STRUCTURAL ANALYSIS

Influence Lines for Beams and Trusses

An influence line shows the variation of an effect (reaction, shear and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect.

Moving Concentrated Load Sets

The absolute maximum moment produced in a beam by a set of "n" moving loads occurs when the resultant "R" of the load set and an adjacent load are equal distance from the centerline of the beam. In general, two possible load set positions must be considered, one for each adjacent load.

Beam Stiffness and Moment Carryover

\[ \theta = \frac{M L}{4 E I} \quad M = \left(\frac{4 E I}{L}\right) \theta = k_{AB} \theta \]

\[ k_{AB} = \text{stiffness} \quad M_B = M_A/2 = \text{carryover} \]

Truss Deflection by Unit Load Method

The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

\[ \Delta_{\text{joint}} = \sum_{i=1}^{\text{members}} f_i (\Delta L)_i \]

where: \( \Delta_{\text{joint}} \) = joint displacement at point of application of unit load ( + in direction of unit load )

\( f_i \) = force in member "i" caused by unit load (+ tension)

Frame Deflection by Unit Load Method

The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

\[ \Delta = \sum_{i=1}^{\text{members}} m_i \int_{x=0}^{x=L} \frac{m_i M_i}{E I} \, dx \]

where: \( \Delta \) = displacement at point of application of unit load ( + in direction of unit load )

\( m_i \) = moment equation in member "i" caused by the unit load

\( M_i \) = moment equation in member "i" caused by loads applied to frame

\( L_i \) = length of member "i"

\( I_i \) = moment of inertia of member "i"

If either the real loads or the unit load cause no moment in a member, that member can be omitted from the summation.

Member Fixed-End Moments (Magnitudes)

\[ FEM_{AB} = FEM_{BA} = \frac{wL^2}{12} \]

\[ FEM_{AB} = \frac{P a b^2}{L^2} \quad FEM_{BA} = \frac{P a^2 b}{L^2} \]
STABILITY, DETERMINANCY, AND CLASSIFICATION OF STRUCTURES

\[ m = \text{number of members} \]
\[ r = \text{number of independent reaction components} \]
\[ j = \text{number of joints} \]
\[ c = \text{number of condition equations based on known internal moments or forces, such as internal moment of zero at a hinge} \]

**Plane Truss**

<table>
<thead>
<tr>
<th>Classification</th>
<th>Static Analysis</th>
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<tr>
<td>Unstable</td>
<td>( m + r &lt; 2j )</td>
</tr>
<tr>
<td>Stable and statically determinate</td>
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<tr>
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Stability also requires an appropriate arrangement of members and reaction components.

**STRUCTURAL DESIGN**

**Live Load Reduction**

The effect on a building member of nominal occupancy live loads may often be reduced based on the loaded floor area supported by the member. A typical model used for computing reduced live load (as found in ASCE 7 and many building codes) is:

\[
L_{\text{reduced}} = L_{\text{nominal}} \left( 0.25 + \frac{15}{K_{\text{LL}}A_T} \right) \geq 0.4L_{\text{nominal}}
\]

where: \( L_{\text{nominal}} \) = nominal live load given in ASCE 7 or a building code
\( A_T \) = the cumulative floor tributary area supported by the member
\( K_{\text{LL}}A_T \) = area of influence supported by the member
\( K_{\text{LL}} \) = ratio of area of influence to the tributary area supported by the member:

\( K_{\text{LL}} = 4 \) (typical columns)
\( K_{\text{LL}} = 2 \) (typical beams and girders)

**Load Combinations using Strength Design (LRFD, USD)**

Nominal loads used in following combinations:

\[ D = \text{dead loads} \]
\[ E = \text{earthquake loads} \]
\[ L = \text{live loads (floor)} \]
\[ L_r = \text{live loads (roof)} \]
\[ R = \text{rain load} \]
\[ S = \text{snow load} \]
\[ W = \text{wind load} \]

Load factors \( \lambda: \lambda_D \) (dead load), \( \lambda_L \) (live load), etc.

Basic combinations

\[ L_r/S/R = \text{largest of } L_r, S, R \]
\[ L \text{ or } 0.8W = \text{larger of } L, 0.8W \]

\[
1.4D
\]
\[
1.2D + 1.6L + 0.5 \left( L_r/S/R \right)
\]
\[
1.2D + 1.6(L_r/S/R) + (L \text{ or } 0.8W)
\]
\[
1.2D + 1.6W + L + 0.5(L_r/S/R)
\]
\[
1.2D + 1.0E + L + 0.2S
\]
\[
0.9D + 1.6W
\]
\[
0.9D + 1.0E
\]
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<tr>
<th>Shape</th>
<th>Area A (in.²)</th>
<th>Depth d (in.)</th>
<th>Web Flange t_w</th>
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<th>Axis Y-Y I r</th>
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<th>Area &amp; Centroid</th>
<th>Area Moment of Inertia</th>
<th>(Radius of Gyration)²</th>
<th>Product of Inertia</th>
</tr>
</thead>
</table>
| ![Figure A](image1.png) | $A = bh/2$  
$x_c = 2b/3$  
$y_c = h/3$ | $I_x = bh^3/36$  
$I_y = b^3h/36$  
$I_z = bh^3/12$  
$I_b = b^3h/4$ | $r_x^2 = h^2/18$  
$r_y^2 = b^2/18$  
$r_z^2 = h^2/6$  
$r_b^2 = b^2/2$ | $I_{xx} = Abh/36 = b^2h^2/72$  
$I_{yy} = Abh/4 = b^2h^2/8$ |
| ![Figure B](image2.png) | $A = bh/2$  
$x_c = b/3$  
$y_c = h/3$ | $I_x = bh^3/36$  
$I_y = b^3h/36$  
$I_z = bh^3/12$  
$I_b = b^3h/12$ | $r_x^2 = h^2/18$  
$r_y^2 = b^2/18$  
$r_z^2 = h^2/6$  
$r_b^2 = b^2/6$ | $I_{xx} = -Abh/36 = -b^2h^2/72$  
$I_{yy} = Abh/12 = b^2h^2/24$ |
| ![Figure C](image3.png) | $A = bh$  
$x_c = b/2$  
$y_c = h/2$ | $I_x = bh^3/12$  
$I_y = b^3h/12$  
$I_z = bh^3/3$  
$I_b = b^3h/3$ | $r_x^2 = h^2/12$  
$r_y^2 = b^2/12$  
$r_z^2 = h^2/3$  
$r_b^2 = b^2/3$ | $I_{xx} = 0$  
$I_{yy} = Abh/4 = b^2h^2/4$ |
| ![Figure D](image4.png) | $A = h(a + b)/2$  
$y_c = h/3$  
$y_c = (2a + b)/3(a + b)$ | $I_x = h^3(a^2 + 4ab + b^2)/36(a + b)$  
$I_y = h^3(3a + b)/12$  
$I_z = h^3(3a + b)/12$ | $r_x^2 = h^2(3a + b)/18(a + b)$  
$r_y^2 = h^2(a + b)/6(a + b)$ | |
| ![Figure E](image5.png) | $A = ab \sin \theta$  
$x_c = (b + a \cos \theta)/2$  
$y_c = (a \sin \theta)/2$ | $I_x = (a^3b \sin^3 \theta)/12$  
$I_y = (ab \sin \theta(a^2 + a^2 \cos^2 \theta))/12$  
$I_z = (a^3 \sin \theta)/3$  
$I_b = (a^3 \sin \theta)/3$ | $r_x^2 = (a \sin \theta)^2/12$  
$r_y^2 = (b^2 + a^2 \cos^2 \theta)/12$  
$r_z^2 = (a \sin \theta)^2/3$  
$r_b^2 = (b + a \cos \theta)^2/3$ | $I_{xx} = (a^3b \sin^2 \theta \cos \theta)/12$ |

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<th>(Radius of Gyration)$^2$</th>
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</tr>
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</table>
| ![Circle](image) | $A = \pi a^2$  
$x_c = a$  
$y_c = a$ | $I_{x_c} = I_{y_c} = \pi a^4/4$  
$I_s = I_y = 5\pi a^4/4$  
$J = \pi a^4/2$ | $r_x^2 = r_y^2 = a^2/4$  
$r_x^2 = r_y^2 = 5a^2/4$  
$r_p^2 = a^2/2$ | $I_{x,y} = 0$  
$I_{xy} = Aa^2$ |
| ![Egg](image) | $A = \pi(a^2 - b^2)$  
$x_c = a$  
$y_c = a$ | $I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$  
$I_s = I_y = 5\pi a^4 - \pi a^2 b^2 - \pi b^4/4$  
$J = \pi(a^4 - b^4)/2$ | $r_x^2 = r_y^2 = (a^2 + b^2)/4$  
$r_x^2 = r_y^2 = (5a^2 + b^2)/4$  
$r_p^2 = (a^2 + b^2)/2$ | $I_{x,y} = 0$  
$I_{xy} = Aa^2$  
$= \pi a^2(a^2 - b^2)$ |
| ![Quarter Circle](image) | $A = \pi a^2/2$  
$x_c = a$  
$y_c = 4a/(3\pi)$ | $I_{x_c} = a^4(\theta^2 - 64)/72\pi$  
$I_{y_c} = \pi a^4/8$  
$I_s = I_y = 5\pi a^4/8$ | $r_x^2 = a^2(\theta^2 - 64)/36\pi^2$  
$r_x^2 = a^2/4$  
$r_x^2 = a^2/4$  
$r_y^2 = 5a^2/4$ | $I_{x,y} = 0$  
$I_{xy} = 2a^4/3$ |
| ![Circular Sector](image) | $A = a^2\theta$  
$x_c = 2a \sin\theta/3$  
$y_c = 0$ | $I_{x_c} = a^4(\theta - \sin\theta \cos\theta)/4$  
$I_{y_c} = a^4(\theta + \sin\theta \cos\theta)/4$ | $r_x^2 = a^2(\theta - \sin\theta \cos\theta)/4$  
$r_x^2 = a^2(\theta + \sin\theta \cos\theta)/4$  
$r_y^2 = 0$ | $I_{x,y} = 0$  
$I_{xy} = 0$ |
| ![Circular Segment](image) | $A = a^2\left[\theta - \sin2\theta/2\right]$  
$x_c = 2a \sin^3\theta/3 - \sin\theta\cos\theta$  
$y_c = 0$ | $I_{x_c} = Aa^2/4 \left[1 - 2\sin^3\theta \cos\theta/30 - 3\sin\theta \cos\theta\right]$  
$I_{y_c} = Aa^2/4 \left[1 + 2\sin^3\theta \cos\theta/\theta - \sin\theta \cos\theta\right]$ | $r_x^2 = a^2/4 \left[1 - 2\sin^3\theta \cos\theta/30 - 3\sin\theta \cos\theta\right]$  
$r_x^2 = a^2/4 \left[1 + 2\sin^3\theta \cos\theta/\theta - \sin\theta \cos\theta\right]$  
$r_y^2 = 0$ | $I_{x,y} = 0$  
$I_{xy} = 0$ |

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<tbody>
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<td><img src="image1.png" alt="Parabola" /></td>
<td>$A = 4ab/3$</td>
<td>$I_{x_c} = I_x = 4ab^3/15$</td>
<td>$r_{x_c}^2 = r_x^2 = b^2/5$</td>
<td>$I_{x,y_c} = 0$</td>
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<td><img src="image2.png" alt="Half a Parabola" /></td>
<td>$A = 2ab/3$</td>
<td>$I_x = 2ab^3/15$</td>
<td>$r_x^2 = b^2/5$</td>
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<tr>
<td><img src="image3.png" alt="n° Degree Parabola" /></td>
<td>$A = bh/(n+1)$</td>
<td>$I_x = bh^3/3(n+1)$</td>
<td>$r_x^2 = h^2(n+1)/3(n+1)$</td>
<td>$r_y^2 = n+1/b^2$</td>
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<tr>
<td><img src="image4.png" alt="n° Degree Parabola" /></td>
<td>$A = n+1/bh$</td>
<td>$I_x = n/(3n+3)b^3h$</td>
<td>$r_x^2 = n+1/3h^2$</td>
<td>$r_y^2 = n+1/3b^2$</td>
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