification. The purpose of this example is to illustrate how interaction formulas work.

**Figure 6.2**

- \( P_D = 35k \)
- \( P_L = 99k \)
- \( W10 \times 49 \)
- A992 steel
- \( Q_D = 5k \)
- \( Q_L = 12k \)
- \( L_{12} \)
- \( L_{12} \)
- \( P \)
- Strong X-X bending

\[
P = 1.2(35) + 1.6(99) = P_n = 200.4k
\]

\( P = 25.2k \)
\[ M_{\text{original}} = \frac{FL}{4} \]

\[ M = \frac{PS_2}{2} \]

\[ M_{\text{added}} = \frac{PS_3}{2} \]

\[ M_0 = \frac{\omega L^2}{8} \]

\[ M = \frac{FL}{4} \]

MOMENT AMPLIFICATION (LATER)
From the column load tables, the axial compressive design strength of a W10 × 49 with \( F_y = 50 \text{ ksi} \) and an effective length of \( K_e L = 1.0 \times 17 = 17 \text{ feet} \) is 
\[
\phi_n P_n = 405 \text{ kips}
\]
Since bending is about the strong axis, the design moment, \( \phi_n M_n \) for \( C_b = 1.0 \) can be obtained from the beam design charts in Part 3 of the Manual.

For an unbraced length \( L_b = 17 \text{ ft} \),
\[
\phi_n M_n = 197 \text{ ft-kips}
\]

For the end conditions and loading of this problem, \( C_b = 1.32 \) (see Figure 5.15c).

For \( C_b = 1.32 \), the design strength is
\[
\phi_n M_n = C_b \times 197 = 1.32 \times 197 = 260 \text{ ft-kips}
\]
This moment is larger than \( \phi_n M_n = 226.5 \text{ ft-kips} \) (also obtained from the beam design charts), so the design moment must be limited to \( \phi_n M_n \). Therefore,
\[
\phi_n M_n = 226.5 \text{ ft-kips}
\]

**Factored loads:**

\[
P_t = 1.2P_D + 1.6P_L = 1.2(35) + 1.6(99) = 200.4 \text{ kips}
\]

\[
Q_y = 1.2Q_D + 1.6Q_c = 1.2(5) + 1.6(12) = 25.2 \text{ kips}
\]

The maximum bending moment occurs at midheight, so
\[
M_n = \frac{25.2(17)}{2} = 107.1 \text{ ft-kips}
\]

Determine which interaction equation controls:
\[
\frac{P}{\phi_n P_n} = \frac{200.4}{405} = 0.4948 > 0.2 \quad \therefore \text{use Equation 6.3 (AISC Eq. H1-1a)}
\]
\[
\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{x x}}{\phi_n M_{x x}} + \frac{M_{y y}}{\phi_n M_{y y}} \right) = \frac{200.4}{405} + \frac{8}{9} \left( \frac{107.1}{226.5} + 0 \right) = 0.915 < 1.0 \quad \text{(OK)}
\]

**Answer**

This member satisfies the AISC Specification.

**ASD Solution**

From the column load tables, the allowable compressive strength of a W10 × 49 with \( F_y = 50 \text{ ksi} \) and \( K_e L = 1.0 \times 17 = 17 \text{ feet} \) is
\[
\frac{P}{\Omega_c} = 270 \text{ kips}
\]

From the design charts in Part 3 of the Manual, for \( L_b = 17 \text{ ft} \) and \( C_b = 1.0 \),
\[
\frac{M_n}{\Omega_b} = 131 \text{ ft-kips}
\]
Table 4-1 (continued)
Available Strength in Axial Compression, kips

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</table>

Properties

- $P_{n_e}$ (kips) = 68.8, 103, 60.1, 90.1, 65.3, 98.0, 54.1, 81.1, 45.2, 67.8
- $P_{n_w}$ (kips/lin.) = 12.3, 18.5, 11.3, 17.0, 11.7, 17.5, 10.5, 15.8, 9.67, 14.5
- $P_{n_e}$ (kips) = 112, 168, 88.5, 130, 94.4, 142, 68.8, 103, 55.7, 80.7
- $P_{n_w}$ (kips) = 20.6, 106, 58.7, 88.2, 71.9, 108, 52.8, 78.3, 35.4, 53.2
- $L_x$ (ft) = 9.04, 8.97, 7.10, 6.99, 6.85
- $L_y$ (ft) = 33.7, 31.6, 26.9, 24.2, 21.8
- $A_y$ (in.²) = 15.8, 14.4, 11.3, 13.3, 11.5, 9.71
- $I_y$ (in.⁴) = 303, 272, 248, 209, 171
- $I_x$ (in.⁴) = 103, 93.4, 53.4, 45.0, 36.6
- $f_y$ (in.) = 2.56, 2.54, 2.01, 1.98, 1.94
- Ratio $f_y/f_y = 1.97, 1.71, 1.45, 1.26, 1.21$
- $P_{n_e}(KSE/10^4) (k-in.²) = 8670, 7790, 7100, 5980, 4890
- $P_{n_w}(KSE/10^4) (k-in.²) = 2950, 2670, 1530, 1290, 1050$

ASD LRFD

Note: Heavy line indicates $K/\gamma$ equal to or greater than 200.

$\phi_e = 0.90$

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<table>
<thead>
<tr>
<th>Load</th>
<th>Lateral Bracing Along Span</th>
<th>$C_b$</th>
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</thead>
<tbody>
<tr>
<td>$P$</td>
<td>None (Load at midpoint)</td>
<td>1.32</td>
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<tr>
<td>$P$</td>
<td>At load point</td>
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<tr>
<td>$P$ $P$</td>
<td>None (Loads at third points)</td>
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<td></td>
<td>At load points</td>
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<tr>
<td></td>
<td>Loads symmetrically placed</td>
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<tr>
<td>$P$ $P$ $P$</td>
<td>None (Loads at quarter points)</td>
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<td>At load points</td>
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<td>Loads at quarter points</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td></td>
<td>1.14</td>
</tr>
</tbody>
</table>

Note: Lateral bracing must always be provided at points of support per AISC Specification Chapter F.
For an unbraced length $L_b = 17$ ft,

$$\phi_b M_n = 197 \text{ ft-kips}$$

For the end conditions and loading of this problem, $C_b = 1.32$ (see Figure 5.15c).

For $C_b = 1.32$, the design strength is

$$\phi_b M_n = C_b \times 197 = 1.32(197) = 260 \text{ ft-kips}$$

This moment is larger than $\phi_b M_p = 226.5$ ft-kips (also obtained from the beam design charts), so the design moment must be limited to $\phi_b M_p$. Therefore,

$$\phi_b M_n = 226.5 \text{ ft-kips}$$

**Factored loads:**

$$P_u = 1.2P_D + 1.6P_L = 1.2(35) + 1.6(99) = 200.4 \text{ kips}$$

$$Q_u = 1.2Q_D + 1.6Q_L = 1.2(5) + 1.6(12) = 25.2 \text{ kips}$$

The maximum bending moment occurs at midheight, so

$$M_u = \frac{25.2(17)}{4} = 107.1 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_b P_n} = \frac{200.4}{405} = 0.4948 > 0.2 \quad \therefore \text{Use Equation 6.3 (AISC Eq. H1-1a).}$$

$$\frac{P_u}{\phi_b P_n} + \frac{8}{9} \left( \frac{M_{uu}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{xy}} \right) = \frac{200.4}{405} \cdot 9 \left( \frac{107.1}{226.5} + 0 \right) = 0.915 < 1.0 \quad \text{(OK)}$$

**Answer**

This member satisfies the AISC Specification.
From the column load tables, the allowable compressive strength of a W10 × 49 with $F_y = 50$ ksi and $K_p L = 1.0 \times 17 = 17$ feet is

$$\frac{P_n}{\Omega_c} = 270 \text{ kips}$$

From the design charts in Part 3 of the Manual, for $L_b = 17$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 131 \text{ ft-kips}$$

From Figure 5.15c, $C_b = 1.32$. For $C_b = 1.32$,

$$\frac{M_n}{\Omega_b} = C_b \times 131 = 1.32(131) = 172.9 \text{ ft-kips}$$

This is larger than $M_p/\Omega_b = 151$ ft-kips, so the allowable moment must be limited to $M_p/\Omega_b$. Therefore,

$$\frac{M_n}{\Omega_b} = 151 \text{ ft-kips}$$

The total axial compressive load is

$$P_a = P_D + P_L = 35 + 99 = 134 \text{ kips}$$

The total transverse load is

$$Q_a = Q_D + Q_L = 5 + 12 = 17 \text{ kips}$$

The maximum bending moment is at midheight

$$M_a = \frac{17(17)}{4} = 72.25 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{134}{270} = 0.4963 > 0.2 \quad \therefore \text{ Use Equation 6.5 (AISC Equation H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left( \frac{M_{ax}}{\Omega_{ax}} + \frac{M_{ay}}{\Omega_{ay}} \right) = \frac{134}{270} + \frac{8}{9} \left( \frac{72.25}{151} + 0 \right) = 0.922 < 1.0 \quad \text{(OK)}$$

**Answer** This member satisfies the AISC Specification.