replacement is made, the load will be concentric, and each fastener can be assumed to resist an equal share of the load, given by \( P_c = P/n \), where \( n \) is the number of fasteners. The fastener forces resulting from the couple can be found by considering the shearing stress in the fasteners to be the result of torsion of a cross section made up of the cross-sectional areas of the fasteners. If such an assumption is made, the shearing stress in each fastener can be found from the torsion formula

\[
\text{Stress} = f_v = \frac{Md_i}{J} = \frac{T_c i}{J} \quad i = ?
\]

(8.1)

where

\[ d = \text{distance from the centroid of the area to the point where the stress is being computed} \]
\[ J = \text{polar moment of inertia of the area about the centroid} \]

and the stress \( f_v \) is perpendicular to \( d \). Although the torsion formula is applicable only to right circular cylinders, its use here is conservative, yielding stresses that are somewhat larger than the actual stresses.

If the parallel-axis theorem is used and the polar moment of inertia of each circular area about its own centroid is neglected, \( J \) for the total area can be approximated as

\[ J = \sum A d_i^2 \]

provided all fasteners have the same area, \( A \). Equation 8.1 can then be written as

\[ f_v = \frac{Md_i}{A \sum d_i^2} \]
8.2 Eccentric Bolted Connections: Shear Only

and the shear force in each fastener caused by the couple is

\[ \text{Moment Force} = P_m = A f_y = A \frac{Md_i}{A \sum d_i^2} = \frac{Md_i}{\sum d_i^2} \]

The two components of shear force thus determined can be added vectorially to obtain the resultant force, \( P \), as shown in Figure 8.3b, where the lower right-hand fastener is used as an example. When the largest resultant is determined, the fastener size is selected so as to resist this force. The critical fastener cannot always be found by inspection, and several force calculations may be necessary.

It is generally more convenient to work with rectangular components of forces. For each fastener, the horizontal and vertical components of force resulting from direct shear are

\[ P_{cx} = \frac{P_x}{n} \quad \text{and} \quad P_{cy} = \frac{P_y}{n} \]

where \( P_x \) and \( P_y \) are the \( x \)- and \( y \)-components of the total connection load, \( P \), as shown in Figure 8.4. The horizontal and vertical components caused by the eccentricity can be found as follows. In terms of the \( x \)- and \( y \)-coordinates of the centers of the fastener areas,

\[ \sum d_i^2 = \sum (x_i^2 + y_i^2) \]

where the origin of the coordinate system is at the centroid of the total fastener shear area. The \( x \)-component of \( P_m \) is

\[ P_{mx} = \frac{My}{\sum (x^2 + y^2)} \]

Similarly,

\[ P_{my} = \frac{Mx}{\sum (x^2 + y^2)} \]

and the total fastener force is

\[ P = \sqrt{\left( \frac{P_{mx}}{x} \right)^2 + \left( \frac{P_{my}}{y} \right)^2} \]

\[ P_x = \sqrt{\left( \frac{P_m}{y} \right)^2 + \left( \frac{P_{mx}}{x} \right)^2} \]

\[ P_y = \sqrt{\left( \frac{P_m}{x} \right)^2 + \left( \frac{P_{my}}{y} \right)^2} \]

**FIGURE 8.4**
where

\[
\begin{align*}
\sum p_x &= p_{cx} + p_{mx} \\
\sum p_y &= p_{cy} + p_{my}
\end{align*}
\]

If \( P \), the load applied to the connection, is a factored load, then force \( p \) on the fastener is the factored load to be resisted in shear and bearing—that is, the required design strength. If \( P \) is a service load, then \( p \) will be the required allowable strength—of the fastener.

**Example 8.1** Determine the critical fastener force in the bracket connection shown in Figure 8.5.

**Solution** The centroid of the fastener group can be found by using a horizontal axis through the lower row and applying the principal of moments:

\[
\frac{2(0) + 2(5) + 2(8) + 2(11)}{8} = 6 \text{ in.}
\]

\[
\Sigma M = 2(0) + 2(5) + 2(8) + 2(11)
\]

The horizontal and vertical components of the load are

\[
P_x = \frac{1}{\sqrt{5}} (50) = 22.36 \text{ kips} \quad \text{and} \quad P_y = \frac{2}{\sqrt{5}} (50) = 44.72 \text{ kips}
\]

Referring to Figure 8.6a, we can compute the moment of the load about the centroid:

\[
M = 44.72 (12 + 2.75) - 22.36 (14 - 6) = 480.7 \text{ in.-kips} \quad \text{clockwise}
\]
8.2 Eccentric Bolted Connections: Shear Only

Figure 8.6b shows the directions of all component bolt forces and the relative magnitudes of the components caused by the couple. Using these directions and relative magnitudes as a guide and bearing in mind that forces add by the parallelogram law, we can conclude that the lower right-hand fastener will have the largest resultant force.

The horizontal and vertical components of force in each bolt resulting from the concentric load are

\[
p_{cx} = \frac{22.36}{8} = 2.795 \text{ kips} \quad \text{and} \quad p_{cy} = \frac{44.72}{8} = 5.590 \text{ kips}
\]

For the couple,

\[
\sum (x^2 + y^2) = 8(2.75)^2 + 2((6)^2 + (1)^2) + (2)^2 + (5)^2 = 192.5 \text{ in}^2
\]

\[
p_{mx} = \frac{My}{\sum (x^2 + y^2)} = \frac{480.7(6)}{192.5} = 14.98 \text{ kips}
\]

\[
p_{my} = \frac{Mx}{\sum (x^2 + y^2)} = \frac{480.7(2.75)}{192.5} = 6.867 \text{ kips}
\]

\[
\sum p_x = 2.795 + 14.98 = 17.78 \text{ kips}
\]

\[
\sum p_y = 5.590 + 6.867 = 12.46 \text{ kips}
\]

\[
p = \sqrt{(17.78)^2 + (12.46)^2} = 21.7 \text{ kips}
\]

(see Figure 8.6c)
### Table 7-1
**Available Shear Strength of Bolts, kips**

<table>
<thead>
<tr>
<th>Nominal Bolt Diameter, ( d ), in.</th>
<th>1/8</th>
<th>1/4</th>
<th>3/8</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Bolt Area, in.°²</td>
<td>0.307</td>
<td>0.442</td>
<td>0.601</td>
<td>0.785</td>
</tr>
<tr>
<td><strong>ASTM Design.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Thread Cond.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fᵥ/Ω (kips)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>φFᵥ (kips)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
</tr>
<tr>
<td>N</td>
<td>27.0</td>
<td>12.4</td>
<td>11.3</td>
<td>17.0</td>
</tr>
<tr>
<td>X</td>
<td>34.0</td>
<td>15.7</td>
<td>15.0</td>
<td>22.5</td>
</tr>
<tr>
<td>X</td>
<td>34.0</td>
<td>15.7</td>
<td>15.0</td>
<td>22.5</td>
</tr>
<tr>
<td>A307</td>
<td>13.5</td>
<td>6.23</td>
<td>5.97</td>
<td>8.97</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nominal Bolt Diameter, ( d ), in.</th>
<th>1/8</th>
<th>1/4</th>
<th>3/8</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Bolt Area, in.°²</td>
<td>0.994</td>
<td>1.23</td>
<td>1.48</td>
<td>1.77</td>
</tr>
<tr>
<td><strong>ASTM Design.</strong></td>
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<tr>
<td><strong>Thread Cond.</strong></td>
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<tr>
<td><strong>Fᵥ/Ω (kips)</strong></td>
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<td><strong>φFᵥ (kips)</strong></td>
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</tr>
<tr>
<td><strong>Loading</strong></td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
<td>LRFD</td>
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<td>33.2</td>
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<tr>
<td>X</td>
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<td>41.8</td>
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<tr>
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<td>34.0</td>
<td>50.7</td>
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<tr>
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<td>62.6</td>
<td>51.7</td>
<td>77.5</td>
</tr>
<tr>
<td>ASD</td>
<td>2.00</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For end loaded connections greater than 38 in., see AISC Specification Table J3.2 footnote b.

**American Institute of Steel Construction**