Review for
Fundamentals of Engineering Exam

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Fluid Mechanics
• Fluid Properties
• Fluid Statics
• Fluid Dynamics
• Fluid Measurements
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Topic VII: Fluid Mechanics
Chapter 22 Fluid Properties
Chapter 23 Fluid Statics
Chapter 24 Fluid Dynamics
Chapter 25 Fluid Measurements and Similitude
states of matter
- solid
- fluid
  - liquid
  - gas

A fluid is a substance that deforms continuously under the action of an applied shear stress.
Fluid Properties

density
specific volume
specific weight
specific gravity
pressure
stress
viscosity
surface tension
capillarity
Mass and Force Units

dimension: mass force

Systeme International (SI) units:

kilogram (kg) newton (N)

British Gravitational System units:

slug pound

English Engineering System units:

pound mass (lbf) pound force (lbf)
Newton’s Second Law:

\[
\text{force} = \text{mass} \cdot \text{acceleration}
\]

\[
F = m \cdot a
\]

weight = mass \cdot \text{acceleration of gravity}

\[
W = m \cdot g
\]

standard acceleration of gravity

\[
g = 9.807 \text{ m/s}^2 = 32.174 \text{ ft/s}^2
\]
SI System

\[ w = m \ g \]

\[ 1 \text{N} = 1 \text{Kg} \cdot \text{m/s}^2 \]

British Gravitational System

\[ m = w / g \]

1 slug = lb / (ft/s^2) = lb \cdot s^2/ft

English Engineering System

\[ F = m \ a / g_c \]

where: \( g_c = 32.174 \text{ ft.lbm/lbf.s}^2 \)

\[ 1 \text{lbf} = \frac{1 \text{ lbm} \times 32.174 \text{ ft/s}^2}{32.174 \text{ ft} \times \text{ lbm} / \text{lbf} \times \text{s}^2} \]
Density (\( \rho \)) and Specific Weight (\( \gamma \))

\[ \rho = \text{mass / unit volume} \]

\((\text{kg/m}^3, \text{slugs/ft}^3, \text{lbm/ft}^3)\)

\[ \gamma = \text{weight / unit volume} \]

\((\text{N/m}^3, \text{lb/ft}^3, \text{lbf/ft}^3)\)

\text{SI and British Gravitational System:}

\[ \gamma = \rho g \]

\text{English Engineering System:}

\[ \gamma = \rho \left( \frac{g}{g_c} \right) \]
For water at temperature of 10°C or 50°F:

\[ \rho = 1,000 \text{ kg/m}^3 \]

\[ = 1.94 \text{ slugs/ft}^3 \]

\[ = 62.4 \text{ lbm/ft}^3 \]

\[ \gamma = 9.80 \text{ kN/m}^3 \]

\[ = 62.4 \text{ lb/ft}^3 \]

\[ = 62.4 \text{ lbf/ft}^3 \]
Specific Gravity (S.G.)

The specific gravity of a fluid is the ratio of the density of the fluid to the density of water at a specified temperature and pressure.

\[ \text{S.G.} = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} \]
Absolute and Gage Pressure

absolute pressure =

gage pressure + atmospheric pressure

\[ P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} \]
Pressure

absolute pressure A

gage pressure A

local atmospheric pressure

gage pressure B (vacuum)

absolute pressure B

absolute zero pressure
(complete vacuum)
Stress

stress ($\tau$) is force per unit area

- normal stress
- tangential (shear) stress
Viscosity

The viscosity of a fluid is a measure of its resistance to flow when acted upon by an external force such as a pressure gradient or gravity.

Newton’s Law of Viscosity

$$\tau = \mu \left( \frac{dv}{dy} \right)$$

$\tau$ - shear stress  
$dv/dy$ - velocity gradient  
$\mu$ - absolute or dynamic viscosity
\[ \nu = \frac{\mu}{\rho} \]

\( \nu \) - kinematic viscosity
\( m^2/s \) or \( \text{ft}^2/s \)

\( \mu \) - dynamic viscosity
\( N\cdot s/m^2 \) or \( \text{lb}\cdot s/\text{ft}^2 \)

\( \rho \) - density
\( \text{kg/m}^3 \) or slug or \( \text{lbm/ft}^3 \)
For water at temperature of

of \(10^\circ\text{C}\) or \(50^\circ\text{F}\):

\[
\mu = 1.307 \times 10^{-3} \text{ N} \cdot \text{s}/\text{m}^2
\]

\[
= 2.735 \times 10^{-5} \text{ lb} \cdot \text{s}/\text{ft}^2
\]

\[
v = 1.306 \times 10^{-6} \text{ m}^2/\text{s}
\]

\[
= 1.410 \times 10^{-5} \text{ ft}^2/\text{s}
\]
Surface Tension

*Surface tension* is property used to describe a phenomena observed at the interface between a gas and liquid where intermolecular cohesive forces form an imaginary film capable of resisting tension.

Capillarity

Capillary action is caused by surface tension between a liquid and solid surface. Water rises in a thin-bore tube.
Fluid Statics

Hydrostatic pressure is the pressure a fluid exerts on an immersed object or on container walls.

hydrostatic pressure distribution
Fluid Statics

\[ P_1 - P_2 = \gamma (z_2 - z_1) \]

\[ P = \gamma h \]

\[ F = pA \]

where:

\[ P_1 = \text{pressure at elevation } z_1 \]

\[ P_2 = \text{pressure at elevation } z_2 \]

\[ h = z_2 - z_1 = \text{head} \]

\[ p = \text{pressure for head } h \]

\[ \gamma = \text{specific weight} \]

\[ F = \text{force} \]

\[ A = \text{area} \]
Example: What is the pressure 10 feet below the surface of a swimming pool?

\[
p = \gamma h = (62.4 \text{ lb/ft}^3) (10 \text{ft}) \\
= 624 \text{ lb/ft}^2
\]
Example: The tank of water has a 3-m column of gasoline (S.G. = 0.73) above it. Atmospheric pressure is 101 kPa. Compute the pressure on the bottom of the tank.

\[
\begin{align*}
\text{gasoline} & \quad (\text{S.G.} = 0.73) \\
\text{water} & \quad (\gamma = 9.81 \text{ kN/m}^3) \\
\h_g &= 3 \text{ m} \\
\h_w &= 2 \text{ m}
\end{align*}
\]
pressure at bottom of tank

\[ P_{gage} = \gamma_w h_w + \gamma_g h_g \]

\[ = (9.81 \text{ kN/m}^3) (2\text{m}) + (0.73) (9.81 \text{ kN/m}^3) (3\text{m}) \]

\[ = 41.1 \text{ kN/m}^2 \]

\[ P_{abs} = P_{gage} + P_{atm} \]

\[ = 41 \text{ kN/m}^2 + 101 \text{ kN/m}^2 \]

\[ = 142 \text{ kN/m}^2 \]

\[ = 142 \text{ kPa} \]
Example: Use the manometer measurements to compute the pressure in the pipe.

\[ P_{\text{pipe}} + \gamma_w h_w - \gamma_m h_m = 0 \]

\[ P_{\text{pipe}} + (62.4 \text{ lb/ft}^3) (1.5\text{ ft}) \]

\[ - (13.6) (62.4 \text{ lb/ft}^3) (2.0 \text{ ft}) = 0 \]

\[ P_{\text{pipe}} = 1,604 \text{ lb/ft}^2 \]
Forces on Submerged Surfaces

Plane Surface

\[ F = \gamma h_c A \]  \text{ magnitude}

\[ l_p - l_c = \frac{I_c}{(l_c A)} \]  \text{ location}

Curved or Plane Surface

\[ F_h = \gamma h_c A \]  \text{ (vertical projection)}

\[ F_v = \gamma V \]  \text{ (weight of fluid)}

Buoyant Force

\[ F = \gamma \]  \text{ (volume displaced)}
Example: Compute the magnitude and location of the resultant force.

moment of inertia $I_c = bh^3/36$
\[ F = \gamma h_c A \]

\[ I_p - I_c = \frac{I_c}{(l_c A)} \]

\[ A = 0.5 \times (2\text{m})(1.5\text{m}) = 1.5 \text{ m}^2 \]

\[ h_c = 2.75 \text{ m} - \left[ \frac{2}{3} \times (1.5\text{m}) \right] \sin 45^\circ \]

\[ = 2.043 \text{ m} \]

\[ l_c = \frac{h_c}{\sin 45^\circ} = \frac{2.043 \text{ m}}{0.7071} \]

\[ = 2.889 \text{ m} \]
moment of inertia \( I_c = bh^3/36 \)

\[ I_c = (2\text{m})(1.5\text{m})^3/36 = 0.1875 \text{ m}^4 \]

\[ F = \gamma h_c A \]

\[ = (9.80 \text{ kN/m}^3)(2.043 \text{ m})(1.5 \text{ m}^2) \]

\[ = 30.0 \text{ kN} \]

\[ I_p - I_c = \frac{I_c}{(I_c A)} \]

\[ = 0.1875 \text{ m}^4/[2.889 \text{ m}) (1.5\text{m}^2)] \]

\[ = 0.0433 \text{ m} \]

\[ I_p = 0.0433 \text{ m} + 2.889 \text{ m} = 2.932 \text{ m} \]
Example: Compute the force on the curved corner for a unit width.
\[ F_H = \gamma h_c A \]
\[ = (9.80 \text{ kN/m}^3) \times (11.5 \text{ m}) \times (3 \text{ m}^2) \]
\[ = 338 \text{ kN} \]

\[ F_v = \gamma V \]

volume \((V) = (10 \text{ m}) \times (3 \text{ m}) \times (1 \text{ m}) + \frac{1}{4\pi} \times (3 \text{ m})^2 \times (1 \text{ m}) \]
\[ = 37.07 \text{ m}^3 \]

\[ F_v = (9.80 \text{ kN/m}^3) \times (37.07 \text{ m}^3) = 363 \text{ kN} \]

\[ F = (F_H^2 + F_V^2)^{0.5} = (338^2 + 363^2)^{0.5} \]
\[ = 496 \text{ kN} \]
Laws of Buoyancy and Flotation

1. A body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced.

2. A floating body displaces its own weight of the liquid in which it floats.
Example: Compute the force in the rope.

\[
\text{wood (} y = 40 \text{ lb/ft}^3) \quad \text{rope}
\]

3 ft 1 ft 1 ft into paper

2 ft
buoyant force \( F_B \) = 

\[ \gamma \text{ (volume displaced)} \]

\[ F_B = (62.4 \text{ lbs/ft}^3) (2 \text{ ft}^3) \]

= 124.8 lbs

weight of wood \( W \)

\[ W = (40 \text{ lbs/ft}^3) (2 \text{ ft}^3) = 80 \text{ lbs} \]

\[ F_B - W - F_{rope} = 0 \]

124.8 lbs - 80lbs - \( F_{rope} \) = 0

\( F_{rope} \) = 44.8 lbs
Alternative Solution

The buoyant force \( F_B \) on a submerged body is the difference between the vertical component of pressure force on its underside and upper side.

\[
F = \gamma h_c A \quad \text{or} \quad F = \gamma V
\]

\[
F_B = (62.4 \text{ lb/ft}^3)(5 \text{ ft})(1 \text{ ft}^2)
- (62.4 \text{ lb/ft}^3)(3 \text{ ft})(1 \text{ ft}^2)
= 124.8
\]
Fluid Dynamics

Conservation Laws
mass
energy
momentum

Flow in pressure conduits
Flow in open channels
Conservation of Mass

(Continuity Equation)

\[ \dot{m}_1 = \dot{m}_2 \]

\[ \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \]

For incompressible fluids \((\rho_1 = \rho_2)\)

\[ A_1 V_1 = A_2 V_2 \]

\[ Q_1 = Q_2 \]
**Example:** Three kN/s of water flows through the pipeline reducer. Determine the flow rate and velocity in the 300 mm and 200 mm pipes.
\[ \gamma_1 = \gamma_2 = 9.8 \text{ kN/m}^3 \]
\[ \gamma = \rho g \]
\[ w = mg \]
\[ \dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \]
\[ \dot{w} = \gamma_1 A_1 V_1 = \gamma_2 A_2 V_2 \]

\[ Q = \frac{\dot{w}}{\gamma} = 3 \text{ kN/s / 9.8 kN/m}^3 \]
\[ Q = 0.306 \text{ m}^3/\text{s} \]

\[ V = \frac{Q}{A} \]
\[ A = 0.25\pi D^2 \]

\[ V_1 = 0.306 \text{ m}^3/\text{s / 0.25}\pi(0.3)^2 \]
\[ V_1 = 4.33 \text{ m/s} \]

\[ V_2 = 0.306 \text{ m}^3/\text{s / 0.25}\pi(0.2)^2 \]
\[ V_2 = 9.74 \text{ m/s} \]
Conservation of Momentum

(Impulse-Momentum Equation)

\[ \sum \vec{F} = \rho \ Q \ (\vec{V}_{out} - \vec{V}_{in}) \]

\[ \sum F_x = \rho Q (V_{2x} - V_{1x}) \]
\[ \sum F_y = \rho Q (V_{2y} - V_{1y}) \]
\[ \sum F_z = \rho Q (V_{2z} - V_{1z}) \]
Example: Water is flowing at 0.884 m$^3$/s through a 15 cm diameter pipe, that has a 90° bend. What is the reaction on the water in the z-direction in the bend?
\[ \rho = 1000 \text{ kg/m}^3 \]

\[ V = \frac{Q}{A} \]
\[ A = 0.25\pi D^2 \]
\[ V = 0.884 \text{ m}^3/\text{s} / 0.25\pi(0.15 \text{ m})^2 \]
\[ V = 50 \text{ m/s} \]

\[ \sum F_z = \rho Q (V_{2z} - V_{1z}) \]
\[ F_z = (1000 \text{ kg/m}^3)(0.884 \text{ m}^3/\text{s})(0-50 \text{ m/s}) \]
\[ F_z = -44,200 \text{ kg} \cdot \text{m/s} \]
\[ F_z = -44,200 \text{ N} \]

Reaction = 44,200 N = 44.2 kN
Conservation of Energy

(Bernoulli and Energy Equations)

\[ \text{total head} = z + \frac{P}{\gamma} + \frac{V^2}{2g} \]

\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \]

\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \]
Pitot Tube

\[ h_s = \frac{p_s}{\gamma} \]

\[ \frac{V^2}{2g} \]

\[ p_0 \]

\[ v, p_s \]

\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \]

\[ 0 + \frac{P_0}{\gamma} + 0 = 0 + \frac{P_s}{\gamma} + \frac{V^2}{2g} \]

\[ \frac{P_0}{\gamma} - \frac{P_s}{\gamma} = \frac{V^2}{2g} \]

\[ \frac{P_0}{\gamma} - \frac{P_s}{\gamma} = h \]

\[ h = \frac{V^2}{2g} \]

\[ V = (2gh)^{0.5} \]
**Example:** The height of water in the pitot tube is measured to be 7.3 cm. What is the velocity at that point in the flow.

\[
V = (2gh)^{0.5}
\]

\[
V = [2(9.81 \text{ m/s}^2)(0.073 \text{ m})]^{0.5}
\]

\[
V = 1.2 \text{ m/s}
\]
slope of energy line \( S = \frac{h_L}{L} \)

\[
\begin{align*}
V_1^2 \quad \frac{V_1^2}{2g} \\
\frac{P_1}{\gamma} \\
Z_1 \\
\text{horizontal datum} \\
\text{pipe length (L)}
\end{align*}
\]

\[
\begin{align*}
V_2^2 \quad \frac{V_2^2}{2g} \\
\frac{P_2}{\gamma} \\
Z_2
\end{align*}
\]

head = energy per unit weight

\[
Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L
\]
Pressure and Pressure Head

\[ p = \gamma h \quad h = \frac{p}{\gamma} \]

for \( p = 1 \text{ psi (lb/inch}^2) \)

\[ h = (1 \text{ lb/in}^2) \left(144 \text{ in}^2/\text{ft}^2\right) / 62.4 \text{ lb/ft}^3 \]

\[ = 2.31 \text{ ft} \]

Pressures in municipal water distribution systems are typically 60 - 80 psi

(139 - 185 ft).

faucet pressures > 5 psi (11.5 ft)

main pressures > 35 psi (80.8 ft)
$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$
Darcy - Weisbach Equation

\[ h_L = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \]

- \( h_L \) = head loss due to pipe friction (feet or meters)
- \( f \) = friction factor (dimensionless)
- \( L \) = length of pipe (ft or m)
- \( D \) = pipe diameter (ft or m)
- \( V \) = flow velocity (ft/sec or m/sec)
- \( g \) = gravitational acceleration constant (32.2 ft/sec\(^2\) or 9.81 m/sec\(^2\))
- \( \epsilon \) = roughness (ft or m)
- \( \epsilon/D \) = relative roughness (dimensionless)
\[ h_f = \frac{fL\nu^2}{2Dg} \]

<table>
<thead>
<tr>
<th>material</th>
<th>ft</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>riveted steel</td>
<td>0.003</td>
<td>0.9-9.0</td>
</tr>
<tr>
<td>concrete</td>
<td>0.001</td>
<td>0.3-3.0</td>
</tr>
<tr>
<td>galvanized iron</td>
<td>0.00085</td>
<td>0.25</td>
</tr>
<tr>
<td>commercial steel or wrought iron</td>
<td>0.00015</td>
<td>0.046</td>
</tr>
<tr>
<td>drawn tubing</td>
<td>0.000005</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Figure 24.6 Moody Friction Factor Chart
Darcy-Weisbach Example:  For a discharge (Q) of 1.0 ft³/s, compute the head loss in 1,500 feet of new 6-inch diameter cast iron pipe (ε = 0.00085 ft). Assume a water temperature of 60° F.

\[ A = \frac{1}{4} \pi D^2 = \frac{1}{4} \pi \left(\frac{6}{12}\right)^2 = 0.196 \text{ ft}^2 \]

\[ V = \frac{Q}{A} = \frac{1.0 \text{ ft}^3/\text{s}}{0.196 \text{ ft}^2} = 5.09 \text{ ft/s} \]

\[ \nu = 1.217 \times 10^{-5} \text{ ft}^2/\text{s} \]

\[ N_R = \frac{VD}{\nu} = \frac{(5.09 \text{ ft/s})(0.5\text{ft})}{1.217 \cdot 10^5 \text{ ft}^2/\text{s}} = 2.09 \cdot 10^5 \]

\[ \frac{\varepsilon}{D} = \frac{0.00085 \text{ ft}}{0.5 \text{ ft}} = 0.0017 \]

\[ f = 0.023 \]

\[ h_L = f \frac{L}{D} \frac{V^2}{2g} \]

\[ h_L = 0.023 \left(\frac{1,500\text{ft}}{0.5\text{ft}}\right) \frac{(5.09\text{ft/s})^2}{2(32.2\text{ft/s}^2)} \]

\[ = 27.8 \text{ ft} \]
Minor Losses

\[ h_L = C \left( \frac{V^2}{2g} \right) \]

<table>
<thead>
<tr>
<th>exit/entrance condition</th>
<th>( C ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>exit, sharp</td>
<td>1.0</td>
</tr>
<tr>
<td>exit, protruding</td>
<td>0.8</td>
</tr>
<tr>
<td>entrance, sharp</td>
<td>0.5</td>
</tr>
<tr>
<td>entrance, rounded</td>
<td>0.1</td>
</tr>
<tr>
<td>entrance, gradual, smooth</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Example: A 6-inch diameter 500 ft long steel pipe ($\epsilon = 0.00015$ ft) conveys flow between two reservoirs which have a difference in water surface elevation of 30 ft. The pipe exit and entrance are square edge. Compute the flow rate.

\[
Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L
\]

\[
30 + 0 + 0 = 0 + 0 + 0 + h_L
\]

\[
h_L = 30 \text{ ft}
\]
Pipe Friction

\[ \frac{\epsilon}{D} = 0.00015 \text{ ft} / 0.5 \text{ ft} = 0.0003 \]

assume \( f = 0.016 \)

\[
h_L = f \left( \frac{L}{D} \right) \frac{V^2}{2g}
\]

\[
h_L = 0.016 \left( \frac{500}{0.5} \right) \frac{V^2}{2g} = 16 \frac{V^2}{2g}
\]

Minor Losses (entrance and exit)

\[
h_{Lm} = 0.5 \frac{V^2}{2g} + 1.0 \frac{V^2}{2g} = 1.5 \frac{V^2}{2g}
\]

Compute \( V \) for assumed \( f \)

\[
h_L = 16 \frac{V^2}{2g} + 1.5 \frac{V^2}{2g} = 17.5 \frac{V^2}{2g} = 30 \text{ ft}
\]

\[ V = 10.5 \text{ ft/s} \]
Check $f$

$$N_R = \frac{VD}{v} = \frac{(10.5 \text{ ft/s}) (0.5 \text{ ft})}{1.22 \cdot 10^{-5} \text{ ft}^2/\text{s}}$$

$$= 4.3 \times 10^5$$

$$f = 0.0164$$

Compute $Q$

$$Q = VA = (10.5 \text{ ft/s}) (0.196 \text{ ft}^2)$$

$$= 2.06 \text{ ft}^3/\text{s}$$
Example: Assume a pump is added to the previous example and the flow direction is reversed. What pump head is required for a discharge of 2.0 cfs.
\[ A = \frac{1}{4} \pi D^2 = \frac{1}{4} \pi (0.5 \text{ ft})^2 = 0.1963 \text{ ft}^2 \]

\[ V = \frac{Q}{A} = 2 \text{ cfs} / 0.1963 \text{ ft}^2 = 10.2 \text{ ft/s} \]

from previous problem:

\[ h_L = 16 \frac{V^2}{2g} + 1.5 \frac{V^2}{2g} = 17.5 \frac{V^2}{2g} \]

\[ h_L = 17.5 \left( \frac{(10.2 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} \right) = 28.2 \text{ ft} \]

\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \]

\[ 0 + 0 + 0 + h_p = 30 + 0 + 0 + 28.2 \]

\[ h_p = 58.2 \text{ feet} \]
Power and Energy

energy - ability to do work

work - force through a distance

power - rate of transferring energy or work per unit of time

\[ \text{power} = \frac{\text{energy}}{\text{time}} = \frac{(\text{weight/time})(\text{energy/weight})}{\text{time}} \]

\[ P = \gamma Q h \]

units of power

kilowatt (kw) - kN\cdot m/s
horsepower (hp) - 550 ft\cdot lb/s
1 hp = 0.746 kw
\[ P = \gamma Q h / \eta \]

- **P** - power (kW = kN·m/s or ft·lb/s)
- **\(\gamma\)** - unit weight (kN/m³ or lb/ft³)
- **Q** - discharge (m³/s or ft³/s)
- **h** - head (m or ft)
- **\(\eta\)** - efficiency

**brake horsepower** - input horsepower delivered to the pump shaft

\[ BHP = \frac{\gamma Q h_p}{550 \eta} \]

- horsepower (hp) - 550 ft·lb/s
Example: Determine the horsepower required for the pump of the previous problem, assuming the pump efficiency is 75%.

\[ Q = 2.0 \text{ ft}^3/\text{s} \]

\[ h_p = 58.2 \text{ ft} \]

\[ \eta = 0.75 \]

\[ BHP = \frac{\gamma Q h_p}{550 \eta} \]

\[ BHP = \left( \frac{62.4 \text{ lb/ft}^3}{(2.0 \text{ ft}^3/\text{s})(58.2 \text{ ft})} \right) \left( \frac{550 \text{ ft} \cdot \text{lb/s}}{h_p} \right) (0.75) \]

\[ = 17.6 \text{ horsepower} \]
Example: Compute the discharge rate that causes the pressure to drop to vapor pressure. The pipe between the reservoir and pump has a length of 1,000 ft, diameter of 3 feet, and friction factor (f) of 0.02. Neglect minor losses. For a water temperature of 80°F, the vapor pressure (P_V) is 0.51 psia.

\[ P_{\text{atm}} = 13.8 \text{ psia} = 1,987 \text{ lb/ft}^2 \]
\[ P_V = 0.51 \text{ psia} = 73.4 \text{ lb/ft}^2 \]
\[ \gamma = 62.22 \text{ lb/ft}^3 \]

\[ h_L = f(L/D) \left( \frac{V^2}{2g} \right) \]
\[ h_L = (0.02)(1000 \text{ft}/3 \text{ft}) \left( \frac{V^2}{[2(32.2 \text{ ft/s}^2)]} \right) \]
\[ h_L = 0.119 V^2 \]
\[ P_{atm} = 1,987 \text{ lb/ft}^2 \]

\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \]

\[ 0 + \frac{P_{atm}}{\gamma} + 0 = Z_P + \frac{P_V}{\gamma} + \frac{V_2^2}{2g} + h_L \]

\[ \frac{1,987 \text{ lb/ft}^2}{62.22 \text{ lb/ft}^3} = 10 \text{ ft} + \]

\[ \frac{73.4 \text{ lb/ft}^2}{62.22 \text{ lb/ft}^3} + \frac{V^2}{2(32.2 \text{ ft/s}^2)} + 0.119V^2 \]

\[ V = 13.2 \text{ ft/s} \]

\[ Q = VA = (13.2 \text{ ft/s})(7.069 \text{ ft}^2) \]
\[ Q = 93.3 \text{ ft}^3/\text{s} \]
\[ Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \]

\[ 0 + \frac{P_{\text{atm}}}{\gamma} + 0 = Z_p + \frac{P_V}{\gamma} + \frac{V_2^2}{2g} + h_L \]

\[ \frac{P_{\text{atm}} - P_V}{\gamma} = Z_p + \frac{V_2^2}{2g} + h_L \]
\[ h_L = K Q^N \]
\[ h_L = K Q^2 \]

Darcy Weisbach

\[ h_f = f(L/D)(1/2A^2g)Q^2 \]
\[ K = f(L/D)(1/2A^2g) \]

Minor Losses

\[ h_L = C \left( Q^2/2gA^2 \right) \]
\[ K = C/2gA^2 \]
FOR PIPES IN SERIES

\[ Q = Q_1 = Q_2 = Q_3 \]

\[ h_L = h_{L1} + h_{L2} + h_{L3} \]

\[ h_{LE} = h_L \]

\[ K_E Q^n = K_1 Q^n + K_2 Q^n + K_3 Q^n \]

\[ K_E = K_1 + K_2 + K_3 \]
FOR PIPES IN PARALLEL

\[ Q = Q_1 + Q_2 \]

\[ h_L = h_{L1} = h_{L2} \]

\[ h_L = KQ^n \quad Q = (h_L/K)^{1/n} \]

\[ (h_{LE}/K_E)^{1/n} = (h_{L1}/K_1)^{1/n} + (h_{L2}/K_2)^{1/n} \]

\[ (1/K_E)^{1/n} = (1/K_1)^{1/n} + (1/K_2)^{1/n} \]
Example: Compute the discharge.
Neglect the minor losses.

- \( \varepsilon = 0.00085 \)
- \( \varepsilon/D = 0.00085/D \)
- \( \varepsilon/D_1 = 0.000425 \)
- \( \varepsilon/D_2 = 0.000850 \)
- \( \varepsilon/D_3 = 0.000283 \)

- \( f_1 = 0.0170 \)
- \( f_2 = 0.0190 \)
- \( f_3 = 0.0160 \)

\( A = \frac{1}{4} \pi D^2 \)
\( A_1 = 3.142 \text{ ft}^2 \)
\( A_2 = 0.7854 \text{ ft}^2 \)
\( A_3 = 7.069 \text{ ft}^2 \)

\( h_L = f_1 (L/D)(Q^2/2gA^2) = KQ^2 \)

\( K = F(L/D)(Q^2/2gA^2) \)
\( K_1 = 0.00669 \)
\( K_2 = 0.38259 \)
\( K_3 = 0.001491 \)
\[ Q = Q_1 = Q_2 = Q_3 \]

\[ h_L = h_{L1} + h_{L2} + h_{L3} = 30 \text{ ft} \]

\[ K_1Q^2 + K_2Q^2 + K_3Q^2 = 30 \text{ ft} \]

\[ 0.00669 Q^2 + 0.38259 Q^2 + 0.001491 Q^2 = 30 \text{ ft} \]

\[ Q = 8.76 \text{ cfs} \]

Check the original estimates of friction factor \((f)\):

\[ V = \frac{Q}{A} \]

\[ V_1 = \frac{8.76 \text{ ft}^3/\text{s}}{3.142 \text{ ft}^2} = 2.79 \text{ ft/s} \]

\[ V_2 = \frac{8.76 \text{ ft}^3/\text{s}}{0.7854 \text{ ft}^2} = 11.15 \text{ ft/s} \]

\[ V_3 = \frac{8.76 \text{ ft}^3/\text{s}}{7.069 \text{ ft}^2} = 1.24 \text{ ft/s} \]

\[ N_R = \frac{DV}{\nu} \]

\[ N_R = 4.57 \times 10^5 \]

\[ N_R = 9.14 \times 10^5 \]

\[ N_R = 3.05 \times 10^5 \]

\[ f = .017 \]

\[ f = .019 \]

\[ f = .017 \]
Example: Compute the discharge in each pipe. Neglect minor losses.

\[ h_L = KQ^2 \]

\[ K = f \left( \frac{L}{D} \right) \left( \frac{1}{2gA^2} \right) \]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>L (feet)</th>
<th>D (inch)</th>
<th>A (ft(^2))</th>
<th>(f)</th>
<th>(K = f(L/D)(1/2gA^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,000</td>
<td>16</td>
<td>1.396</td>
<td>0.020</td>
<td>0.359</td>
</tr>
<tr>
<td>2</td>
<td>3,000</td>
<td>12</td>
<td>0.785</td>
<td>0.022</td>
<td>1.663</td>
</tr>
<tr>
<td>3</td>
<td>2,000</td>
<td>8</td>
<td>0.349</td>
<td>0.020</td>
<td>7.649</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>16</td>
<td>1.396</td>
<td>0.020</td>
<td>0.120</td>
</tr>
</tbody>
</table>
\[ h_{L2} = h_{L3} \]
\[ K_2 Q_2^2 = K_3 Q_3^2 \]
\[ 1.663 Q_2^2 = 7.649 Q_3^2 \]
\[ Q_2 = 2.145 Q_3^3 \]

\[ Q_1 = Q_2 + Q_3 \]
\[ Q_1 = 2.145 Q_3 + Q_3 \]
\[ Q_1 = 3.145 Q_3 \]
\[ Q_3 = 0.318 Q_3 \]

\[ h_L = h_{L1} + h_{L4} + h_{L3} \]

\[ 25 = K_1 Q_1^2 + K_4 Q_4^2 + K_3 Q_3^2 \]
\[ 25 = K_1 Q_1^2 + K_4 Q_4^2 + K_3 (0.318 Q_1)^2 \]
\[ 25 = 1.252 Q_1^2 \]
\[ Q_1 = 4.47 \text{ cfs} \]

\[ Q_4 = Q_1 = 4.47 \text{ cfs} \]

\[ Q_3 = 0.318 Q_1 = 0.318 (4.47 \text{ cfs}) = 1.42 \text{ cfs} \]

\[ Q_2 = 2.145 Q_3 = 2.145 (1.42 \text{ cfs}) = 3.05 \text{ cfs} \]
EXAMPLE: Two reservoirs are connected by a 850 feet long 6-inch diameter pipe \( (f=0.020) \). A pump with the given characteristic curves is used to lift water from one reservoir to the other. Determine the discharge rate.
\[
Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_1^2}{2g} + h_L
\]

\[z_1 + 0 + 0 + h_p = z_2 + 0 + 0 + h_L\]

\[h_p = (z_2 - z_1) + h_L\]

\[h_p = \Delta z + h_L\]
System Head Curve

\[ h_p = \Delta z + h_L \]
\[ h_p = \Delta z + f(L/D) \left( \frac{Q^2}{2gA^2} \right) \]
\[ h_p = 100 + 0.020 \left( \frac{850}{0.5} \right) \left( \frac{Q^2}{2(32.2)(0.19635)^2} \right) \]
\[ h_p = 100 + 13.694 Q^2 \]

<table>
<thead>
<tr>
<th>Q (gpm)</th>
<th>Q (cfs)</th>
<th>( h_p ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100.0</td>
</tr>
<tr>
<td>200</td>
<td>0.445</td>
<td>102.7</td>
</tr>
<tr>
<td>400</td>
<td>0.891</td>
<td>110.9</td>
</tr>
<tr>
<td>500</td>
<td>1.114</td>
<td>117.0</td>
</tr>
<tr>
<td>600</td>
<td>1.336</td>
<td>124.4</td>
</tr>
<tr>
<td>700</td>
<td>1.559</td>
<td>133.3</td>
</tr>
<tr>
<td>800</td>
<td>1.782</td>
<td>143.5</td>
</tr>
<tr>
<td>900</td>
<td>2.004</td>
<td>155.0</td>
</tr>
<tr>
<td>1000</td>
<td>2.227</td>
<td>167.9</td>
</tr>
</tbody>
</table>
\[ Q = 630 \text{ gpm} \]

\[ \eta = 83 \% \]
Open Channel Flow
Flow Classification

- uniform - flow characteristics (discharge, velocity, depth) are constant along the length of channel

- nonuniform (varied) - flow characteristics vary along the length of channel
  - gradually varied
  - rapidly varied

- steady - flow characteristics are constant over time

- unsteady - flow characteristics change with time
Uniform Flow

1. The depth, cross-sectional area, velocity, and discharge at every section are constant.

2. The energy line, water surface, and channel bottom are parallel. Their slopes are equal.

normal depth \((y_n)\) - depth at which uniform flow occurs.
Manning Formula

English Units

\[ V = \frac{1.486}{n} R^{2/3} S^{1/2} \]

\[ Q = \frac{1.486}{n} AR^{2/3} S^{1/2} \]

Metric Units

\[ V = \frac{1}{n} R^{2/3} S^{1/2} \]

\[ Q = \frac{1}{n} AR^{2/3} S^{1/2} \]

Q = discharge (ft\(^3\)/s, m\(^3\)/s)
V = Q/A = velocity (ft/s, m/s)
A = cross-sectional area (ft\(^2\), m\(^2\))
R = A/P = hydraulic radius (ft, m)
S = slope
Z = AR\(^{2/3}\) = section factor
n = roughness coefficient
Geometric Elements of Channel Section

example: trapezoidal section

\[
T = 2(2\text{(6 ft)}) + 20\text{ ft} = 44\text{ ft}
\]

flow area
\[
A = \frac{44\text{ ft} + 20\text{ ft}}{2} \times 6\text{ ft} = 192\text{ ft}^2
\]

wetted perimeter
\[
P = 20\text{ ft} + 2\sqrt{(6\text{ ft})^2 + (12\text{ ft})^2} = 46.8\text{ ft}
\]

hydraulic radius
\[
R = \frac{A}{P} = \frac{192\text{ ft}^2}{46.8\text{ ft}} = 4.10\text{ ft}
\]

hydraulic depth
\[
D = \frac{A}{T} = \frac{192\text{ ft}^2}{44\text{ ft}} = 4.36\text{ ft}
\]
Example: Compute the discharge in a concrete \((n = 0.015)\) channel with the previous cross-section and slope of 0.10%.

\[
Q = \frac{1.486}{n} AR^{2/3} S^{1/2}
\]

\[
Q = \frac{1.486}{0.015}(192\text{ft}^2)(4.10\text{ft})^{2/3}(0.001)^{1/2}
\]

\[
Q = 1,540 \text{ ft}^3/\text{s}
\]
Example: Compute the normal depth and velocity.

\[ Q = 400 \text{ cfs} \]
\[ S = 0.0016 \]
\[ n = 0.025 \]

\[ Q = \frac{1.486}{n} AR^{2/3} S^{1/2} \]
\[ AR^{2/3} = \frac{Q n}{1.486 S^{1/2}} = \frac{400 (0.025)}{1.486 (0.0016)^{1/2}} \]
\[ AR^{2/3} = 168.2 \]

\[ A = y \left( \frac{b + T}{2} \right) = y \left( \frac{20 + 20 + 2(2y)}{2} \right) = y(20 + 2y) \]

\[ P = b + 2 \sqrt{y^2 + (2y)^2} = 20 + 2 \sqrt{y^2 + (2y)^2} \]
\[ = 20 + 4.472y \]

\[ R = \frac{A}{P} = \frac{y(20 + 2y)}{20 + 4.472y} \]
\[ AR^{2/3} = 168.2 \]

\[ Y(20 + 2Y) \left( \frac{Y(20 + 2Y)}{20 + 4.472Y} \right)^{2/3} = 168.2 \]

by trial and error

\[ Y = Y_n = 3.36 \text{ ft} \]

\[ V = \frac{Q}{A} = \frac{Q}{Y(20 + 2Y)} \]

\[ V = \frac{400}{3.36(20 + 2(3.36))} \]

\[ = 4.46 \text{ ft/s} \]