6.3 METHODS OF ANALYSIS FOR REQUIRED STRENGTH

The foregoing approach to the analysis of members subjected to both bending and axial load is satisfactory so long as the axial load is not too large. The presence of the axial load produces secondary moments, and unless the axial load is relatively small, these additional moments must be accounted for. For an explanation, refer to Figure 6.3, which shows a beam-column with an axial load and a transverse uniform load. At an arbitrary point O, there is a bending moment caused by the uniform load and an additional moment $P_y$, caused by the axial load acting at an eccentricity from the longitudinal axis of the member. This secondary moment is largest where the deflection is largest—in this case, at the centerline, where the total moment is $wL^2/8 + P\delta$. Of course, the additional moment causes an additional deflection over and above that resulting from the transverse load. Because the total deflection cannot be found directly, this problem is nonlinear, and without knowing the deflection, we cannot compute the moment.

In addition to the secondary moments caused by member deformation ($P-\delta$ moments, shown in Figure 6.4a), additional secondary moments are present when one end of the member translates with respect to the other. These moments are called $P-\Delta$ moments and are illustrated in Figure 6.4b. In a braced frame, the member ends do not undergo translation, so only the $P-\delta$ moments are present. In an unbraced frame, the additional moment, $P\Delta$, increases the end moment. The distribution of moments in the member is therefore a combination of the primary moment, the $P-\delta$ moment, and the $P-\Delta$ moment.

An unbraced rigid frame depends on the transfer of moments at its joints for stability. For this reason, unbraced frames are frequently referred to as moment frames. Multistory buildings can consist of a combination of braced frames and moment frames.

For all but the simplest of structures, a computerized frame analysis is required to obtain the bending moments and axial loads. The analysis gives the required strength of members. As covered in Chapter 4 of this book, the available strength of

FIGURE 6.3

![Diagram of beam-column with axial and transverse loads](image-url)
We will mainly use
1) Direct Analysis method
2) First-order moment and loads
3) Thansamplid

compression members takes into account member out-of-straightness and inelasticity. The analysis for the required strength should account for the displaced geometry, member out-of-plumbness (deviation from vertical), and inelasticity.

Ordinary structural analysis methods that do not take the displaced geometry into account are called first-order methods. Iterative analyses that account for these effects are referred to as second-order methods.

AISC Specification Chapter C, "Design for Stability," provides three approaches for determining the required flexural and axial compressive strength: the direct analysis method, the effective length method, and the first-order analysis method.

1. The direct analysis method is a second-order analysis that considers both $P$-$\delta$ and $P$-$A$ effects. As an alternative, an approximate second-order analysis, as given in Appendix 8, can be used. This approach uses amplified first-order moments and axial loads. Both the second-order analysis and the approximate second-order analysis are considered direct analysis methods. In the direct analysis method, member stiiffnesses are reduced, and an effective length factor of $K = 1$ is used both in the analysis and in computing the available strength from AISC Chapter 4. (Part 4) $[4-1]$

2. The effective length method of analysis is covered in Appendix 7. It also requires a second-order or approximate second-order analysis. Computation of the corresponding available strength has been discussed in Chapter 4, "Compression Members." As the name implies, an effective length factor, $K$, must be determined. Member stiffnesses are not reduced.

3. The first-order analysis method is a simplified version of the direct analysis method that can be used when certain conditions are satisfied. It is covered in Appendix 7. For the available strength, an effective length factor of $K = 1$ is used. Member stiffnesses are not reduced.

All columns in real structures are subject to initial displacements that result from member out-of-plumbness. In each of the three analysis methods, member out-of-plumbness is accounted for by including fictitious lateral loads, called notional loads, in the load combinations.
APPENDIX 8
APPROXIMATE SECOND-ORDER ANALYSIS (SPRECS)

This appendix provides, as an alternative to a rigorous second-order analysis, a procedure to account for second-order effects in structures by amplifying the required strengths indicated by a first-order analysis.

The appendix is organized as follows:

8.1. Limitations
8.2. Calculation Procedure

8.1. LIMITATIONS

The use of this procedure is limited to structures that support gravity loads primarily through nominally vertical columns, walls or frames, except that it is permissible to use the procedure specified for determining P-δ effects for any individual compression member.

8.2. CALCULATION PROCEDURE

The required second-order flexural strength, \( M_r \), and axial strength, \( P_r \), of all members shall be determined as follows:

\[
M_r = B_1 M_{nt} + B_2 M_{lt}
\]

\[
P_r = P_{nt} + B_2 P_{lt}
\]

where

- \( B_1 \) = multiplier to account for P-δ effects, determined for each member subject to compression and flexure, and each direction of bending of the member in accordance with Section 8.2.1. \( B_1 \) shall be taken as 1.0 for members not subject to compression.
- \( B_2 \) = multiplier to account for P-Δ effects, determined for each story of the structure and each direction of lateral translation of the story in accordance with Section 8.2.2
- \( M_{nt} \) = first-order moment using LRFD or ASD load combinations, due to lateral translation of the structure only, kip-in. (N-mm)
- \( M_{lt} \) = first-order moment using LRFD or ASD load combinations, with the structure restrained against lateral translation, kip-in. (N-mm)
- \( M_r \) = required second-order flexural strength using LRFD or ASD load combinations, kip-in. (N-mm)
- \( P_{nt} \) = first-order axial force using LRFD or ASD load combinations, due to lateral translation of the structure only, kips (N)
- \( P_{lt} \) = first-order axial force using LRFD or ASD load combinations, with the structure restrained against lateral translation, kips (N)
\( P_r \) = required second-order axial strength using LRFD or ASD load combinations, kips (N)

**User Note:** Equations A-8-1 and A-8-2 are applicable to all members in all structures. Note, however, that \( B_1 \) values other than unity apply only to moments in *beam-columns*; \( B_2 \) applies to moments and axial forces in components of the **lateral force resisting system** (including *columns*, *beams*, *bracing members* and *shear walls*). See Commentary for more on the application of Equations A-8-1 and A-8-2.

1. **Multiplier \( B_1 \) for \( P-\delta \) Effects**

The \( B_1 \) multiplier for each member subject to compression and each direction of bending of the member is calculated as follows:

\[
B_1 = \frac{C_m}{1 - \frac{P_r}{P_{e1}}} \geq 1 \quad C_m \leq 1.0 \quad (A-8-3)
\]

where

- \( \alpha = 1.00 \) (LRFD); \( \alpha = 1.60 \) (ASD)
- \( C_m \) = coefficient assuming no lateral translation of the frame determined as follows:
  
  (a) For *beam-columns* not subject to transverse loading between supports in the plane of bending

  \[
  C_m = 0.6 - 0.4(M_1/M_2) \quad (A-8-4)
  \]

  where \( M_1 \) and \( M_2 \), calculated from a *first-order analysis*, are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. \( M_1/M_2 \) is positive when the member is bent in *reverse curvature*, negative when bent in *single curvature*.

  (b) For beam-columns subject to transverse loading between supports, the value of \( C_m \) shall be determined either by analysis or conservatively taken as 1.0 for all cases.

\( P_{e1} \) = elastic critical *buckling strength* of the member in the plane of bending, calculated based on the assumption of no lateral translation at the member ends, kips (N)

\[
P_{e1} = \frac{\pi^2 E I *}{(K_1 L)^2} \quad (A-8-5)
\]

where

- \( E I^* = \) flexural rigidity required to be used in the analysis (= 0.8 \( \tau_s E I \) when used in the *direct analysis method* where \( \tau_s \) is as defined in Chapter C)
- \( E = \) modulus of elasticity of steel = 29,000 ksi (200,000 MPa)
CALCULATION PROCEDURE

\[ (306c) \]

16.1-239

\[ \]

\( I \) = moment of inertia in the plane of bending, \( \text{in.}^4 \) (mm^4)

\( L \) = length of member, \( \text{in.} \) (mm)

\( K_1 \) = \textit{effective length factor} in the plane of bending, calculated based on the assumption of no lateral translation at the member ends, set equal to 1.0 unless analysis justifies a smaller value

It is permitted to use the first-order estimate of \( P_r \) (i.e., \( P_r = P_{mf} + P_h \)) in Equation A-8-3.

2. \textbf{Multiplier} \( B_2 \) for \( P-\Delta \) Effects

The \( B_2 \) multiplier for each story and each direction of lateral translation is calculated as follows:

\[ B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e,story}}} \geq 1 \quad \text{(A-8-6)} \]

where

\( \alpha = 1.00 \) (LRFD); \( \alpha = 1.60 \) (ASD)

\( P_{story} \) = total vertical load supported by the story using LRFD or ASD load combinations, as applicable, including loads in columns that are not part of the lateral force resisting system, kips (N)

\( P_{e,story} \) = elastic critical buckling strength for the story in the direction of translation being considered, kips (N), determined by sidesway buckling analysis or as:

\[ P_{e,story} = R_M \frac{HL}{\Delta_H} \quad \text{(A-8-7)} \]

where

\( R_M = 1 - 0.15 \left( \frac{P_{mf}}{P_{story}} \right) \quad \text{(A-8-8)} \)

\( L \) = height of story, \( \text{in.} \) (mm)

\( P_{mf} \) = total vertical load in columns in the story that are part of moment frames, if any, in the direction of translation being considered (\( \leq 0 \) for braced frame systems), kips (N)

\( \Delta_H \) = first-order interstory drift, in the direction of translation being considered, due to lateral forces, \( \text{in.} \) (mm), computed using the stiffness required to be used in the analysis (stiffness reduced as provided in Section C2.3 when the direct analysis method is used). Where \( \Delta_H \) varies over the plan area of the structure, it shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift.

\( H \) = story shear, in the direction of translation being considered, produced by the lateral forces used to compute \( \Delta_H \), kips (N)

\textbf{User Note:} \( H \) and \( \Delta_H \) in Equation A-8-7 may be based on any lateral loading that provides a representative value of story lateral stiffness, \( H/\Delta_H \).
stiffness to account for inelastic softening prior to the members reaching their design strength. The \( \tau_b \) factor is similar to the inelastic stiffness reduction factor implied in the column curve to account for loss of stiffness under high compression loads \((\alpha P_c > 0.5P_y)\), and the 0.8 factor accounts for additional softening under combined axial compression and bending. It is a fortuitous coincidence that the reduction coefficients for both slender and stocky columns are close enough, such that the single reduction factor of 0.8 \( \tau_b \) works over the full range of slenderness.

The use of reduced stiffness only pertains to analyses for strength and stability limit states. It does not apply to analyses for other stiffness-based conditions and criteria, such as for drift, deflection, vibration and period determination.

For ease of application in design practice, where \( \tau_b = 1 \), the reduction on \( EI \) and \( EA \) can be applied by modifying \( E \) in the analysis. However, for computer programs that do semi-automated design, one should ensure that the reduced \( E \) is applied only for the second-order analysis. The elastic modulus should not be reduced in nominal strength equations that include \( E \) (for example, \( M_n \) for lateral-torsional buckling in an unbraced beam).

As shown in Figure C-C2.5, the net effect of modifying the analysis in the manner just described is to amplify the second-order forces such that they are closer to the

![Diagram](image-url)

(a) Effective length method

![Diagram](image-url)

(b) Direct analysis method

Fig. C-C2.5. Comparison of in-plane beam-column interaction checks for (a) the effective length method and (b) the direct analysis method.

Specification for Structural Steel Buildings, June 22, 2010
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
APPENDIX 8

APPROXIMATE SECOND-ORDER ANALYSIS

(COMMENTARY)

Section C2.1(2) states that a second-order analysis that captures both P-Δ and P-δ effects is required. As an alternative to a rigorous second-order analysis, the amplification of first-order analysis forces and moments by the approximate procedure in this Appendix is permitted. The main approximation in this technique is that it evaluates P-Δ and P-δ effects separately, through separate multipliers $B_2$ and $B_1$, respectively, considering the influence of P-δ effects on the overall response of the structure (which, in turn, influences P-Δ) only indirectly, through the factor $R_M$. A rigorous second-order elastic analysis is recommended for accurate determination of the frame internal forces when $B_1$ is larger than 1.2 in members that have a significant effect on the response of the overall structure.

This procedure uses a first-order elastic analysis with amplification factors that are applied to the first-order forces and moments so as to obtain an estimate of the second-order forces and moments. In the general case, a member may have first-order load effects not associated with sidesway that are multiplied by a factor $B_1$, and first-order load effects produced by sidesway that are multiplied by a factor $B_2$. The factor $B_1$ estimates the P-δ effects on the nonsway moments in compression members. The factor $B_2$ estimates the P-Δ effects on the forces and moments in all members. These effects are shown graphically in Figures C-C2.1 and C-A-8.1.

The factor $B_2$ applies only to internal forces associated with sidesway and is calculated for an entire story. In building frames designed to limit $\Delta H/L$ to a predetermined value, the factor $B_2$ may be found in advance of designing individual members by using the target

![Diagram of moment amplification](image)

*Fig. C-A-8.1. Moment amplification.*

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### TABLE C-A-8.1
Amplification Factors $\psi$ and $C_m$ $P_r = P_u$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\psi$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
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<td><img src="image1" alt="Case 1" /></td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td><img src="image2" alt="Case 2" /></td>
<td>$\alpha = 1.0$ [1 - 0.4 \frac{\alpha P_r}{P_{el}}]</td>
<td>$P_r = P_u$ $P_{el} = \text{Euler}$</td>
</tr>
<tr>
<td><img src="image3" alt="Case 3" /></td>
<td>$\alpha = 0.4$ [1 - 0.4 \frac{\alpha P_r}{P_{el}}]</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>$\alpha = 0.3$ [1 - 0.3 \frac{\alpha P_r}{P_{el}}]</td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Case 6" /></td>
<td>$\alpha = 0.2$ [1 - 0.2 \frac{\alpha P_r}{P_{el}}]</td>
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</tr>
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</table>
Table 4-21
Stiffness Reduction Factor

<table>
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<tr>
<th>ASD</th>
<th>LRFD</th>
<th>$f_y$, ksi</th>
</tr>
</thead>
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<tr>
<td>$\frac{P_y}{A_y}$</td>
<td>$\frac{P_o}{A_o}$</td>
<td>35</td>
</tr>
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<td>ASD</td>
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<td>ASD</td>
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<tr>
<td>45</td>
<td>-</td>
<td>-</td>
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</tr>
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<tr>
<td>5</td>
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</table>

*Indicates the stiffness reduction parameter is not applicable because the required strength exceeds the available strength for $KLr = 0$.*
APPENDIX 7

ALTERNATIVE METHODS OF DESIGN FOR STABILITY
(SPECS)

This appendix presents alternatives to the direct analysis method of design for stability defined in Chapter C. The two alternative methods covered are the effective length method and the first-order analysis method.

The appendix is organized as follows:

7.1. General Stability Requirements
7.2. Effective Length Method
7.3. First-Order Analysis Method

7.1. GENERAL STABILITY REQUIREMENTS

The general requirements of Section C1 shall apply. As an alternative to the direct analysis method (defined in Sections C1 and C2), it is permissible to design structures for stability in accordance with either the effective length method, specified in Section 7.2, or the first-order analysis method, specified in Section 7.3, subject to the limitations indicated in those sections.

7.2. EFFECTIVE LENGTH METHOD

1. Limitations

The use of the effective length method shall be limited to the following conditions:

(1) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.

(2) The ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations) in all stories is equal to or less than 1.5.

User Note: The ratio of second-order drift to first-order drift in a story may be taken as the $B_2$ multiplier, calculated as specified in Appendix 8.

2. Required Strengths

The required strengths of components shall be determined from analysis conforming to the requirements of Section C2.1, except that the stiffness reduction indicated in Section C2.3 shall not be applied; the nominal stiffnesses of all structural steel components shall be used. Notional loads shall be applied in the analysis in accordance with Section C2.2b.
User Note: Since the condition specified in Section C2.2b(4) will be satisfied in all cases where the effective length method is applicable, the notional load need only be applied in gravity-only load cases.

3. Available Strengths

The available strengths of members and connections shall be calculated in accordance with the provisions of Chapters D, E, F, G, H, I, J and K, as applicable.

The effective length factor, \( K \), of members subject to compression shall be taken as specified in (a) or (b), below, as applicable.

(a) In braced frame systems, shear wall systems, and other structural systems where lateral stability and resistance to lateral loads does not rely on the flexural stiffness of columns, the effective length factor, \( K \), of members subject to compression shall be taken as 1.0, unless rational analysis indicates that a lower value is appropriate.

(b) In moment frame systems and other structural systems in which the flexural stiffnesses of columns are considered to contribute to lateral stability and resistance to lateral loads, the effective length factor, \( K \), or elastic critical buckling stress, \( F_e \), of those columns whose flexural stiffnesses are considered to contribute to lateral stability and resistance to lateral loads shall be determined from a side-sway buckling analysis of the structure; \( K \) shall be taken as 1.0 for columns whose flexural stiffnesses are not considered to contribute to lateral stability and resistance to lateral loads.

Exception: It is permitted to use \( K = 1.0 \) in the design of all columns if the ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations) in all stories is equal to or less than 1.1.

User Note: Methods of calculating the effective length factor, \( K \), are discussed in the Commentary.

Bracing intended to define the unbraced lengths of members shall have sufficient stiffness and strength to control member movement at the braced points.

User Note: Methods of satisfying the bracing requirement are provided in Appendix 6. The requirements of Appendix 6 are not applicable to bracing that is included in the analysis of the overall structure as part of the overall force-resisting system.
7.3. FIRST-ORDER ANALYSIS METHOD

1. Limitations

The use of the first-order analysis method shall be limited to the following conditions:

(1) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.
(2) The ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations) in all stories is equal to or less than 1.5.

User Note: The ratio of second-order drift to first-order drift in a story may be taken as the $B_2$ multiplier, calculated as specified in Appendix 8.

(3) The required axial compressive strengths of all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure satisfy the limitation:

$$\alpha P_r \leq 0.5P_y$$  \hspace{1cm} (A-7-1)

where

\begin{align*}
\alpha &= 1.0 \text{ (LRFD)}; \quad \alpha = 1.6 \text{ (ASD)} \\
P_r &= \text{required axial compressive strength under LRFD or ASD load combinations, kips (N)} \\
P_y &= F_y A = \text{axial yield strength, kips (N)}
\end{align*}

2. Required Strengths

The required strengths of components shall be determined from a first-order analysis, with additional requirements (1) and (2) below. The analysis shall consider flexural, shear and axial member deformations, and all other deformations that contribute to displacements of the structure.

(1) All load combinations shall include an additional lateral load, $N_i$, applied in combination with other loads at each level of the structure:

$$N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_l$$  \hspace{1cm} (A-7-2)

where

\begin{align*}
\alpha &= 1.0 \text{ (LRFD)}; \quad \alpha = 1.6 \text{ (ASD)} \\
Y_i &= \text{gravity load applied at level } i \text{ from the LRFD load combination or ASD load combination, as applicable, kips (N)} \\
\Delta/L &= \text{maximum ratio of } \Delta \text{ to } L \text{ for all stories in the structure} \\
\Delta &= \text{first-order interstory drift due to the LRFD or ASD load combination, as applicable, in. (mm). Where } \Delta \text{ varies over the plan area of the structure, } \Delta \text{ shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift.} \\
L &= \text{height of story, in. (mm)}
\end{align*}
The additional lateral load at any level, \( N_i \), shall be distributed over that level in the same manner as the gravity load at the level. The additional lateral loads shall be applied in the direction that provides the greatest destabilizing effect.

**User Note:** For most building structures, the requirement regarding the direction of \( N_i \) may be satisfied as follows: For load combinations that do not include lateral loading, consider two alternative orthogonal directions for the additional lateral load, in a positive and a negative sense in each of the two directions, same direction at all levels; for load combinations that include lateral loading, apply all the additional lateral loads in the direction of the resultant of all lateral loads in the combination.

(2) The nonsway amplification of beam-column moments shall be considered by applying the \( B_1 \) amplifier of Appendix 8 to the total member moments.

**User Note:** Since there is no second-order analysis involved in the first-order analysis method for design by ASD, it is not necessary to amplify ASD load combinations by 1.6 before performing the analysis, as required in the *direct analysis method* and the *effective length method*.

3. **Available Strengths**

The *available strengths* of members and connections shall be calculated in accordance with the provisions of Chapters D, E, F, G, H, I, J and K, as applicable.

The *effective length factor*, \( K \), of all members shall be taken as unity.

*Bracing* intended to define the *unbraced lengths* of members shall have sufficient *stiffness* and strength to control member movement at the braced points.

**User Note:** Methods of satisfying this requirement are provided in Appendix 6. The requirements of Appendix 6 are not applicable to bracing that is included in the analysis of the overall structure as part of the overall force-resisting system.
APPENDIX 7

ALTERNATIVE METHODS OF DESIGN FOR STABILITY
(COMMENTARY)

The effective length method and first-order analysis method are addressed in this Appendix as alternatives to the direct analysis method, which is presented in Chapter C. These alternative methods of design for stability can be used when the limits on their use as defined in Appendix Sections 7.2.1 and 7.3.1, respectively, are satisfied.

Both methods in this Appendix utilize the nominal geometry and the nominal elastic stiffnesses \((EI, EA)\) in the analysis. Accordingly, it is important to note that the sidesway amplification \(\Delta_{2nd-order}/\Delta_{1st-order}\) or \(B_2\) limits specified in Chapter C and Appendix 7 are different. For the direct analysis method in Chapter C, the limit of 1.7 for certain requirements is based upon the use of reduced stiffnesses \((EI^* \text{ and } EA^*)\). For the effective length method and first-order analysis method, the equivalent limit of 1.5 is based upon the use of unreduced stiffnesses \((EI \text{ and } EA)\).

7.2. EFFECTIVE LENGTH METHOD

The effective length method (though it was not formally identified by this name) has been used in various forms in the AISC Specification since 1961. The current provisions are essentially the same as those in Chapter C of the 2005 Specification for Structural Steel Buildings (AISC, 2005a), with the following exceptions:

These provisions, together with the use of a column effective length greater than the actual length for calculating available strength in some cases, account for the effects of initial out-of-plumbness and member stiffness reductions due to the spread of plasticity. No stiffness reduction is required in the analysis.

The effective length, \(KL\), for column buckling based upon elastic (or inelastic) stability theory, or alternatively the equivalent elastic column buckling load, \(F_e = \pi^2EI/(KL)^2\), is used to calculate an axial compressive strength, \(P_c\), through an empirical column curve that accounts for geometric imperfections and distributed yielding (including the effects of residual stresses). This column strength is then combined with the flexural strength, \(M_c\), and second-order member forces, \(P_r\) and \(M_r\), in the beam-column interaction equations.

**Braced Frames**

Braced frames are commonly idealized as vertically cantilevered pin-connected truss systems, ignoring any secondary moments within the system. The effective length factor, \(K\), of components of the braced frame is normally taken as 1.0, unless a smaller value is justified by structural analysis and the member and connection design is consistent with this assumption. If connection fixity is modeled in the analysis, the resulting member and connection moments must be accommodated in the design.

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If $K < 1$ is used for the calculation of $P_n$ in braced frames, the additional demands on the stability bracing systems and the influence on the second-order moments in beams providing restraint to the columns must be considered. The provisions in Appendix 6 do not address the additional demands on bracing members from the use of $K < 1$. Generally, a rigorous second-order elastic analysis is necessary for calculation of the second-order moments in beams providing restraint to column members designed based on $K < 1$. Therefore, design using $K = 1$ is recommended, except in those special situations where the additional calculations are deemed justified.

**Moment Frames**

Moment frames rely primarily upon the flexural stiffness of the connected beams and columns. Stiffness reductions due to shear deformations may require consideration when bay sizes are small and/or members are deep.

When the *effective length method* is used, the design of all beam-columns in moment frames must be based on an effective length, $KL$, greater than the actual length, $L$, except when specific exceptions based upon high structural stiffness are met. When the sidesway amplification ($\Delta_{2nd-order}/\Delta_{1st-order}$ or $B_2$) is equal to or less than 1.1, the frame design may be based on the use of $K = 1.0$ for the columns. This simplification for stiffer structures results in a 6% maximum error in the in-plane beam-column strength checks of Chapter H (White and Hajjar, 1997a). When the sidesway amplification is larger, $K$ must be calculated.

A wide range of methods has been suggested in the literature for the calculation of $K$-factors (Kavanagh, 1962; Johnston, 1976; LeMessurier, 1977; ASCE Task Committee on Effective Length, 1997; White and Hajjar, 1997b). These range from simple idealizations of single columns as shown in Table C-A.7.1 to complex buckling solutions for specific frames and loading conditions. In some types of frames, $K$-factors are easily estimated or calculated, and are a convenient tool for stability design. In other types of structures, the determination of accurate $K$-factors is determined by tedious hand procedures, and system stability may be assessed more effectively with the direct analysis method.

The most common method for determining $K$ is through use of the *alignment charts*, which are shown in Figure C-A.7.1 for frames with sidesway inhibited and Figure C-A.7.2 for frames with sidesway uninhibited (Kavanagh, 1962). These charts are based on assumptions of idealized conditions, which seldom exist in real structures, as follows:

1. Behavior is purely elastic.
2. All members have constant cross section.
3. All joints are rigid.
4. For columns in frames with sidesway inhibited, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.
5. For columns in frames with sidesway uninhibited, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending.
6. The stiffness parameter $L\sqrt{P/El}$ of all columns is equal.
TABLE C-A-7.1
Approximate Values of Effective Length Factor, K

<table>
<thead>
<tr>
<th>Buckled shape of column is shown by dashed line</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical K value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design value when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>End condition code</td>
<td>Rotation fixed and translation fixed</td>
<td>Rotation free and translation fixed</td>
<td>Rotation fixed and translation free</td>
<td>Rotation free and translation free</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(7) Joint restraint is distributed to the column above and below the joint in proportion to $EI/L$ for the two columns.
(8) All columns buckle simultaneously.
(9) No significant axial compression force exists in the girders.

The alignment chart for sidesway inhibited frames shown in Figure C-A-7.1 is based on the following equation:

$$\frac{G_A G_B}{4} \left( \frac{\pi}{K} \right)^2 + \left( \frac{G_A + G_B}{2} \right) \left( 1 - \frac{\pi}{K} \frac{\pi}{\tan(\pi/K)} \right) + \frac{2 \tan(\pi/2K)}{(\pi/K)} - 1 = 0 \quad (C-A-7.1)$$

The alignment chart for sidesway uninhibited frames shown in Figure C-A-7.2 is based on the following equation:

$$\frac{G_A G_B}{6(G_A + G_B)} \frac{\pi/K}{\tan(\pi/K)} = 0 \quad (C-A-7.2)$$
where

\[
G = \frac{\sum (E_c I_c / L_c)}{\sum (E_g I_g / L_g)} = \frac{\Sigma (EI / L)_{c}}{\Sigma (EI / L)_{g}}
\]  \hspace{1cm} (C-A-7-3)

The subscripts A and B refer to the joints at the ends of the column being considered. The symbol \( \Sigma \) indicates a summation of all members rigidly connected to that joint and located in the plane in which buckling of the column is being considered. \( E_c \) is the elastic modulus of the column, \( I_c \) is the moment of inertia of the column, and \( L_c \) is the unsupported length of the column. \( E_g \) is the elastic modulus of the girder, \( I_g \) is the moment of inertia of the girder, and \( L_g \) is the unsupported length of the girder or other restraining member. \( I_c \) and \( I_g \) are taken about axes perpendicular to the plane of buckling being considered. The alignment charts are valid for different materials if an appropriate effective rigidity, \( EI \), is used in the calculation of \( G \).

It is important to remember that the alignment charts are based on the assumptions of idealized conditions previously discussed and that these conditions seldom exist in real structures. Therefore, adjustments are often required, such as:

Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).
Adjustments for Columns With Differing End Conditions. For column ends supported by, but not rigidly connected to, a footing or foundation, $G$ is theoretically infinity but unless designed as a true friction-free pin, may be taken as 10 for practical designs. If the column end is rigidly attached to a properly designed footing, $G$ may be taken as 1.0. Smaller values may be used if justified by analysis.

Adjustments for Girders With Differing End Conditions. For sidesway inhibited frames, these adjustments for different girder end conditions may be made:

(a) If the far end of a girder is fixed, multiply the $(EI/L)_{g}$ of the member by 2.
(b) If the far end of the girder is pinned, multiply the $(EI/L)_{g}$ of the member by $1\frac{1}{2}$.

For sidesway uninhibited frames and girders with different boundary conditions, the modified girder length, $L'_{g}$, should be used in place of the actual girder length, where

$$L'_{g} = L_{g} \left(2 - \frac{M_{F}}{M_{N}}\right)$$  \hspace{1cm} (C-A-7-4)

$M_{F}$ is the far end girder moment and $M_{N}$ is the near end girder moment from a first-order lateral analysis of the frame. The ratio of the two moments is positive if the girder is in reverse curvature. If $M_{F}/M_{N}$ is more than 2.0, then $L'_{g}$ becomes negative.

---

Fig. C-A-7.2. Alignment chart sidesway—uninhibited (moment frame).
7.3. FIRST-ORDER ANALYSIS METHOD

1. Limitations

The use of the first-order analysis method shall be limited to the following conditions:

(1) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.

(2) The ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations) in all stories is equal to or less than 1.5.

User Note: The ratio of second-order drift to first-order drift in a story may be taken as the $B_2$ multiplier, calculated as specified in Appendix 8.

(3) The required axial compressive strengths of all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure satisfy the limitation:

$$\alpha P_r \leq 0.5 P_y$$

where

$\alpha = 1.0$ (LRFD); $\alpha = 1.6$ (ASD)

$P_r =$ required axial compressive strength under LRFD or ASD load combinations, kips (N)

$P_y = F_y A =$ axial yield strength, kips (N)

2. Required Strengths

The required strengths of components shall be determined from a first-order analysis, with additional requirements (1) and (2) below. The analysis shall consider flexural, shear and axial member deformations, and all other deformations that contribute to displacements of the structure.

(1) All load combinations shall include an additional lateral load, $N_i$, applied in combination with other loads at each level of the structure:

$$N_i = 2.1 \alpha (\Delta/L) Y_i \geq 0.0042 Y_i$$

where

$\alpha = 1.0$ (LRFD); $\alpha = 1.6$ (ASD)

$Y_i =$ gravity load applied at level $i$ from the LRFD load combination or ASD load combination, as applicable, kips (N)

$\Delta/L =$ maximum ratio of $\Delta$ to $L$ for all stories in the structure

$\Delta =$ first-order interstory drift due to the LRFD or ASD load combination, as applicable, in. (mm). Where $\Delta$ varies over the plan area of the structure, $\Delta$ shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift.

$L =$ height of story, in. (mm)
The additional lateral load at any level, \( N_i \), shall be distributed over that level in the same manner as the gravity load at the level. The additional lateral loads shall be applied in the direction that provides the greatest destabilizing effect.

**User Note:** For most building structures, the requirement regarding the direction of \( N_i \) may be satisfied as follows: For load combinations that do not include lateral loading, consider two alternative orthogonal directions for the additional lateral load, in a positive and a negative sense in each of the two directions, same direction at all levels; for load combinations that include lateral loading, apply all the additional lateral loads in the direction of the resultant of all lateral loads in the combination.

(2) The nonway amplification of *beam-column* moments shall be considered by applying the \( B_1 \) amplifier of Appendix 8 to the total member moments.

**User Note:** Since there is no second-order analysis involved in the first-order analysis method for design by ASD, it is not necessary to amplify ASD load combinations by 1.6 before performing the analysis, as required in the *direct analysis method* and the *effective length* method.

3. **Available Strengths**

The *available strengths* of members and connections shall be calculated in accordance with the provisions of Chapters D, E, F, G, H, I, J and K, as applicable.

The *effective length factor, \( K \)* of all members shall be taken as unity.

*Bracing* intended to define the *unbraced lengths* of members shall have sufficient *stiffness* and strength to control member movement at the braced points.

**User Note:** Methods of satisfying this requirement are provided in Appendix 6. The requirements of Appendix 6 are not applicable to bracing that is included in the analysis of the overall structure as part of the overall force-resisting system.
In the effective length method, the intersection of the second-order elastic analysis curve with the $P_{nKL}$ interaction curve determines the member strength. The plot in Figure C-C2.5(a) shows that the effective length method is calibrated to give a resultant axial strength, $P_c$, consistent with the actual response. For slender columns, the calculation of the effective length, $KL$, (and $P_{nKL}$) is critical to achieving an accurate solution when using the effective length method.

One consequence of the procedure is that it underestimates the actual internal moments under the factored loads, as shown in Figure C-C2.5(a). This is inconsequential for the beam-column in-plane strength check since $P_{nKL}$ reduces the effective strength in the correct proportion. However, the reduced moment can affect the design of the beams and connections, which provide rotational restraint to the column. This is of greatest concern when the calculated moments are small and axial loads are large, such that $P$-$\Delta$ moments induced by column out-of-plumbness can be significant.

The important difference between the direct analysis method and the effective length method is that where the former uses reduced stiffness in the analysis and $K = 1.0$ in the beam-column strength check, the latter uses nominal stiffness in the analysis and $K$ from a side-way buckling analysis in the beam-column strength check. The direct analysis method can be more sensitive to the accuracy of the second-order elastic analysis since analysis at reduced stiffness increases the magnitude of second-order effects. However, this difference is important only at high side-way amplification levels; at those levels the accuracy of the calculation of $K$ for the effective length method also becomes important.

7.3. **FIRST-ORDER ANALYSIS METHOD**

This section provides a method for designing frames using a first-order elastic analysis with $K = 1.0$, provided the limitations in Appendix 7, Section 7.3.1 are satisfied. This method is derived from the direct analysis method by mathematical manipulation (Kuchenbecker et al., 2004) so that the second-order internal forces and moments are determined directly as part of the first-order analysis. It is based upon a target maximum drift ratio, $\Delta/L$, and assumptions, including:

1. The side-way amplification $\Delta_{2nd\ order}/\Delta_{1st\ order}$ (or $B_2$) is assumed equal to 1.5.
2. The initial out-of-plumbness in the structure is assumed as $\Delta_0/L = 1/500$, but the initial out-of-plumbness does not need to be considered in the calculation of $\Delta$.

The first-order analysis is performed using the nominal (unreduced) stiffness; stiffness reduction is accounted for solely within the calculation of the amplification factors. The non-way amplification of beam-column moments is addressed within the procedure specified in this Section by applying the $B_1$ amplifier of Appendix 8, Section 8.2.1 conservatively to the total member moments. In many cases involving beam-columns not subject to transverse loading between supports in the plane of bending, $B_1 = 1.0$.

The target maximum drift ratio, corresponding to drifts under either the LRFD strength load combinations or 1.6 times the ASD strength load combinations, can
The direct analysis method is the preferred method. If the appropriate software is available, a second-order analysis is the method of choice. If a second-order analysis is not available, the moment amplification method, which is an acceptable direct analysis approach, can be used. In this book, the results of structural analyses will be given in all examples and problems. The reader will not be required to perform an analysis. If the bending moment and axial force are from a second-order analysis, you can go straight to the interaction equations from AISC Specification Chapter H. If the required strengths are from first-order analyses, the moment amplification method, an approximate second-order analysis given in Appendix 8, can be used. This method will be covered in detail in the following sections.

6.4 THE MOMENT AMPLIFICATION METHOD

The moment amplification method entails computing the maximum bending moment resulting from flexural loading (transverse loads or member end moments) by a first-order analysis, then multiplying by a moment amplification factor to account for the secondary moment. An expression for this factor will now be developed.

Figure 6.5 shows a simply supported member with an axial load and an initial out-of-straightness. This initial crookedness can be approximated by

$$y_0 = e \sin \frac{\pi x}{L}$$

where $e$ is the maximum initial displacement, occurring at midspan. For the coordinate system shown, the moment–curvature relationship can be written as

$$\frac{d^2 y}{dx^2} = -\frac{M}{EI}$$

The bending moment, $M$, is caused by the eccentricity of the axial load, $P$, with respect to the axis of the member. This eccentricity consists of the initial crookedness, $y_0$, plus additional deflection, $y$, resulting from bending. At any location, the moment is:

$$M = P (y_0 + y)$$

**FIGURE 6.5**
Substituting this equation into the differential equation, we obtain

\[
\frac{d^2 y}{dx^2} = -\frac{P}{EI} \left( \frac{y''}{\sin \frac{\pi x}{L}} + y' \right)
\]

Rearranging gives

\[
\frac{d^2 y}{dx^2} + \frac{P}{EI} y = -\frac{Pe}{EI} \sin \frac{\pi x}{L}
\]

which is an ordinary, nonhomogeneous differential equation. Because it is a second-order equation, there are two boundary conditions. For the support conditions shown, the boundary conditions are

At \( x = 0, y = 0 \) and at \( x = L, y = 0 \).

That is, the displacement is zero at each end. A function that satisfies both the differential equation and the boundary conditions is

\[
y = B \sin \frac{\pi x}{L}
\]

where \( B \) is a constant. Substituting into the differential equation, we get

\[
-\frac{\pi^2}{L^2} B \sin \frac{\pi x}{L} + \frac{P}{EI} B \sin \frac{\pi x}{L} = -\frac{Pe}{EI} \sin \frac{\pi x}{L}
\]

Solving for the constant gives

\[
B = \frac{Pe}{EI} = \frac{-e}{\frac{P}{EI} - \frac{\pi^2}{L^2}} = \frac{e}{P - \frac{\pi^2 EI}{PL^2}} = \frac{e}{P \left( \frac{Pe}{P} - 1 \right)}
\]

where

\[
P_e = \frac{\pi^2 EI}{L^2} = \text{the Euler buckling load}
\]

\[
\therefore y = B \sin \frac{\pi x}{L} = \left[ \frac{e}{(P_e/P) - 1} \right] \sin \frac{\pi x}{L}
\]

\[
M = P(y_0 + y) = P \left\{ e \sin \frac{\pi x}{L} + \left[ \frac{e}{(P_e/P) - 1} \right] \sin \frac{\pi x}{L} \right\}
\]
The maximum moment occurs at $x = L/2$:

$$M_{\text{max}} = P \left[ e + \frac{e}{(P_e/P) - 1} \right] = P e \left[ \frac{(P_e/P) - 1 + 1}{(P_e/P) - 1} \right] = M_0 \left[ \frac{1}{1 - (P/P_e)} \right]$$

where $M_0$ is the unamplified maximum moment. In this case, it results from initial crookedness, but in general, it can be the result of transverse loads or end moments. The moment amplification factor is therefore

$$\frac{1}{1 - (P/P_e)}$$

Because the member deflection corresponds to a buckled shape, the axial load corresponds to a failure load—that is, a load corresponding to an LRFD formulation. Therefore, the amplification factor should be written as

$$\frac{1}{1 - (P_e/P_u)}$$

(6.7)

where $P_u$ is the factored axial load. The form shown in Expression 6.7 is appropriate for LRFD. For ASD, a different form, to be explained later, will be used.

As we describe later, the exact form of the AISC moment amplification factor can be slightly different from that shown in Expression 6.7.

**EXAMPLE 6.2**

Use Expression 6.7 to compute the LRFD amplification factor for the beam-column of Example 6.1. Since the Euler load $P_e$ is part of an amplification factor for a moment, it must be computed for the axis of bending, which in this case is the $x$-axis. In terms of effective length, the Euler load can be written as

$$P_e = \frac{\pi^2 E I_x}{(K L)^2} = \frac{\pi^2 E I_x}{(K_e L)^2} = \frac{\pi^2 (29,000)(272)}{(1.0 \times 17 \times 12)^2} = 1871 \text{ kips}$$

$200.4 \text{ k}$

**SOLUTION**

Since the Euler load $P_e$ is part of an amplification factor for a moment, it must be computed for the axis of bending, which in this case is the $x$-axis. In terms of effective length, the Euler load can be written as

$$P_e = \frac{\pi^2 E I_x}{(K L)^2} = \frac{\pi^2 E I_x}{(K_e L)^2} = \frac{\pi^2 (29,000)(272)}{(1.0 \times 17 \times 12)^2} = 1871 \text{ kips}$$

$200.4 \text{ k}$

$$8.5'$$

$25.2 \text{ k}$

$25.2 \text{ k}$

Top View
Axis of bending = x - x due to transverse load

\[ P_e \text{ for use in } \frac{1}{1 - (Pu/Pex)}, \]

use \( Pex \)

Axis of bending = y - y due to a transverse load

\[ P_e \text{ for use in } \frac{1}{1 - (Pu/Pey)}, \]

use \( Pey \)
From the LRFD solution to Example 6.1, \( P_u = 200.4 \text{ kips} \), and

\[
\frac{1}{1 - (P_u/P_e)} = \frac{1}{1 - (200.4/1871)} = 1.12
\]

which represents a 12% increase in bending moment. The amplified primary LRFD moment is

\[1.12 \times M_u = 1.12(107.1) = 120 \text{ ft-kips} \]

**ANSWER** Amplification factor = 1.12.

---

### 6.5 BRACED VERSUS UNBRACED FRAMES

As explained in Section 6.3, "Methods of Analysis for Required Strength," there are two types of secondary moments: \( P-\delta \) (caused by member deflection) and \( P-\Delta \) (caused by the effect of sway when the member is part of an unbraced frame [moment frame]). Because of this, two amplification factors must be used. The AISC Specification covers this in Appendix 8, "Approximate Second-Order Analysis." The approach is the same as the one used in the ACI Building Code for reinforced concrete (ACI, 2008). Figure 6.4 illustrates these two components of deflection. In Figure 6.4a, the member is restrained against sidesway, and the maximum secondary moment is \( P\delta \), which is added to the maximum moment within the member. If the frame is actually unbraced, there is an additional component of the secondary moment, shown in Figure 6.4b, that is caused by sidesway. This secondary moment has a maximum value of \( P\Delta \), which represents an amplification of the end moment.

To approximate these two effects, two amplification factors, \( B_1 \) and \( B_2 \), are used for the two types of moments. The amplified moment to be used in design is computed from the loads and moments as follows (\( x \) and \( y \) subscripts are not used here; amplified moments must be computed in the following manner for each axis about which there are moments):

\[
M = B_1M_{nt} + B_2M_{tt}
\]

(AISC Equation A-8-1)

where

\[
M_t = \text{required moment strength}
\]

\[
= M_u \text{ for LRFD}
\]

\[
= M_s \text{ for ASD}
\]

\[
M_{nt} = \text{maximum moment assuming that no sidesway occurs, whether the frame is actually braced or not (the subscript nt is for "no translation"). } M_{nt} \text{ will be a factored load moment for LRFD and a service load moment for ASD.}
\]
\( M_{lt} \) = maximum moment caused by sidesway (the subscript \( lt \) is for "lateral translation"). This moment can be caused by lateral loads or by unbalanced gravity loads. Gravity load can produce sidesway if the frame is unsymmetrical or if the gravity loads are unsymmetrically placed. \( M_{lt} \) will be zero if the frame is actually braced. For LRFD, \( M_{lt} \) will be a factored load moment, and for ASD, it will be a service load moment.

\( B_1 \) = amplification factor for the moments occurring in the member when it is braced against sidesway (\( P-\delta \) moments).

\( B_2 \) = amplification factor for the moments resulting from sidesway (\( P-\Delta \) moments).

In addition to the required moment strength, the required axial strength must account for second-order effects. The required axial strength is affected by the displaced geometry of the structure during loading. This is not an issue with member displacement (\( \delta \)), but it is with joint displacement (\( \Delta \)). The required axial compressive strength is given by

\[
P_r = P_m + B_2 P_{lt}
\]

(AISC Equation A.8-2)

where

- \( P_m \) = axial load corresponding to the braced condition
- \( P_{lt} \) = axial load corresponding to the sidesway condition

We cover the evaluation of \( B_1 \) and \( B_2 \) in the following sections.

6.6 MEMBERS IN BRACED FRAMES

The amplification factor given by Expression 6.7 was derived for a member braced against sidesway—that is, one whose ends cannot translate with respect to each other. Figure 6.6 shows a member of this type subjected to equal end moments producing single-curvature bending (bending that produces tension or compression on
### Table 3-23

#### Shears, Moments and Deflections

1. **SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD**

   - Total Eqv. Uniform Load: \( \frac{w l}{2} \)
   - \( R = V = \frac{w l}{2} \)
   - \( V_x = \frac{w}{2} \left( 1 - \frac{x}{l} \right) \)
   - \( M_{max} \) (at center): \( \frac{w l^2}{8} \)
   - \( M_x \) (at center): \( \frac{w x}{2} \left( l - x \right) \)
   - \( \Delta_{max} \) (at center): \( \frac{5 w l^4}{384 E I} \)
   - \( \Delta_x \) (at center): \( \frac{w x}{24 E I} \left( 3 - 21 x^2 + x^4 \right) \)

2. **SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO ONE END**

   - Total Eqv. Uniform Load: \( \frac{16 W}{9 \sqrt{3}} = 1.03 W \)
   - \( R_1 = V_1 = \frac{W}{3} \)
   - \( R_2 = V_2 = V_{max} = \frac{2W}{3} \)
   - \( V_x = \frac{W}{3} \left( 1 - \frac{x^2}{l^2} \right) \)
   - \( M_{max} \) (at \( x = \frac{l}{\sqrt{3}} \)): \( \frac{2Wl}{9 \sqrt{3}} = 0.128 WL \)
   - \( M_x \) (at \( x = \frac{6}{15} - \frac{0.519 l}{15} \)): \( \frac{W x^3}{3 l^2} - \frac{2 W l^2}{9 \sqrt{3}} \)
   - \( \Delta_{max} \) (at \( x = \frac{l}{15} \)): \( \frac{0.0125 W l^3}{E I} \)
   - \( \Delta_x \) (at \( x = \frac{l}{15} \)): \( \frac{W x}{180 E I l^2} \left( 3 l^4 - 10 l^2 x^2 + 7 x^4 \right) \)

3. **SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO CENTER**

   - Total Eqv. Uniform Load: \( \frac{4W}{3} \)
   - \( R = V = \frac{W}{2} \)
   - \( V_x \) (when \( x < \frac{l}{2} \)): \( \frac{W}{2} \left( 2 - 4 x^2 \right) \)
   - \( M_{max} \) (at center): \( \frac{W l}{6} \)
   - \( M_x \) (when \( x < \frac{l}{2} \)): \( \frac{W l^3}{8} \left( \frac{1}{2} x^2 - \frac{3}{2} l^2 \right) \)
   - \( \Delta_{max} \) (at center): \( \frac{W l^3}{60 E I} \)
   - \( \Delta_x \) (when \( x < \frac{l}{2} \)): \( \frac{W x}{480 E I l^2} \left( 6 l^4 - 6 x^2 \right)^2 \)
Table 3-23 (continued)

Shears, Moments and Deflections

32. BEAM — UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS

\[
R_x = V_x = \frac{w l}{2} \left( M_1 - \frac{M_2}{l} \right)
\]

\[
M_x = \begin{cases} 
\frac{w}{2} \left( \frac{l - x}{l} \right) M_1 \quad \text{(at } x = \frac{l}{2} \text{)} \\
\frac{w}{2} \left( \frac{l - x}{l} \right) \frac{M_1 - \frac{M_2}{l}}{l} \quad \text{(when } x < \frac{l}{2} \text{)} \\
\frac{w}{2} \left( \frac{l - x}{l} \right) \frac{(M_1 - \frac{M_2}{l})^2}{l} \quad \text{(when } x > \frac{l}{2} \text{)}
\end{cases}
\]

\[
\Delta_x = \frac{w l^3}{24 E I} \left[ x^3 \left( \frac{2}{w} \frac{4 M_1}{w l} \frac{4 M_2}{w l} \right) x^2 + \frac{12 M_1}{w} x^3 - \frac{8 M_1}{w} \frac{4 M_2}{w} \right]
\]

33. BEAM — CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS

\[
R_x = V_x = \frac{P}{2} \left( M_1 - \frac{M_2}{l} \right)
\]

\[
M_x = \begin{cases} 
\frac{P}{4} \frac{M_1 + M_2}{2} \quad \text{(at center)} \\
\frac{P}{2} \left( \frac{M_1 - \frac{M_2}{l}}{l} \right) x - M_1 \quad \text{(when } x < \frac{l}{2} \text{)} \\
\frac{P}{2} \left( \frac{l - x}{l} \right) \frac{(M_1 - \frac{M_2}{l}) x}{l} - M_1 \quad \text{(when } x > \frac{l}{2} \text{)}
\end{cases}
\]

\[
\Delta_x = \frac{P x}{48 E I} \left[ 3 l^2 - 4 x^2 - \frac{8 (l - x)}{p l} \left( M_1 (2 l - x) + M_2 (l + x) \right) \right]
\]

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one side throughout the length of the member). Maximum moment amplification occurs at the center, where the deflection is largest. For equal end moments, the moment is constant throughout the length of the member, so the maximum primary moment also occurs at the center. Thus the maximum secondary moment and maximum primary moment are additive. Even if the end moments are not equal, as long as one is clockwise and the other is counterclockwise there will be single-curvature bending, and the maximum primary and secondary moments will occur near each other.

That is not the case if applied end moments produce reverse-curvature bending as shown in Figure 6.7. Here the maximum primary moment is at one of the ends, and maximum moment amplification occurs between the ends. Depending on the value of the axial load $P$, the amplified moment can be either larger or smaller than the end moment.

The maximum moment in a beam-column therefore depends on the distribution of bending moment within the member. This distribution is accounted for by a factor, $C_m$, applied to the amplification factor given by Expression 6.7. The amplification
factor given by Expression 6.7 was derived for the worst case, so \( C_m \) will never be greater than 1.0. The final form of the amplification factor is

\[
B_1 = \frac{C_m}{1 - (\alpha P_r/P_{el})} \geq 1 \quad C_m \leq 1 \quad \alpha = 1.00
\]

(AISC Equation A-8-3)

\[
B_1 \geq 1
\]

where

\[
P_r = \text{required unamplified axial compressive strength } (P_{n1} + P_{a})(C_{V\text{EN} 305})
\]

\[
= P_u \text{ for LRFD}
\]

\[
= P_r \text{ for ASD}
\]

\[
\alpha = 1.00 \text{ for LRFD}
\]

\[
= 1.60 \text{ for ASD}
\]

\[
P_{el} = \frac{\pi^2 EI^*}{(K/L)^2}
\]

(AISC Equation A-8-5)

\[
EI^* = \text{flexural rigidity}
\]

In the direct analysis method, \( EI^* \) is a reduced stiffness obtained as

\[
EI^* = 0.8 \tau_b EI
\]

\[
= \frac{16.1-238}{306b}
\]

(6.8)

where

\[
\tau_b = \text{a stiffness reduction factor}
\]

\[
\tau_b = \begin{cases} 
1.0 & \text{when } \alpha P_r/P_y \leq 0.5 \\
0.5 & \text{when } \alpha P_r/P_y > 0.5 
\end{cases}
\]

(AISC Equation C2-2a)

\[
\begin{cases} 
4 \left( \alpha P_r/P_y \right) & \text{when } \alpha P_r/P_y \leq 0.5 \\
\left( \frac{\alpha P_r}{P_y} \right) \left( 1 - \alpha P_r/P_y \right) & \text{when } \alpha P_r/P_y > 0.5 
\end{cases}
\]

(AISC Equation C2-2b)

This stiffness reduction factor is the same one used in Chapter 4 in connection with the alignment charts for inelastic columns. Under certain conditions, \( \tau_b \) can be taken as 1.0 even when \( \alpha P_r/P_y > 0.5 \). As mentioned in Section 6.3, acceptable frame analysis methods, including the direct analysis method, require the application of notional loads to account for initial out-of-plumbness of columns. AISC C2.3(3) permits the use of \( \tau_b = 1.0 \) if a small additional notional load is included. In this book, we assume that that is the case, and we will use \( \tau_b = 1.0 \). Keep in mind that in this book, we do not perform any structural analyses; we only use the results of the analyses.

In the effective length and first-order methods, the flexural rigidity is unreduced, and \( EI^* = EI \). The moment of inertia \( I \) and the effective length factor \( K_l \) are for the axis of bending, and \( K_l = 1.0 \) unless a more accurate value is computed (AISC C3). Note that the subscript 1 corresponds to the braced condition and the subscript 2 corresponds to the unbraced condition.
stiffness to account for inelastic softening prior to the members reaching their design strength. The $\tau_{th}$ factor is similar to the inelastic stiffness reduction factor implied in the column curve to account for loss of stiffness under high compression loads ($\alpha P_{cr} > 0.5 P_y$), and the 0.8 factor accounts for additional softening under combined axial compression and bending. It is a fortuitous coincidence that the reduction coefficients for both slender and stocky columns are close enough, such that the single reduction factor of 0.8$\tau_{th}$ works over the full range of slenderness.

The use of reduced stiffness only pertains to analyses for strength and stability limit states. It does not apply to analyses for other stiffness-based conditions and criteria, such as for drift, deflection, vibration and period determination.

For ease of application in design practice, where $\tau_{th} = 1$, the reduction on $EI$ and $EA$ can be applied by modifying $E$ in the analysis. However, for computer programs that do semi-automated design, one should ensure that the reduced $E$ is applied only for the second-order analysis. The elastic modulus should not be reduced in nominal strength equations that include $E$ (for example, $M_n$ for lateral-torsional buckling in an unbraced beam).

As shown in Figure C-C2.5, the net effect of modifying the analysis in the manner just described is to amplify the second-order forces such that they are closer to the

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**Fig. C-C2.5. Comparison of in-plane beam-column interaction checks for (a) the effective length method and (b) the direct analysis method.**
notional

1. pertaining to or expressing a notion or idea.
2. of the nature of a notion or idea: a notional response to the question.
3. abstract, theoretical, or speculative, as reflective thought.
4. not real or actual; ideal or imaginary: to create a notional world for oneself.
5. given to or full of foolish or fanciful ideas or moods.

Sprint® Official Site
www.sprint.com/Education
Teach & Connect With Mobile Learning Solutions At Sprint®!
Instant Grammar Checker
www.Grammarrly.com
Correct All Grammar Errors And Enhance Your Writing. Try Now!

Example sentences
Employers make contributions, too, but these are notional since the taxpayer.
The government charges departments a notional amount each year to cover the
Theoretical lobby with putative elevator leads to five notional floor-thru
The last elected seat went to an independent, giving the centre-left a notional

H&R Block
All Home
QUICKER IS BETTER.
It's your money, why wait?

Notional | Define Notional at Dictionary.com
RISA News provides information about upcoming education opportunities, program information/updates and technical solutions for commonly asked questions. Click the title of each individual entry for more information.

May 23, 2011

How to Apply Notional Loads in RISA-3D

In RISA-3D you can automatically apply notional loads to your structure to comply with your steel code (such as AISC 360). Notional loads take into account a building’s actual out-of-plumbness by adding de-stabilizing lateral loads. The AISC 360 recommends either 0.2% or 0.3% of the vertical loads be applied at each floor level.
Specifications, Codes and Standards - AISC
https://www.aisc.org/content.asp... American Institute of Steel Construction
AISC's current specifications, codes and standards are available for free as ...
2010 Specification for Structural Steel Buildings (ANSI/AISC 360-10), Second ...
Seismic Provisions for ... - Prequalified Connections for ...

[PDF] AISC 360
Mar 9, 2005 - Approved by the AISC Committee on Specifications and issued by the ...
(This Preface is not part of ANSI/AISC 360-05. Specification for ...

ANSI-AISC 360-10 Specification for Structural Steel Buildings...
https://docs.google.com/open?id=0B...
Google Docs
You are using an unsupported browser. Some features may not work ...

AISC 360 Specification for Structural Steel Buildings - Wikipe...
en.wikipedia.org/.../AISC_360_Specification_for_Structural_S...
Wikipedia
AISC 360 is a specification that gives provision for the determination of the nominal ...
and available strengths of members, connections and other components of ...

New AISC Specification Now Available for Free Downloading
www.modernsteel.com/SteelInTheNews/?p=979
Jan 27, 2011 - The 2010 AISC standard, Specification for Structural Steel Buildings ...
(ANSI/AISC 360-10), is now available as a free download at ...