3.7 Threaded Rods & Cables

- Often used as tension members
- Rods: solid; used in suspended roof systems; hangers or suspension members in bridges; also used in bracing systems

\[ P_n = A_s \cdot F_u = 0.75A_b \cdot F_u \]

Lower bound of \( \frac{A_{thread}}{A_b} \)

Where: \( A_b = \frac{\pi}{4} d_b^2 \) (nominal, unthreaded area)

\[ \varphi = 0.75 \]

\[ \varphi P_n = 0.75 (0.75 A_b \cdot F_u) \]

\[ \varphi P_n = 0.75 (0.75 A_b \cdot F_u) \]

* See example 3.14, p. 61 of Segui text.

Note: \( d_b > \frac{5}{8} \) in. to prevent damage during construction.

- Cables: strand or wire rope, see Fig. 3.26, p. 61 of Segui; used when flexibility is needed, also increased redundancy with respect to fracture. No AISC provision - design provisions published by manufacturers.
Chapter 4: Compression Members

4.1 Introduction - members subject to pure axial compressive loads.

Applicable limit states:
1) \( f = \frac{P}{A} \) (yielding)
2) Buckling \( \rightarrow \) almost always elastic controls, inelastic provision account for both in Chap. E

4.2 Column Theory:

Elastic buckling - "Euler" column (perfect)
1) pinned ends, 2) prismatic - \( \frac{I}{A} \) are constant over \( L \),
3) homogeneous - \( E \) constant over \( L \),
4) perfectly straight, initially

Diagram:

\[ \text{Per.} \quad \text{Y} \quad X \quad \text{Per.} \]

Buckled shape

\[ \text{Per.} \quad \text{L} \quad \text{Per.} \]

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Cut at distance "x" and draw F.B.D. of displaced shape:

\[ P_{cr} = P \]

\[ M = -Py \]

\[ \Sigma M_{cut} = 0 \]

From CVEN 305, 345:

\[ M = \frac{d^2y}{dx^2} EI \]

\[ \frac{d^2y}{dx^2} EI = -Py \]

\[ \frac{d^2y}{dx^2} EI + Py = 0 \]

(1) \[ y'' + \frac{P}{EI} y = 0 \] (2nd order, linear, O.D.E.)
The solution to equation (1):

(2) \( y = A \sin \omega x + B \cos \omega x \)

Where: \( \omega = \sqrt{\frac{P}{EI}} \)

Boundary Conditions:

a) @ \( x = 0, \quad y = 0 \)

b) @ \( x = L, \quad y = 0 \)

Substitute B.C. a) into (2):

\[
A \sin \omega 0 + B \cos \omega 0 = 0
\]

\[
A \sin \omega L + B \cos \omega L = 0
\]

\[
B = 0
\]

Substitute B.C. b) into (2):

\[
A \sin \omega L = 0
\]

\[
A \neq 0 \quad \text{(trivial solution)}
\]

\[
\sin \omega L = 0
\]
\[ \sin \omega L = 0 \]
\[ \omega L = n \pi \]

Substituting for \( \omega \):
\[ \sqrt{\frac{P}{N EI}} = L = n \pi \]
\[ P = \frac{n^2 \pi^2 EI}{L^2} \]

For \( P_{\text{min}} = P_{\text{cr}} \), \( n = 1 \)
\[ P_{\text{cr}} = \frac{\pi^2 EI}{L^2} \tag{4.3} \]

(Euler's buckling formula)

For various values of \( n \):
- \( n = 0 \), \( P_{\text{cr}} = P_e \)
- \( n = 1 \), \( P_{\text{cr}} = 4P_e \)
- \( n = 2 \), \( P_{\text{cr}} = 9P_e \)
From CVEN 305: \[ I = r^2A \]

Where, \( r \) = radius of gyration
\( A \) = cross-sectional area.

\[ P_{cr} = \frac{\pi^2 E (r^2A)}{L^2} = \frac{\pi^2 EA}{(4r)^2} \]

\[ F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(4r)^2} \quad (4.4) \]

Where: \( L/r \) = slenderness ratio.

Eqns. (4.3); (4.4), Euler's buckling formula is valid only for elastic buckling: \( F_{cr} \leq P_{yield} \).

\[ F_{cr} = \frac{\pi^2 E}{(4r)^2} \]

\[ \frac{1}{2} \left( \frac{L}{r} \right)_{yield} \]

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Eqns. (4.3) & (4.4) are also based on the assumptions:

1) Column is perfectly straight
2) Load is purely axial (no eccentricity)
3) Column is pinned at both ends.
4) No initial or residual stresses.
5) Factural \leq F_y (elastic behavior only)

In hot rolled steel sections residual stresses exist because of the differential cooling in the cross-section after the member is formed. The elements of the cross-section that cool last will be in residual tension while those that cool first will be in residual compression. These stresses may be as large as 50% of \( F_y \).
Consider of W-section:

\[ f_r \]

\[ + = \text{tension} \]  \[ - = \text{compression} \]

Flanges

Web

Assuming perfectly elastic/plastic behavior and idealized \( f_r \) (flanges only):

Perfectly elastic/plastic:

\[ \frac{F_y}{2} \]

\[ \frac{F_y}{2} \]

\[ f_r \text{ (residual)} \]

\[ \frac{F_y}{2} \]

\[ \frac{F_y}{2} \]

\[ f_a \leq \frac{P}{A} \text{ (applied)} \]

Resultant (if \( f_a > \frac{F_y}{2} \))

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When \( f_a > \frac{F_y}{2} \) parts of the cross-section reach yield and become inelastic so that \( E = 0 \) in these parts. As the load increases (and \( f_a \) also increases) these areas cannot carry any more stress (load) thus the stiffness of the member is effectively reduced: \( EI_{\text{inelastic}} < EI_{\text{elastic}} \).

The result is:

\[
F_e r = \frac{\pi^2 E}{(4r_{\text{in}})^2}
\]

\[
F_e r = \frac{\pi^2 E}{(4r)^2}
\]

- Inelastic buckling
- Elastic buckling
Effective Length:

Equation (4.4) can be re-written as

\[ P_{cr} = \frac{\pi^2 EA}{\left(\frac{KL}{R}\right)^2} \]  (4.6a)

Where, \( KL = \) "effective length"

\[ K = f(\text{support condition at column ends and/or intermediate brace points}) \]

See p. 16.1-240 of AISC Manual

Table C-C2.2 for \( K_{\text{THEO}} \) and \( K_{\text{DESIGN}} \)

for various end conditions.

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<thead>
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<th>( K_{\text{THEO}} )</th>
<th>1.0</th>
<th>0.50</th>
<th>0.70</th>
<th>1.0</th>
<th>2.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{\text{DESIGN}} )</td>
<td>1.0</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
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