Pile Integrity and Capacity Determined by Redriving
L’Intégrité et la Capacité du Pieu Déterminé par Reconduire

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SYNOPSIS A procedure of redriving 0.36m square prestressed concrete piles up to 82m long was developed where low resistance to driving for substantial depths within the bearing stratum was experienced. Instrumentation and load tests demonstrated the static capacity of the piles in excess of twice the 889kN (100-ton) design load. Dynamic instrumentation during driving was used to locate broken piles and establish redriving requirements for determining the integrity and the adequacy of the piles.

INTRODUCTION

Piles driven through 62 to 77m of low strength clay into consistently compact fine sand encountered highly variable and unpredictable driving resistances at the Metropolitan Syracuse Water Pollution Control Plant. Piles met refusal after only 1m of penetration into the sand and a short distance away penetrated up to 23m with little increase in driving resistance. Driving all 4200 piles to the required resistance in the compact sand layer would have resulted in a large increase in the planned 328km of piling and excessive waste due to unpredictable pile tip levels. Piezometers located in the sand 1.5m or less from piles recorded temporary excess pore pressures that approached the effective vertical overburden stress during driving of piles to low final resistance (Lacy, 1979). Load tests demonstrated adequate static capacity for those piles driven to low final resistance (Lacy, 1979). Additional borings were made to more clearly define the variations in character and the level of the top of the bearing stratum. Very low final driving resistances raised questions of pile integrity even though most had experienced no hard driving.

SOIL PROFILE

The Syracuse, New York site at the southeast end of Onondaga Lake is underlain by 1 to 4.5m of granular fill and compressible chemical waste over loose silty fine sand. Deep deposits of normally consolidated soft to stiff silty clay contain occasional random layers of loose fine sand. Compression of these deep deposits under fill loading has caused up to 1m settlement of the surface and shallow supported structures over the last 20 years. Bearing strata underlying the clay consist of an upper layer of very compact silty fine to medium sand in varying thicknesses, underlain by a very compact silt varved with hard clay. A lower very compact silty fine to medium sand extends at least 18m below the silt layer. The sand strata contains zones of medium sand with traces of coarse sand and gravel. In some areas, the upper sand layer becomes very thin or is not present in the 290 by 420m site. Soil properties have been described previously (Lacy, 1979).

PILE TEST PROGRAM

Theoretical analysis of static pile capacity indicated adequate capacity with less than 3m penetration into the bearing stratum. This was confirmed by load test. The piles were prestressed using seven or eight strands of 1.27cm reinforcement to 6.0 and 6.5k'(880 and 940 psi). Moment resistance splices were spaced generally at 26m intervals. The piles were driven with a Link Belt model 660 diesel hammer with a rated energy of 61kNm (45,000 ft-lb). When low driving resistance was experienced, the hammer energy was reduced to 37kNm (27,000 ft-lb) to prevent damage from reflected tension waves. Original driving requirements were a minimum of 50 blows per 0.3m at full energy for 3m within the bearing stratum and 8 blows per 2.5m for the last 10m.

Drive Test Piles

Initially-driven piles at wide spacing identified areas where little or no increase in driving resistance was observed as the pile tip penetrated the sand bearing stratum. It was found that an interruption in driving of 10 to 120 minutes while only 3m into the bearing stratum resulted in an increase in driving resistance by as much as 30 times. The decrease in temporary excess pore pressure within the bearing stratum following pile driving corresponded well with the increase in driving resistance.

DYNAMIC INSTRUMENTATION

Uncertainties concerning the integrity of a number of piles driven to low resistance prompted
the use of dynamic testing. Tests were performed on 136 previously driven piles (Goble & Associates, 1976 unpublished report) to determine the condition and extent of pile damage or to demonstrate a pile was undamaged. The tests were performed several weeks after the piles were initially driven using a pile hammer for redriving. The test procedure consisted of attaching two accelerometers and two strain transducers to gauges connected to a pile driving analyzer for immediate evaluation, with signals recorded on an analog magnetic tape for further processing. This equipment records strain and acceleration waveforms as they pass through the pile following impact with a pile hammer plus returning waves that rebound from the pile tip. The strain and acceleration waves are usually integrated and presented as force and velocity waves. These waves pass through concrete at a constant known speed. Completely broken piles reflect downward velocity waves in a much shorter time than a longer unbroken pile. A broken pile within the clay layer reflects a markedly reduced return force wave or even a tension wave as there is little tip resistance to pile penetration and a high reflected positive or downward velocity wave. (Rausche, 1979). Fig. 1 shows a pile broken at 12m based on measurements of the distance between positive peaks and a force wave that drops to zero.

![Fig. 1 Record from Completely Broken Pile](image)

Fig. 2 illustrates an unbroken 63m pile with no bearing resistance in the compact sand. The reflected velocity wave is negative (upward) and the reflected force wave is pronounced and impressive at the pile top.

![Fig. 2 Record of an Unbroken Pile Showing Location Where Total Resistance is Maximum](image)

Redriving of instrumented piles was several ks after installation, pile deformation was initiated by adhesion of the reconsolidated surrounding clays. Broken and damaged piles sometimes experienced high redrive resistance. It necessary to use the full hammer energy for waves to penetrate to the bearing strata to limit the number of blows to prevent penetration to the pile top. It is also possible to determine the degree of damage by comparing the relative difference between the reflected force and velocity decrease at the depth of a discontinuity. These damaged areas could be distinguished from minor discontinuities across splices even though damage was often close to the splice.

Results of Tests

While most piles were driven several weeks before testing, two piles had instruments attached before the last section of pile was driven. A continuous record was made of one pile that became damaged and broke during six hammer blows while the pile tip was passing suddenly from compact sand into compact silt at decreasing driving resistance. The pile broke at a depth of 23m or 3m below the upper splice. The data indicates that structural deterioration originated from the splice. The second pile was monitored as it was driven without damage through compact sand into compact silt at low resistance and then redriven one hour later at high resistance as shown in Fig. 3.

![Fig. 3 Resistance Distribution for Pile 33A Tested During Driving](image)

Most of the piles tested were previously suspected of being broken. Several piles that initially drove to high resistance were also tested as controls. The test program was also validated by examining results of those examined were significantly damaged or broken. Studies to evaluate the number of severely damaged piles appear in Fig. 4.

![Fig. 4 Damage Frequency Versus Depth](image)
cause of damage showed the predominate location of damage was near the upper splice as illustrated in Fig. 4. Where driving resistance is low, the reflected tension wave is indicated at the pile bottom by a sharp reduction in compression wave travelling down the pile. By the time the reflected wave reaches the upper splice, this section of the pile is in tension. A computer analysis method (Rausche, 1973) was used to estimate tension stresses during driving. This wave equation analysis uses an iterative procedure of comparing computed response based on assumed soil properties with measured response. Fig. 5 illustrates estimated tensile stresses in pile concrete as a pile approaches and penetrates the bearing stratum.

PILE CONCRETE TENSILE STRESSES:

<table>
<thead>
<tr>
<th>Depth Below Pile Top in Meters</th>
<th>Tension (psi)</th>
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<tbody>
<tr>
<td>10</td>
<td>59</td>
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<tr>
<td>20</td>
<td>67</td>
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<td>50</td>
<td>61</td>
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Fig. 5 Pile Tension Versus Depth for Pile 333A

Tension is maximum in the upper part of the pile and increases near the upper splice as the pile tip passes from compact sand into compact silt. Each depth is also shown in Fig. 3 for the same pile along with the soil stratification. This pile was driven successfully to good bearing.

Estimates of Pile Support

Using measured reflected waves, and estimated static and dynamic soil parameters, analyses were made of the distribution of load during driving. Soil parameters were adjusted until the match of computed and measured forces is close, yielding the stress distribution illustrated on Fig. 3. A low pile compressive load was distributed uniformly before the pile tip reached the bearing stratum. A much higher pile force resulted when the pile tip was in the middle of the upper sand layer with most of the load carried in end bearing. As the pile started to punch through the upper sand layer at a depth of 57m, end bearing decreased markedly. The pile load reduced substantially as the pile penetrated the silt layer at a depth of 68m and increased gradually to the final depth of 70m. The pile was redriven after one hour, with an increase in end bearing resistance with very little change in side friction. Pile driving resistance is shown for the depths analyzed. "Hit adhesion for the clays above the bearing stratum estimated from these analyses of dynamic loading averaged over the pile length from 16.8 to 19.2 kPa (350 to 400 psi) and in one case as high as 43.1 kPa (500 psi). Mobilized unit adhesion under static loading was somewhat higher for similar times after driving. Analysis of the load distribution along several other undamaged piles redriven several weeks after installation produced somewhat similar results:

(1) Piles founded in the middle of the upper sand layer had high end bearing and high side friction in the sand with low mobilized dynamic shear resistance in the clay.

(11) Piles founded in the varved silt layer had low end bearing, low side friction in the sand layer above and slightly higher mobilized shear resistance in the lower part of the thick upper clay layer, but adequate total capacity which was less than (1).

(111) Piles extending into the lower sand received primary support from the upper sand and low total mobility resistance from the varved silt and end bearing in the lower sand. These piles had the highest total resistance and considerable reserve capacity is indicated in the lower sand layer.

DEVELOPMENT OF REDRIVE PROCEDURE

While dynamic instrumentation was used initially to determine whether previously driven piles were unsatisfactory, a method was needed for the large majority of piles remaining to be installed to establish the integrity of piles whenever driving was terminated at low resistance.

Correlation with Driving Record and Instrumentation

Instrumented dynamic testing has demonstrated that some of the piles driven to low final resistance were broken. Review of the driving records revealed certain correlations with the results of the instrumented testing:

(1) Most of the damaged or broken piles had been driven to a final resistance of less than one blow per 2.5 cm.

(11) Significantly out-of-plumb piles were commonly broken.

(111) Piles experiencing a very rapid and large drop in driving resistance as the pile tip passed from the upper compact sand into the compact varved silt were frequently broken.

(1iv) Piles that experienced a marked drop in driving resistance when the pile tip was at other levels and the decrease in resistance differed from the performance of adjacent piles were usually broken.

(v) Piles that had been redriven, within a day after initial driving, to resistances above six blows per 2.5 cm were unbroken and founded in adequate bearing soil.

(vi) Piles redriven several days or weeks after initially driven were sometimes damaged or broken even though redriving resistances
were greater than six blows per 2.5cm.

**Rapid Adhesion of Clays Above Bearing Stratum**

It was observed that the deep clay, which became disturbed around the pile during driving, rapidly developed adhesion to the pile after driving was stopped. This was particularly noticeable after the first two sections of pile were driven to a 50cm depth six to 21m above the bearing stratum. Piles were left for several hours or overnight while adjacent piles were driven. The piles had initially penetrated the clay at 3 to 6m per blow increasing to 5 to 10 blows per 0.31m at the 50m depth. When driving resumed, much higher resistances were observed, especially for the first 0.31 to 0.92m. Driving resistances were plotted versus the time period over which driving was interrupted for a number of piles driven to this level as shown on Fig. 6.

**PILE DRIVING RESISTANCE BLOWS/0.35m**

![Graph showing pile driving resistance](image)

**Approximate Range for First 0.31m After Redrive Commences**

![Graph showing time of interruption in pile driving](image)

**Fig. 6 Redrive Resistance Versus Time With Pile Tip in Clay**

Even a delay of 15 minutes to an hour while the pile was spliced resulted in a marked increase in driving resistance for the first 0.31m of driving with progressively reduced driving resistances for continued driving, eventually approaching the original low values. The wide variation in driving resistance is a result of erratically positioned layers of sand and silt that tend to reconsolidate around piles much more rapidly than the dominant clayey material. Criteria were developed to permit acceptance of piles driven into the bearing stratum at resistances less six blows per 2.5cm. These requirements are shown on Fig. 6. Redriven piles had to obtain at least ten blows per 2.5cm between two and 18 hours. A minimum total of 50 blows was required for redriving between two and six hours while a minimum of 150 blows was required for redriving between six and 18 hours. The higher number of blows permitted piles driven at a rate of ten blows per 2.5cm to be driven more than 0.31m. Prolonged redriving often resulted in pile breakage.

**Production Pile Driving**

Pile driving continued during testing, evaluation and several modifications in drive and redrive requirements using three to five rigs.

The site was divided into three areas of similar foundation characteristics to permit pile driving to proceed in areas of consistent conditions.

**CONCLUSIONS**

Development of redrive criteria for this site has:

1. permitted the use of piles driven to low resistances by redriving the piles to show, primarily, that the piles were not damaged or broken;
2. demonstrated that redriving piles, to confirm increased static resistance following dissipation of pore pressures from pile driving and to determine pile integrity, must be based on rational criteria resulting from tests and analyses;
3. shown at this site that the deep deposits of low strength clay have a substantial time dependent effect on redrive resistance;
4. illustrated the effectiveness of instrumented dynamic testing in understanding pile driving and redriving records. This testing was critical to development of the redrive procedure;
5. provided an example of highly variable pile driving resistance in compact soils. Development of satisfactory methods of installing reliable foundations required close cooperation between the contractor, owner and engineers permitting the project to be completed economically with minimum possible rejected piles and without significant delay.

The completed structures have been in service for two years with measured settlement well within the range of elastic compression of these concrete piles between the pile top and the bearing stratum with insignificant differential settlement.

**REFERENCES**


ABSTRACT

Large diameter piling are increasingly being used in offshore structures to house the well conductors. In order to predict the drivability of these large diameter members, it is desirable to utilize the wave equation theory developed by Smith and others, which has been used for a number of years to examine the drivability of conventional piling. This paper examines the questions: (1) "Does the wave equation theory accurately predict the drivability of large diameter piling?" and (2) "If so, what variations from the standard input data are required?"

Three recently installed structures were analyzed and the predicted blow counts and driving stresses were compared to those measured in the field. The first of the structures was a 96- to 168-inch diameter, tapered, self-supporting caisson, and the other two were mudslide platforms with 125-inch diameter piling. Two variations from the standard Smith input data were required to predict the drivability of these piles. The first variation was a result of the fact that large diameter piling will usually require a tapered drive head because of the smaller diameter of the standard pile cap. In addition, large diameter caisson well protectors are usually tapered above the mudline to make more efficient use of the steel. Formulas were derived to simplify calculation of the stiffness and weight of a tapered, nonprismatic pipe segment. Then a study was made comparing the use of tapered drive heads with standard pile caps to the use of a special large diameter pile cap. The second variation from the standard input data is related to the problem of soil skin frictional "set-up" and end bearing resistance. The wave equation theory was found to accurately predict the drivability of large diameter piling with only a slight variation from the standard Smith method of analysis.

INTRODUCTION

Special precautions are sometimes taken to protect well conductors in an offshore drilling platform. One way of protecting the well is by housing them inside a large diameter member. In a mudslide structure, the piling may serve the dual purpose of supporting the platform and housing the wells to protect them from unstable soil movements. In some situations a self-supporting caisson well protector can be used in lieu of a platform to support the well cluster when dynamic or fatigue problems exclude the use of self-supporting well conductors. The caisson may be a constant diameter pipe or the more efficient tapered member.

The design engineer must determine the answer to two questions. He must decide whether or not the pile can be driven to the design penetration or if drilling or jetting will be required. Secondly, he must be able to predict the driving stresses induced in the pile. New massive hammers are available and the engineer needs to know whether a large hammer is required or if a smaller and cheaper conventional hammer will suffice. Naturally, the engineer would like to utilize the one-dimensional wave equation for the evaluation of the drivability of large diameter piling.

E. A. L. Smith developed this method in the early 1950's to predict the dynamic behavior of piles. More recently, Samson, Hirsh and Lowery updated and refined Smith's method for a computer solution resulting in the Texas A&M wave equation computer programs. Subsequent investigations by Hirsh and Edwards resulted in the refinement of the input data for a more accurate solution of the problem.

The question raised is, "Does the wave equation theory accurately predict the drivability of large diameter members?" The data base for large diameter pile driving is small due to their relatively recent use. Mobil Oil Exploration & Producing Southeast Inc., recently installed three structures with large diameter members in the Gulf of Mexico. The first of these structures was a 96-inch to 168-inch diameter, tapered, self-supporting caisson in 150 feet of water driven to 150 feet of penetration. The second and third structures were mudslide platforms with 125-inch diameter piling in 120 and 140 feet of water, driven to 340 and 360 feet of penetration.

"Common sense" would indicate that a large hammer would be required to drive these piles because of their high resistance. In order to determine the hammer requirements, estimate the blow counts and predict the
driving stresses, these large diameter piles were modeled on the computer using the wave equation theory. The problems encountered in the models, the predictions and the field results are the subject of this paper.

THE MODEL

The wave equation computer analysis will yield the following information:

1. The blow count for a given resistance.
2. The resulting driving stresses.
3. The maximum penetration that a pile can be driven.
4. The maximum resistance that a hammer can overcome.

The wave equation can also be used to examine the following variables:

1. The effects of various hammers and efficiencies.
2. The effects of various cushions, pile caps and other accessories.
3. The effect of varying the pile wall thickness and diameter.

The model used in the wave equation analysis is adequately described in the previously mentioned references; however, it will be reviewed here briefly. The pile is driven as a result of a force pulse (stress wave) which is generated by the ram impact on the pile (Fig. 1). This stress wave travels down the pile until it is reflected upward at the tip. This process continues until all the energy is dissipated by losses in the hammer and cushion, soil deformation and damping, and internal damping in the pile. The resulting displacement of the pile is referred to as the set. The inverse of the set for one blow is the blow count per unit length.

Smith modeled the pile-hammer system as a series of weights and weightless springs (Fig. 2). He derived a technique of describing the motions of this system in a manner numerically equivalent to the one-dimensional wave equation. The solution of the differential equation for pile driving is difficult, if not impossible. Therefore, a finite difference solution using a computer is employed. Since the solution is based on a stress wave, stresses are a ready byproduct of the analysis. Maximum axial tensile and compressive stresses can be determined at any location in the pile.

The ram and pile cap are modeled as rigid masses while the cushion is modeled as a weightless spring. The pile is divided into segments consisting of masses interconnected with a series of springs with stiffness equal to that of the pile segments. The soil skin frictional resistance is modeled by elasto-plastic springs and linear dashpots acting in parallel with the pile stiffness springs. The end bearing is modeled as a spring and dashpot connected to the end of the pile. This system is particularly adaptable to the variety of pile driving systems available.

The end result of a wave equation analysis is a prediction of the blow count. This is obtained by making a series of computer runs for various soil resistances and plotting the driving resistance versus the blow count (Fig. 3). The resulting curve will approach an asymptote which is the value for absolute refusal. The predicted blow count is obtained by choosing the value corresponding to the dynamic soil resistance for a given penetration.

APPLICATION OF THE WAVE EQUATION

The offshore pile creates some special problems which must be considered. The drivability of conventional diameter piles has been adequately covered in several publications. The computer program used for this study is the Texas A&M "TTI" program. It requires input including the weight and impact velocity of the hammer ram, cushion stiffness, pile cap weight, weight and stiffness of the pile segments, material damping, and maximum (skin frictional and end bearing) location of the mudline, ultimate elastic displacement of the soil, and soil damping. Tables 1 and 2 illustrate the hammer and cushion properties and the soil parameters used in this paper.

For the most part, the input for the large diameter pile is similar to the conventional pile; only the magnitudes are larger. However, there are some differences necessary to obtain an accurate solution. The most important factor in predicting pile drivability is the soil resistance. Because of the dynamic nature of the driving system, the use of static soil resistance is incorrect. A value for the dynamic value of the skin frictional resistance is required. In lieu of test values, this dynamic value may be estimated by dividing the static value (obtained from the soil report) by a soil "set-up" factor. This factor may be estimated as the ratio of the in situ to the remolded shear strength for cohesive soil. It is usually assumed to be 1.0 for cohesionless soils. Skin friction is developed on the outside of the pile and not on the inside if a drive shoe is used. This is due to the gap formed between the soil and pile because of the reduced wall thickness above the shoe.

End bearing resistance must be estimated for the analysis. It is very unlikely that a large diameter pile will totally plug; therefore, it would be incorrect to use the static end bearing from the soils report. The piles in this paper were assumed to be totally unplugged and to develop end bearing resistance on the steel area only. The value of this resistance may be obtained by multiplying the static plugged end bearing value by the ratio of the steel area to the total gross area of the pile. In the case of sand, this value should be multiplied by a "dynamic crushing factor" which is obtained from tests. In lieu of tests, a value of 2.5 was used in this paper.

A break in the driving will cause the soil to begin to "set-up". The dynamic value of soil resistance will begin to approach the static value. For this reason, the last few pile add-on segments should be made as long as possible and the breaks in driving should be kept to a minimum.

As mentioned previously, the wave equation analysis models the pile as a series of weights and springs. The stiffness and weight values for a constant diameter pipe are:

\[ K = \frac{AE}{L} \frac{\pi EDt}{H} \]  
\[ W = \gamma V = \gamma AL = \gamma \rho \delta \]  

(1)  
\( (2) \)
where \( K \) = Stiffness of Pipe  
A = Cross Sectional Area of Segment  
D = Median Diameter of Cross Sectional Area of Segment  
t = Wall Thickness  
L = Length of Segment  
H = Vertical Height of Pipe  
E = Modulus of Elasticity  
W = Weight of Pipe  
V = Volume of Pipe  
γ = Density of Material

This formula is correct for prismatic, constant diameter piling; however, large diameter piling will usually require a tapered driving head because the pile cap may have a diameter smaller than that of the pile. Also, large diameter caisson well protectors are usually tapered above the mudline to make more efficient use of the steel.

Equations (1) and (2) are invalid for these nonprismatic, tapered segments since the area, \( A \), varies with the length. The value for the stiffness, \( K \), can be estimated by using the electrical analogy between springs and resistors. Kirchoff's law is applicable in this case by using the formula:

\[
\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2} + \cdots + \frac{1}{K_N}
\]

where \( K_T \) = Stiffness of the Total Segment  
\( K_1, K_2, \ldots, K_N \) = Stiffness of a Cylindrical Segment

\[
\frac{K}{H} = \frac{\pi D E P}{E D P}
\]

where \( D \) = Average Median Diameter of Cross Sectional Area  
and \( H \) = Height of Segment

for several small constant diameter segments. Of course, the use of this method can be cumbersome for several segments. A mathematically exact formula has been derived. The formula is:

\[
K = \frac{\pi b}{H} \log_e \left( \frac{b}{a} \right)
\]

where \( a \) = Smaller Median Diameter of Cross Sectional Area  
\( b \) = Larger Median Diameter of Cross Sectional Area  
\( \theta \) = Taper Angle

and the other terms are the same as in formula (1). The derivation of this formula can be found in Appendix A. A similar derivation in Appendix B results in the following formula for the weight of a tapered pipe:

\[
W = \frac{\pi H (a + b)}{2 \cos^2 \theta}
\]

Since the wave equation model is one-dimensional, only the axial stiffness of the tapered cone is accounted for. Secondary effects such as bending and hoop stresses due to the angle change are ignored. It is believed that these effects will be minor for a small taper angle.

The question often raised is, "Which is better, the use of a tapered drive head with a standard pile cap, or the use of a special large diameter pile cap?" Formulas (4) and (5) allow a comparative study to be made. Fig. 4 illustrates the results of a study comparing three different length drive heads with a standard pile cap to the use of a special pile cap for a 125-inch diameter pile. The driving resistance versus blow count curve shows that there is not much difference in the driving ability of these different systems. Therefore, the decision on which system to use should be based on the relative economics of obtaining a special pile cap versus the fabrication of the tapered drive heads.

**RESULTS**

Although the wave equation is applicable to large diameter piling in theory, the only way this theory can be verified is by field measurements. As mentioned previously, Mobil installed three structures with large diameter members. The first structure, a 95-inch to 168-inch diameter, tapered caisson in 150 feet of water, was driven to 150 feet of penetration into a cohesive soil. This caisson makeup, soils data and the resistance versus blow count curve are illustrated in Fig. 5a and 5b. This caisson was instrumented with a set of strain gauges during driving. The measured blow count versus penetration curve is shown in Fig. 5c.

The second structure had 125-inch diameter piling in 120 feet of water driven to 340 feet of penetration through sand and clay layers. The piling makeup, soils data and resistance versus blow count curve are shown in Fig. 6a and 6b. These two piles were instrumented and the measured blow count versus penetration curves are shown in Fig. 6c and 6d.

The third structure had 125-inch piling in 140 feet of water driven to 360 feet of penetration through a sand layer into a cohesive soil. The piling makeup, soils data and resistance versus blow count curves are shown in Fig. 7a and 7b. These two piles were instrumented and the measured blow count versus penetration curves are shown in Fig. 7c and 7d. The results of the measurements for these three cases and a comparison to the predicted values are illustrated in Table 3. With the exception of cushion stiffnesses, the predicted and measured values are remarkably comparable.

The high values of measured cushion stiffness indicates that the cushions were worn out. When new cushions were installed, much lower values were obtained.

**CONCLUSIONS**

Although "common sense" would indicate large hammers and/or high blow counts for large diameter piling, the wave equation has shown this not to be the case for the examples cited. The values measured by instrumentation of the piling and the measured blow counts seem to verify the applicability of the wave equation theory to large diameter offshore piles. The large mass and stiffness of a large diameter member seem to transmit the energy very well.

Only two variations from the standard input data were found. These are the use of end bearing on the steel area of the pile only and the calculation of the stiffness and weight of a tapered member.

It should be noted that a precise, exact solution
cannot be obtained from a wave equation analysis for any pile. A considerable amount of engineering judgment must be exercised to evaluate pile drivability. There are several physical uncertainties associated with a pile driving operation, including soil strengths, soil "set-up" factors, hammer efficiency, and the condition of the cushion and other accessories. Although uncertainties are involved, the wave equation analysis combined with engineering judgment should yield an adequate solution to the pile drivability problem.

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REFERENCES


6. Fenwke, C.W., "Use of the Wave Equation to Estimate Pile Drivability", Presented at the ASCE Texas Section Spring Meeting, Fort Worth, Texas, April 1976.


APPENDIX A

I. Derivation of the stiffness of a tapered pipe (reference fig. 11).

\[ K = \text{Stiffness of Cone} \]

\[ P = \text{Total Vertical Load} \]

\[ \Delta = \text{Total Vertical Deflection} \]

\[ D = \text{Median Diameter of Cross Sectional Area of Segment} \]

\[ t = \text{Wall Thickness} \]

\[ \phi = \text{Taper Angle} \]

\[ a = \text{Smaller Median Diameter of Cross Sectional Area} \]

\[ b = \text{Larger Median Diameter of Cross Sectional Area} \]

\[ L = \text{Slant Length of Pipe} \]

\[ E = \text{Modulus of Elasticity} \]

\[ H = \text{Vertical Height of Pipe} \]

\[ \sigma = \text{Axial Stress on Segment} \]

\[ \varepsilon = \text{Axial Strain on Segment} \]

\[ l = \text{Slant Length of Segment} \]

\[ \sigma \cos \theta = \pi D t = P \]

\[ D = a + \frac{(b-a)l}{L} \]

\[ \Delta = \frac{L}{0} \cos \theta \frac{dL}{0} = \frac{L}{0} \frac{\sigma}{E} \cos \theta \frac{dL}{0} \]

\[ \Delta = \frac{L}{0} \frac{P}{E} \cos \theta \frac{dL}{0} = \frac{P}{E} \frac{L}{0} \frac{dL}{0} \]

\[ \Delta = \left( \frac{P}{L} \right) \frac{L}{0} \frac{dL}{0} \frac{u}{1+v} \]

where \( u = \frac{(b-a)}{L} \), \( v = a \)

\[ \Delta = \frac{P}{E} \left( \frac{1}{u} \log_e \left( \frac{(a+1)}{v} \right) \right) \frac{L}{0} \]

\[ \Delta = \frac{PL}{E(t-b-a)} \log_e \left( \frac{(b-a)L}{a} \right) \]
\[ \Delta = \frac{PL \log_e \left( \frac{b}{a} \right)}{\pi Et(b-a)} \]

\[ K = \rho = \frac{\pi Et(b-a)}{L \log_e \left( \frac{b}{a} \right)} \]

\[ L \cos \theta = H \]

\[ K = \frac{\pi Et(b-a)\cos \theta}{H \log_e \left( \frac{b}{a} \right)} \]

Check

The formula should converge to that for a prismatic non-tapered pipe if we let \( a = b \) and \( \theta = 0 \).

\[ K = 2\pi Et(b-b) \cos(0) = \frac{0}{H \log_e \left( \frac{b}{a} \right)} = 0 \]

This value is undefined, but the limit can be evaluated by L'Hopital's rule.

\[
\lim_{a \to b} \frac{\pi Et(b-a)\cos \theta}{H \log_e \left( \frac{b}{a} \right)} = \frac{\frac{d}{db} \left( \pi Et(b-a)\cos \theta \right)}{\frac{d}{db} \left( H \log_e \left( \frac{b}{a} \right) \right)}
\]

\[ = \frac{\pi Et \cos(0)}{H} = \frac{\pi Et}{H} \]

\[ = \frac{AE}{L} \quad \text{for a non-tapered pipe} \checkmark \]

therefore the equation for a tapered pipe converges to that of a non-tapered pipe when \( a = b \).

II. Derivation of the weight of a tapered pipe (reference fig. 11).

\[ W = \text{Weight of Pipe} \]
\[ V = \text{Volume of Pipe} \]
\[ A = \text{Cross Sectional Area of Segment} \]
\[ \gamma = \text{Density of Material} \]
\[ D = \text{Median Diameter of Cross Sectional Area of Segment} \]
\[ t = \text{Wall Thickness} \]
\[ \theta = \text{Taper Angle} \]
\[ a = \text{Smaller Median Diameter of Cross Sectional Area} \]
\[ b = \text{Larger Median Diameter of Cross Sectional Area} \]
\[ H = \text{Vertical Height of Pipe} \]
\[ E = \text{Modulus of Elasticity} \]
\[ x = \text{Vertical Height of Segment} \]

\[ A = \frac{\pi D t}{\cos \theta} \]

\[ D = a + \left( \frac{b-a}{H} \right) x \]

\[ V = \int_0^H A \, dx \]

\[ V = \int_0^H \frac{\pi D t}{\cos \theta} \, dx \]

\[ V = \frac{\pi t}{\cos \theta} \int_0^H \left( a + \left( \frac{b-a}{H} \right) x \right) \, dx \]

\[ V = \frac{\pi t}{\cos \theta} \int_0^H u + vx \, dx \]

where \( u = a \), \( v = \left( \frac{b-a}{H} \right) \)

\[ V = \int_0^H \left( ux + \frac{vx^2}{2} \right) \, dx \]

\[ V = \frac{\pi t}{\cos \theta} \left( ah + \left( \frac{b-a}{H} \right) \frac{H^2}{2} \right) \]

\[ V = \frac{\pi t H (a+b)}{2 \cos \theta} \]

\[ W = \gamma V \]

\[ W = \frac{\gamma t H (a+b)}{2 \cos \theta} \]

Check

The formula should converge to that for a non-tapered pipe if we let \( a = b = D \) and \( \theta = 0 \).

\[ W = \frac{\gamma H t (D+D)}{2 \cos(0)} \]

\[ W = \gamma H t D \]

\[ W = \gamma AL \quad \text{for a non-tapered pipe} \]
### TABLE 1

**HAMMER AND CUSHION PROPERTIES**

<table>
<thead>
<tr>
<th>HAMMER</th>
<th>RAM WEIGHT (KIPS)</th>
<th>PILE CAP WEIGHT (KIPS)</th>
<th>STROKE (FT.)</th>
<th>EFFICIENCY (PER CENT)</th>
<th>CUSHION TYPE</th>
<th>STIFFNESS (KIPS/INCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VULCAN 060</td>
<td>60</td>
<td>42.6</td>
<td>3.0</td>
<td>66</td>
<td>WIRE ROPE</td>
<td>28,530</td>
</tr>
<tr>
<td>VULCAN 3100</td>
<td>100</td>
<td>45.9</td>
<td>3.0</td>
<td>66</td>
<td>WIRE ROPE</td>
<td>42,140</td>
</tr>
<tr>
<td>VULCAN 560</td>
<td>62.5</td>
<td>46.5**</td>
<td>5.0</td>
<td>66</td>
<td>WIRE ROPE</td>
<td>31,320</td>
</tr>
<tr>
<td>VULCAN 5100</td>
<td>100</td>
<td>107.4**</td>
<td>5.0</td>
<td>66</td>
<td>WIRE ROPE</td>
<td>54,560</td>
</tr>
<tr>
<td>MEMCO 3000</td>
<td>66.14</td>
<td>33.07**</td>
<td>4.92</td>
<td>66</td>
<td>BONGASSI</td>
<td>55,820</td>
</tr>
</tbody>
</table>

*STANDARD PILE CAP
**CONTRACTOR’S SPECIAL PILE CAP

COEFFICIENTS OF RESTITUTION

WIRE ROPE - 0.8, BONGASSI - 0.75

MODULUS OF ELASTICITY (KSI)

WIRE ROPE - 330, STEEL - 29000, BONGASSI - 210

### TABLE 2

**SOIL PARAMETERS**

<table>
<thead>
<tr>
<th>SOIL TYPE</th>
<th>SIDE DAMPING (SEC./FT.)</th>
<th>POINT DAMPING (SEC./FT.)</th>
<th>SHAKE (IN.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAY</td>
<td>0.20</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>SAND</td>
<td>0.05</td>
<td>0.15</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### TABLE 3

**MEASURED VS. PREDICTED RESULTS**

<table>
<thead>
<tr>
<th>CASE</th>
<th>HAMMER</th>
<th>EFFICIENCY (PER CENT)</th>
<th>FINAL BLOW COUNT (PER FOOT)</th>
<th>MEASURED STRESS (KSI)</th>
<th>COEFFICIENT OF RESTITUTION</th>
<th>CUSHION STIFFNESS (KIPS/INCH)</th>
<th>CONE STIFFNESS (KIPS/INCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>MENCO 3000</td>
<td>70, 60</td>
<td>31-40, 35</td>
<td>24.0, 23.2</td>
<td>.75</td>
<td>55,820</td>
<td>79,530</td>
</tr>
<tr>
<td>2.</td>
<td>VULCAN 5100</td>
<td>55, 66</td>
<td>24-230, 56</td>
<td>20.1, 23.5</td>
<td>.73</td>
<td>159,000</td>
<td>150,000</td>
</tr>
<tr>
<td>3.</td>
<td>VULCAN 560</td>
<td>64, 66</td>
<td>123-138, 130</td>
<td>21.4, 20.4</td>
<td>.76</td>
<td>129,000</td>
<td>223,000</td>
</tr>
</tbody>
</table>

*MEASURED VALUES PREDICTED VALUES

**MEASURED BLOW COUNTS FOR THE FINAL 5FT. OF PENETRATION.

***ONE PILE MET REFUSAL DUE TO SET-UP INCURRED AFTER HAMMER BREAKDOWN. ITS RESULTS ARE OMITTED.

****DISCREPANCY BELIEVED TO BE DUE TO THE STRAIN GAUGES’ PROXIMITY TO A RING STIFFENER.
\[ \frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} \]

\[ \sqrt{\frac{E}{\rho}} = \text{STRESS WAVE VELOCITY} \]

Fig. 1 - One-dimensional wave equation.

Fig. 2 - Hammer-pile-soil system representation.

Fig. 3 - Typical pile driveability curve.

Fig. 4 - Effect of tapered drive head.
Fig. 5a - Pile makeup and soil properties for 96 to 168 inch diameter caisson.

Fig. 5b - Drivability curve for 96 to 168 inch diameter caisson.

Fig. 5c - Pile driving record for 96 to 168 inch diameter caisson.
Fig. 6a - Pile makeup and soil properties for 125 inch diameter pile Case 1.

Fig. 6b - Drivability curve for 125 inch diameter pile Case 1.

Fig. 6c - Pile driving record for 125 inch diameter pile A-3 Case 1.
Fig. 6d - Pile driving record for 125 inch diameter pile B-3 case 1.

Fig. 7a - Pile makeup and soil properties for 125 inch diameter pile Case 2.
Fig. 7b - Drivability curve for 125 inch diameter pile Case 2.

Fig. 7c - Pile driving record for 125 inch diameter pile A-3 Case 2.

Fig. 7d - Pile driving record for 125 inch diameter pile B-3 Case 2.
Fig. 8 - Tapered nonprismatic pipe segment.

\[ a = \text{smaller median diameter} \]
\[ b = \text{larger median diameter} \]
\[ L = \text{slant length of segment} \]
\[ H = \text{height of segment} \]
\[ t = \text{wall thickness} \]
CASE METHOD

by

Garland E. Likins and Frank Rausche

Introduction

The Case Method of pile capacity determination has been used with increasing frequency during the past ten years for both design and construction control of impact driven piles. As an increasing amount of static load test data became available for correlation the computation methods were modified, improved and refined.

The previous derivation of the fundamental Case Method equation has generally been preceded by the assumption that the pile be treated as a rigid body. The expression resulting from the application of Newton's Second Law has then been modified without a complete derivation being presented with only a general reference to wave equation analysis. This type of presentation has caused the method to be criticized unjustly as being based on a rigid body assumption. It will be developed here in a brief way that will, perhaps, be more readable. Also the various modifications which are necessary to account for different soil and pile types are discussed.

In order to illustrate many of the characteristics of pile mechanics a typical force and velocity record for an impact driven pile is shown in Figure 1, where the velocity was obtained by integrating the measured acceleration over time. Also, for ease of plotting the velocity was multiplied by a constant, EA/c (Young's modulus times cross sectional area divided by the wave speed; all pile quantities). The time scale is given in milliseconds and in L/c units, i.e. in units of thdt time which a stress wave needs to travel along a pile of length L.

This record was obtained from a 100 ft. long concrete pile driven by a Kobe K-22 Diesel hammer. The static resistance as determined by a load test was 470 kips which is low compared to the force at impact. As a result the pile top force decreases at time 2L/c after impact, i.e.
when the impact wave returns after a (tension) reflection at the pile bottom.

2 Wave Mechanics

In the subsequent discussion it is assumed that the bar or pile is of uniform cross section. The derivation and solution of the one dimensional wave equation, a linear, second order differential equation, is available from other sources and will not be presented here.

Figure 1 shows one important phenomenon of wave mechanics, namely the fact that force and velocity at a point on a bar are proportional as long as stress waves at this point travel in only one direction. A wave in a rod which is free at top and bottom always has the same direction of velocity (while the sign of stress changes upon each reflection). The proportionality between the two curves is destroyed as soon as waves caused by soil resistance forces reach the pile top. However, for a completely free pile the top velocity due to a pile top force, $F_t(t)$, can be written

$$v_{t,F}(t) = \frac{c}{EA} F_t(t) \quad 1$$

for $0 < t < \frac{2L}{c}$

At times later than $2L/c$ the effect of waves reflected from the pile bottom is felt at the top. Since a free top is assumed, the velocity will be doubled under reflection and will always be positive.

Therefore

$$v_{t,F}(t) = \frac{2c}{EA} \left( \frac{1}{2} F_t(t) + F_t(t - \frac{2L}{c}) + F_t(t - \frac{4L}{c}) + \ldots \right) \quad 2$$

Resistance forces create a somewhat more complex wave behavior in a pile, since they, in general, act at intermediate locations along the pile. A suddenly applied force at such an intermediate location produces
two waves. If the force, as in the case of soil resistance at a location \( x \), \( R_x(t) \), is directed upwards then the upwards traveling wave will be in compression and the downwards traveling one in tension. For reasons of continuity and equilibrium, together with the proportionality requirement between force and velocity, the forces in each wave must have a magnitude that is one half of the applied force. The velocities in both waves at the point of force application, \( x_i \), are:

\[
v_{t,F_i}(t) = \frac{1}{2} \frac{c}{EA} R_{x_i}(t)
\]

Dealing again with a free pile on which this resistance force acts, the velocities will always be directed upwards. The pile top velocity when either of the two waves arrives and reflects will be twice the magnitude of that in Equation 3. The only difference between the two waves is in the arrival time. The upwards traveling wave arrives earlier than the other wave which is first reflected from the bottom.

Assuming that the distributed soil resistance is concentrated at \( n \) locations, \( x_i \), \( i = 1, \ldots, n \) (\( x_i \) measured from the top) whose magnitudes are \( R_i \), \( i = 1, \ldots, n \) and which are of the ideal plastic type such that

\[
R_{x_i}(t) = R_i H(t - \frac{x_i}{c})
\]

where \( H(t - a) \) is the Heaviside step function which is 0 for \( t < a \) (negative arguments) and 1 for \( t \geq a \). With \( t = 0 \) being the time of impact this soil resistance law implies that the resistance forces act only after the impact wave has reached their respective location and that they are constant thereafter.

The effect of the upwards traveling wave caused by \( R_{x_i}(t) \) is felt at the top with a time delay \( x_i/c \), and that of the downwards traveling wave with a delay \( (2L - x_i)/c \) (after reflection at the bottom). This means that the first effect of \( R_{x_i}(t) \) carried by the downwards traveling
wave is felt together with the bottom reflected impact wave at
time $2L/c$ after impact.

For all times one can write the top velocity due to the
upwards traveling velocity caused by $R_i(t)$ as

$$v_{t,i}(t) = -\frac{c}{EA} R_i \left\{ H(t - \frac{2x_i}{c}) + H(t - \frac{2x_i + 2L}{c}) + H(t - \frac{2x_i + 4L}{c}) + \ldots \right\}$$

remembering again that because of the reflection at the free top the
wave velocity has doubled.

The downwards traveling wave causes a top velocity

$$v_{t,i}(t) = -\frac{c}{EA} R_i \left\{ H(t - \frac{2L}{c}) + H(t - \frac{4L}{c}) + H(t - \frac{6L}{c}) + \ldots \right\}$$

The total velocity caused by the impact force, $F_t(t)$, and all $n$ resis-
tance forces $R_i$, $i = 1, 2, \ldots, n$ can be written as

$$v_t(t) = \frac{c}{EA} (F_t(t) + 2 \sum_{j=1}^{m} F_t(t - \frac{j2L}{c}))$$

$$- \sum_{i=1}^{n} R_i \left[ H(t - \frac{2x_i}{c}) + \sum_{j=1}^{m} H(t - \frac{2x_i + j2L}{c}) + \sum_{j=1}^{m} H(t - \frac{j2L}{c}) \right]$$

where $m$ indicates the time interval after impact:

$$\frac{m2L}{c} < t < \frac{(m + 1)2L}{c}$$

3 Evaluation of Measurements

If this simplified soil model for the resistance force were correct
then $v_t(t)$ would be equal to $v_M(t)$ (i.e. the measured velocity) when
the measured force $F_M(t)$ was substituted for $F_t(t)$ in Equation. 7. Thus,
\[
\frac{EA}{c} \cdot v_M(t) = F_M(t) + 2 \sum_{j=1}^{m} F_M(t - \frac{j2L}{c})
\]
\[
- \sum_{i=1}^{n} R_i \left[ H(t - \frac{2x_i}{c}) + \sum_{j=1}^{m} H(t - \frac{2x_i + j2L}{c}) + \sum_{j=1}^{m} H(t - \frac{j2L}{c}) \right]
\]

The third term in Equation 8, the summation over all resistance forces is shown graphically in Figure 2. The first portion of the term is due to the first arrival at the pile top of the upward traveling wave due to the resistance at the location \(x_i\). The full value is felt at \(\frac{2x_i}{c}\) after impact and stays on for the remainder of the blow as shown by the solid bar in Figure 2. The second portion of the term is due to the same wave after reflections from the top and bottom. Thus, for the first and succeeding time intervals \(m\) the effects are felt \(2L/c\) after the proceeding interval \(m-1\), and are represented by the striped bars in Figure 2. The third portion of the third term is due to the effect of the downward traveling wave. The wave arrives at \(2L/c\) after impact and again remains on for the remainder of the blow. Due to subsequent reflections the same effect is felt every \(2L/c\). This portion is depicted by the white bars in Figure 2. Since all of these effects are of equal magnitude it can be seen from Figure 2 that the third term becomes

\[
- \sum_{i=1}^{n} R_i \left[ 2m + H(t - \frac{2x_i + 2mL}{c}) \right]
\]

where \(m\) is the time interval. If the measured velocity is taken at any time \(t^*\)

\[
(2L/c < t^* < (m+1) \frac{2L}{c})
\]

and if the measured velocity at a time \(2L/c\) later is subtracted, then the result is
\[
\frac{EA}{c} \left( v_M(t^*) - v_M(t^* + \frac{2L}{c}) \right) = F_M(t^*) + 2 \sum_{j=1}^{m} F_M(t^* - \frac{j2L}{c})
\]

\[-F(t^* + \frac{2L}{c}) - 2 \sum_{j=1}^{m+1} F_M(t^* + \frac{2L}{c} - \frac{j2L}{c}) \]

\[-\sum_{i=1}^{n} R_i \left[ 2m + H(t^* - \frac{2x_i + 2mL}{c}) - 2(m + 1) \right] \]

\[-H(t^* + \frac{2L}{c} - \frac{2x_i + 2(m+1)L}{c}) \]

Since the arguments of the Heaviside function terms both have the same value their effects cancel. In addition the first element of the fourth term becomes

\[-2 F_M(t^* + \frac{2L}{c} - \frac{2L}{c}) \]

and combines with \( F_M(t^*) \) to produce \( -F_M(t^*) \). The simplified expression then becomes

\[
\frac{EA}{c} \left( (v_M(t^*) - v_M(t^* + \frac{2L}{c}) \right) = -F_M(t^*) \quad F_M(t^* + \frac{2L}{c}) \quad -\sum_{i=1}^{n} R_i \left[ 2m - 2(m+1) \right]
\]

and with:

\[
\sum_{i=1}^{n} R_i = R
\]

i.e. the total resistance, yields

\[
R = \frac{1}{2} \left( (F_M(t^*) + F_M(t^* + \frac{2L}{c})) + \frac{EA}{c} \left( v_M(t^*) - v_M(t^* + \frac{2L}{c}) \right) \right)
\]
For a uniform rod with

\[ c = \sqrt{\frac{E}{\rho}} \]

and \( M = LA_0 \) (\( \rho \) is the mass density, \( M \) the total pile mass) direct substitution gives

\[ \frac{EA}{c} = \frac{Mc}{L} \]

Thus, Equation 13 can also be written as

\[ R = \frac{1}{2} \left( F_M(t^*) + F_M(t^* + \frac{2L}{c}) \right) + M \frac{v_M(t^*) + v_M(t^* + \frac{2L}{c})}{2L/c} \]

which shows that the prediction \( R \) can be considered as an average of two force values \( 2L/c \) apart plus an inertia term using an average acceleration over the same time period. Equation 13a reduces to Newton's Second Law if \( L \) becomes small enough and the pile can be approximated by a rigid body. For larger lengths, \( L \), where the pile must be considered an elastic rod, Equation 13a is valid.

While the pile elastic properties and the distribution of resistance forces were properly considered in the above derivation, the resistance force versus time variations were neglected. These variations exist because both dynamic resistance forces (soil damping) and unloading (pile rebound) occur.

Originally it was proposed to choose the time \( t^* \), yet to be decided upon, at the time when the pile top velocity became zero. Then, it was argued, soil damping forces would have become small. Results obtained in this way were usually conservative when compared with the ultimate capacity of the pile as determined in a static load test. There was an indication, however, that a correct correlation was to be made with a penetration related capacity (less than ultimate). This approach has been abandoned as it is difficult to use in construction control.
For steel pipe piles in granular soils, empirical correlation with ultimate capacity was best when choosing \( t^* \) at the time when the first relative maximum of velocity was reached (here called the time of impact).

4 Derivation of Damping Estimate

Equation 13a has been derived using a very simple model for soil behavior. An improvement in this model and a commonly used approximation is to assume that the forces \( R_i \) are really made up of two portions. The first portion is due to static resistance forces \( R_{i,s} \) and their sum \( RS \) is then the actual failure load. The second portion \( R_{i,d} \) is due to dynamic resistance forces (damping) which are usually treated as being proportional to velocity. The sum of these dynamic forces \( RD \) is important only during the driving of the pile and is of no further practical value. Thus, the total driving resistance \( RT \) can be broken up into two distinct portions

\[
RT = RS + RD
\]

In order to more closely approximate the pile's static capacity from dynamic measurements, an estimate of the total damping forces \( RD \) must be made.

Because the damping effects of soil and unloading due to pile rebound diminish the force and velocity waves in a rather short time period, only the first \( 2L/c \) period is usually available for estimation of the maximum damping resistance \( RD \). Further, for most pile types the majority of both the static soil resistance and dynamic forces have their origin at the pile tip rather than in frictional side resistance. This hypothesis has been confirmed by using a wave equation analysis on the dynamically measured data and also through strain gages located along the pile length, where readings were taken during the static load tests. The bottom velocity of the pile for the free pile solution after the impact arrives and reflects is

\[
v_B(t) = 2 \, v_c(t - \frac{L}{c})
\]

for \( \frac{L}{c} \leq t \leq \frac{3L}{c} \).
The effect of the downwards traveling wave caused by \( R_{X_i}(t) \) on the bottom velocity is given by twice the magnitude of Equation 3 due to reflection.

\[
v_{B,i}^d(t) = -\frac{c}{EA} R_i H(t - \frac{L - X_i}{c}) = -\frac{c}{EA} R_i
\]

for \( \frac{L}{c} < t < \frac{3L}{c} \).

The pile top velocity characteristically shows a relative maximum in the beginning of the blow (impact) and then diminishes in magnitude with time. In cases of very easy driving, the large tip penetrations and tensile reflections cause a large increase in pile velocity at 2L/c after impact. In several cases the top velocity after the impact reflection from the weak tip can become significantly larger than the pile velocity at impact. Because piles are rarely considered acceptable for capacity in such easy driving, they are not of great importance. However, in the cases where they have been encountered by the project the velocity estimates are satisfactory. The bottom velocity will reach a relative maximum value at time \( t = t_{max} + \frac{L}{c} \) (\( t_{max} \) is the time of impact) and is given by

\[
v_{B,\text{max}}(t_{max} + \frac{L}{c}) = 2v(t_{max}) - \frac{c}{EA} \sum_{i=1}^{n} R_i
\]

(The effect of the upward traveling wave caused by \( R_{X_i} \) will be zero at time L/c because all Heavyside step functions \( H(t - \frac{L + 2X_i}{c}) \) are zero).

If the damping force is treated as proportional to this bottom velocity then the maximum damping force becomes

\[
RD = b_v v_B = J_c \frac{EA}{c} v_B
\]

where \( J_c \) is a dimensionless damping parameter (see Appendix A). Use of Equation 13a with \( t^* \) equal to \( t_{max} \) gives the maximum driving resistance RT.
Rearranging Equation 16 and using Equations 1, 12, 19 and 20 gives the maximum static resistance $RS$ as

$$RS = RT - J_c \left( F(t_{\text{max}}) + \frac{EA}{c} v_t(t_{\text{max}}) - RT \right)$$

Use of Equation 21 can then be made with the measured force and velocity functions of time and the actual failure load of the pile as determined from a static load test (LT) to determine the correct value of $J_c$ for any particular pile

$$J_c = \frac{(RT - LT)}{(F(t_{\text{max}}) + \frac{EA}{c} v_t(t_{\text{max}}) - RT)}$$

The Heaviside step functions used in the preceding derivations serve only to simplify the algebraic manipulations. The same equations (13 and 21) for capacity and damping could be obtained using an elastic plastic resistance law (versus the rigid plastic one used here) as commonly found in the classical lumped mass, numerical wave equation analyses now used extensively in the United States. The only necessary restriction is the displacement at the time $E_x$ for location $x$ below the measurements

$$t_x = t^* + \frac{x}{c}$$

(where $t^*$ is the first time in Equation 13a) is greater than the soil quake $q_x$ (the displacement where the elastic plastic soil law becomes plastic) at location $x$. The time $t^*$ can be increased if necessary to allow time for extra displacement and the condition to be satisfied. In most cases the time $t_{\text{max}}$ does indeed provide the maximum resistance $RT$. In some cases (using little cushioning or steel capblocks, small hammers used for high resistances or other very hard driving cases, or if the soil quakes are large, etc.) this maximum $RT$ will occur at some time after the peak input velocity. Automatic searches for the maximum $RT$ capacity are now included as standard procedure.

Later, it was found, when analyzing data from piles whose capacity was large compared to the hammer driving capability, or which had long lengths $L(t^* + 2L/c$ was then at a very late time) that rebound sometimes
occurred before 2L/c after impact implying that unloading had occurred. A technique to estimate the amount of unloading has since been included (see Appendix B).

The only assumptions used in the derivations above for ultimate pile capacity are

a) the pile is a uniform elastic rod with length much greater than diameter

b) that the soil quake is exceeded at every point along the pile

c) static resistance is related to pile displacement and damping resistance to pile velocity

Violation of condition b will result in very low observed permanent set per blow. If a meaningful rate of penetration is not achieved, the Case Method (or any other dynamic analysis technique, i.e. dynamic formula, wave equations, CAPWAP) can only be expected to indicate the soil resistance actually mobilized.

5 Results of Damping Approach

For most piles, if the damping can be assumed to be concentrated at the pile tip, the actual damping resistance was shown in Equation 21 to be proportional to the pile properties (EA/c), bottom velocity (which can be calculated from the top velocity, pile properties and total driving resistance), and a damping constant $J_c$ which is related to the soil type at the pile tip.

Data had been obtained on over 100 test piles (as of 1977) where static load test capacity, sufficient soil borings, and total driving resistance $RT$ are available.

For each pile a damping constant $J_c$ was calculated which produced a prediction which was equal to the static load test value. In addition, damping constants which produce only 20 percent error in Case Method prediction were determined. Any damping constant $J_c$ chosen between these two limiting values will give a Case Method prediction which will be in error from the static load test value by less than 20 percent. Negative damping constants are physically meaningless and are therefore set to zero, should they occur. A plot of the non-negative damping constant $J_c$ within 20 percent
of the load test value is given as a function of soil type regardless
of pile type in Figure 3. For piles with an ultimate capacity less than
150 kips, the acceptable error used was 30 kips. Measurement errors in
the dynamic and static tests as well as the type and interpretation of
the static load test failure (the Davisson failure criteria for rapid
static testing was used as the failure definition) appear to dictate,
for these low capacity piles, that an error range of 30 kips instead of
20 percent be considered acceptable. This additional range in the accept-
able damping constant value is indicated in Figure 3 with the dashed lines.
It can be seen that as the soil grains become finer the damping constant,
$J_C$, must become larger. This result appeared logical.

For any given soil type on any job site where a static test is also
run, a dynamic test on the static test pile will give the correct damping
constant which can then be used on all remaining dynamically tested piles
driven to the same soil stratum (see Figure 1 for sample $J_C$ adjustment).
For job sites with no static test to correlate with, the previous experi-
ences shown in Figure 3 can then serve as a guide in choosing the proper
damping constant which should yield a Case Method prediction within 20 per-
cent of the static test result with a good degree of confidence. Recommended
values are $J_C$ equal to 0.1 for sand, 0.15 for silty sand, 0.2 for sand silt,
0.25 to 0.4 for silt, 0.4 to 0.6 for silty clays and clayey silt, and 0.6
to 1.0 for clay.

It should be noted from Equation 21 that as the
damping constant is increased, the resulting static prediction becomes more
conservative. Thus, to assure that the Case Method is conservative, a higher
damping constant than would normally be associated with the soil type need
only be selected. It can be seen that the above recommended values are
within 20 percent or are at least conservative in all but three cases.
Two of these cases were for very low capacitivities in silty clay which were
not production piling but rather were driven especially for project personnel
in the early stages of the project, when measurement techniques were not as
advanced. The third pile was within 25 percent. In general the results of
the Case Method using damping proportional to the pile cross section pro-
properties $EA/c$ appear very realistic. A plot of the predicted versus measured
capacities using the Case damping constants from soil borings and Case Method
force and velocity measurements is shown in Figure 4 and uses the above recommended damping constants. Even better, essentially perfect agreement could have been obtained using Equation 21 to obtain the proper $J_c$ for each specific pile site. Equation 21 can also be used with total resistance derived from CAPWAP analysis to obtain $J_c$ for a site if a load test is unavailable and the soils are fine grained.

The Case Method capacity indicates the soil resistances at the time of testing. Setup or relaxation effects which commonly occur in soils are evaluated by testing at the end of driving and during restrike some time later. Care has been taken to only include cases where resistance changes affecting the damping constant empirical correlation have been minimized (i.e. for end of driving by running a static test as quickly as possible using a CRP or other short interval loading program; or by restrike testing after sufficient wait time to allow resistance changes to occur if correlating with lengthy load sequences). Restrike testing is always recommended on at least one pile per job site to assess the soil's strength changes with time and the long term service load of the pile.

6 Energy

The standard Case Method measurements of force and acceleration can be used to determine the energy transferred into the pile from the driving system. After integrating the acceleration to obtain velocity, the energy can then be computed from

$$E = \int F(t)v(t)dt$$

By comparing this measured value with the theoretical potential, manufacturer's rated or kinetic energy just before impact, an energy transfer ratio may be obtained. This ratio is the efficiency of the entire driving system after all losses (i.e. friction, in elastic collisions, pile cushions, capblocks and helmets, and gas compressions) and is not the actual hammer efficiency. It is unusual for the measured transfer ratio to exceed 70%. Values of 40 to 60% are normal. A measured transfer ratio below 30% usually indicates a hammer in need of maintenance.
Sometimes transferred energy can be calculated by force or velocity alone, making use of Equations 1 and 24. This approach is valid only if reflections from soil resistances or cross section changes are not yet present in the data. If reflections are present, Equation 24 is the only correct equation. This usually limits this technique to only very long, uniform section, offshore piles with most of their length above the mudline.

7 Other Uses

The usefulness of measured force and velocity has been extended to other uses as well as capacity and energy determinations. For example, the measured forces can be inspected for their maximum compression and tension values. The force-velocity measurements can also be used to compute the maximum tension in the pile below the point of measurements from

\[
T(x) = \frac{1}{2}(I v(\frac{2L}{c}) - F(\frac{2L}{c}) - I v(t_3) - F(t_3))
\]

where \( t_3 = t_{\text{max}} + 2(L - x)/c \) and \( I = EA/c \). Investigations of stresses are useful to prevent excessive pile damage or to improve driveability if stresses are too low.

Case Method measurements have often been used to inspect the structural integrity of the pile. Increases in velocity relative to the force at times before the computed \( 2L/c \) of the pile are the result of reduced cross sectional areas or stiffnesses, which for a uniform section pile is an unmistakable indication of damage. The magnitude of this relative velocity increase can be related to the amount of damage.
Derivation of $J_c$

Replacing the pile by a mass-spring-damper system Newton's Second Law becomes

$$m \ddot{x}(t) + b \dot{x}(t) + k x(t) = F(t) \quad (A1)$$

If we now define (26)

$$\zeta = \frac{b}{2m \omega_n} \quad \text{and} \quad \omega_n^2 = \frac{k}{m} \quad (A2)$$

where $\zeta$ is the viscous damping factor and $b$ is the coefficient of viscous damping, then

$$\zeta = \frac{b}{\sqrt{mk}} \quad (A3)$$

where, for the pile, the mass $m$ is $\rho AL$ and the stiffness $k$ is $\frac{EA}{L}$. Recalling Equation 3.14, the value for $\zeta$ becomes

$$\zeta = \frac{bc}{2EA} \quad (A4)$$

and is critically damped when

$$b_{cr} = \frac{2EA}{c} \quad (A5)$$

Introducing $J_c = 2\zeta$ we get

$$b = J_c \frac{EA}{c} \quad (A6)$$

for $0 < J_c < 2$. 
APPENDIX B

UNLOADING CORRECTION FOR THE CASE METHOD OF CAPACITY PREDICTION

The Case Method of capacity prediction "measures" the resistance (capacity) acting simultaneously. For long piles having a significant portion of resistance coming from shaft friction, the Case Method may underpredict during hard driving, i.e. when the pile top rebounds. The pile top velocity becomes negative before the stress wave returns at time $2L/c$. When this happens, the pile top is moving upwards and some skin resistance begins to unload.

The Case Method can be corrected in the following manner. (Refer to the accompanying figure.) First determine the difference between the time that the pile top velocity becomes zero and the stress wave return at $2L/c$ after impact. (Note that impact must be defined at the first velocity maxima.) This time, $t_u$, multiplied by the wavespeed $c$ and divided by 2 represents the length of pile over which the unloading has occurred, $l_u$. To estimate the resistance that has unloaded, one measures the skin friction activated over the length, $l_u$. This is done on the force velocity plots by taking one half the difference between the force and velocity at time $t_u$ after impact. In the example the unloading resistance, $UN$, is 468 kips. Adding $UN$ to the RT ($J=0$) prediction of 767 kips gives a total driving resistance of 1235 kips. The dynamic component is then subtracted.

$$RS = RT + UN - J(2F1 - RT - UN)$$

Scales: Force 900 kips/inch
Velocity times $EA/c$ 900 kips/inch
Time 12.5 msec/inch
14 in. P.S.C. (OCTAGONAL)
L = 100 ft
SOIL AT TIP
FINE SAND
Jc = 0.1

\[ P_4 = \frac{F_1 + F_2}{2} + \frac{M_c}{2L} (V_1 - V_2) \]
\[ = \frac{(601 + 17)}{2} + \frac{(589 - 111)}{2} \]
\[ = 309 + 239 \]
\[ = 548 \text{ KIPS} \]

\[ D = J_c \cdot (2F - P_4 - \text{STAT}) \]
\[ = J_c \cdot (1202 - 548 - 12) \]
\[ = J_c \cdot 642 \]

\[ R_u = P_4 - D \]
\[ = 548 - 0.1 \cdot (642) \]
\[ = 484 \text{ KIPS} \]

STATIC = 470 KIPS

\[ R_u = P_4 - D \]
470 = 548 - Jc (642)

\[ J_c = 0.121 \]
\[ V(t) = \frac{-c}{EA} \sum_{i=1}^{n} R_i \left[ 2m + H(t - \frac{2x_i + 2mL}{c}) \right] \]

Figure 2: Velocity effect at pile top caused by a resistance step force \( R_i \) at location \( x_i \) from top.
Figure 3:
Case damping constant $J_c$ giving Case Method prediction within 20% of the static test versus soil type at the pile tip.
FIGURE 4: Case Method Prediction using Case damping technique versus static load test for all pile types
HIGH TENSION STRESSES IN CONCRETE PILES
DURING HARD DRIVING
By Garland E. Likins, Jr.

It is a well understood principle of one dimensional wave propagation that compression impact loads, such as those occurring in pile driving, cause tension reflections from the pile toe if little or no soil resistance is present. As soil resistances increase, this tension reflection decreases. If the pile is short, the continuing input compressive wave superimposed on the upwards travelling reflected tension wave results in little or no net tension. As piles become longer in relation to the input pulse length, harmful net tensions can result. These tension waves can be particularly harmful to concrete piles.

As concrete piles become longer, additional measures are then necessary to prevent damage. For example, piles are often prestressed, thus superimposing a residual compressive stress to any tensile reflections. By pre-drilling, the pile end is often placed in more competent, higher resistance soils even before driving begins. In more difficult soils, the peak input compressive stress and subsequent tension reflection is limited by reducing the ram stroke. Increasing the cushion thickness or ram weight are ways to lengthen the pulse width and reduce the peak of the compressive input wave to be superimposed on the tension reflection.

Many concrete pile jobs have avoided potentially dangerous tension stresses by successfully using a wave equation analysis prior to construction to investigate and control the driving stresses, both tensile and compressive. In many cases dynamic testing has been used during construction to verify stresses from this theoretical analysis.
With the continued development of dynamic pile testing techniques and analysis procedures, much has been learned about the dynamic resistance properties of soils during pile driving. The CAPWAP computer analysis program (1) processes the measured force and acceleration data to determine the soil parameters. CAPWAP uses the measured acceleration and wave equation type pile and soil models to compute a force curve which is then compared with the measured force. Adjustments are made in the soil model until the computed and measured force curves match. Output results are then the ultimate static load and its distribution, skin and toe damping values, and skin and toe quakes (i.e. the displacement at which the elastic plastic static soil model goes plastic).

Prior to this analysis technique and based on parameter studies of the standard wave equation with the quake between 0.05 and 0.3 inches, it was concluded that the quake value did not change any of the basic wave equation results (2). Based on relatively recent experiences using dynamic pile measurements and CAPWAP, it has become apparent that soil quakes far in excess of previously considered values frequently exist and do in fact significantly alter the wave equation results. Authier and Fellenius (3) and Thompson (4) have both demonstrated that these large quakes can cause very hard driving while having low static capacities.

The case histories by Authier and Fellenius were both glacial soils, one being dense sandy silty till and the other dense clayey silty till. Thompson adds that the same observations have also been encountered in more coarse grained materials. The author has observed many such "high quake" cases (toe quakes between 0.5 and 1.0 inches) in a wide variety of soil conditions. The only apparent distinctive feature in the soils is that they
are saturated. In most every case, displacement type piles have been involved and excess pore water pressure buildup during the cyclic pile driving has been suspected. Dissipation of this excess pore pressure does not necessarily result in improved soil friction or lowered static quakes.

The occurrence of large toe quakes has complex effects on pile driving which have great practical importance. First the ultimate capacity which a given hammer attains at refusal driving will be reduced, often requiring the use of a larger hammer. As demonstrated by Fellenius, a reduction in capacity by a factor of three is easily obtained. This reduction becomes larger as the quake increases.

As a secondary effect of the reduced resistance, tension reflections are generated even at refusal driving. If minimum penetrations are required, continued driving at these increased blow counts accompanied by high tension stresses again compound the structural damage potential to long concrete piles.

These tension reflections are further increased due to the slow response of soils with large quakes. With typical pile top cushioning, displacements at the time of arrival of the peak input velocity at every point along the pile are at least equal or comparable to the normal quakes, even in refusal conditions. Thus the full resistance effects are mobilized at the time of the first reflection at the pile tip. Under normal conditions this is enough to prevent damaging tension stresses from occurring. In the large quake case not only is the ultimate resistance achievable reduced but the displacement at the toe at the arrival of the first input peak can be considerably less than the quake. This of course implies that only a fraction of the reduced toe resistance is initially mobilized and even higher tension reflections are
generated. Only after considerable delay is the full displacement and resistance achieved.

The effects of high input stresses associated with prolonged hard driving, reduced resistance and delayed soil response from high quakes combine to often produce unexpected pile damage.

Results obtained at three different sites are presented demonstrating the effects of large soil quakes on tension stresses in concrete piles.

Case 1 - Seattle

Several 24 inch (610mm) octagonal prestressed concrete piles were installed in March 1980. The piles were hollow, having a cross sectional area of 300 square inches (1935cm²) and were 70 feet (21.3m) in length. Below 27 feet the soil was classified as glacial deposits of hard silty clay. After predrilling the first 12 feet (3.7m), the pile had been driven to a penetration of 45 feet (13.7m) with a Kobe K45 open end diesel hammer (rated at 91 kip feet (124kJ) based on a 9.2 foot (3.6m) stroke) with a 10 inch (250mm) plywood cushion and 3.5 inch (89mm) Fosterlon capblock. The pile was redriven and tested dynamically after a wait of three days. Blow counts steadily increased to over 250 blows/foot (100 blows/82mm) at 57 feet (17.4m) penetration. Driving was stopped when the blow count exceeded 50 blows per inch (2 blows/mm). The cushion was then reduced to only four inches (100mm) of plywood and blow counts reduced to 45 blows per two inches (50mm) at a ram stroke observed to be 7.7 feet (2.35m).
Figure 1 shows data taken at the end of driving with the four inch (100mm) cushion. Of special interest is the relative force minimum and velocity maximum at a time 2L/c after the peak input (the time required for the wave to travel the length of the pile, reflect and return to the measuring location which was 60 feet (18.3m) above the pile toe). Ordinarily this would indicate a pile with low resistance, as compared with the peak input and structural capacity of the pile. Using techniques previously developed for the calculation of peak tension in the pile from top measurements (5,6), a tension force of 368 kips (1640kN) is found. This corresponds to a stress of 1.2 ksi (8.46MPa).

CAPWAP was used to further investigate the soil response of this pile. Figure 2 shows the final force and velocity matches (Fig. 2a uses acceleration as input to compute force, Fig. 2b uses force as input to compute velocity) and both are considered quite good. The total predicted capacity was 500 kips (2227kN). The skin friction is distributed rather uniformly with 350 kips (1560kN) indicated at the pile tip. However the indicated toe quake of 0.42 inches (10.7mm) was equal to the calculated maximum toe displacement, thus accounting for the high blow count. The toe displacement at the arrival time of the first input peak was 0.14 inches (3.6mm) and therefore mobilized only about half of the available resistance at the first reflection time. The maximum computed tension force from CAPWAP was 375 kips (1670kN).

An equally good CAPWAP match could be obtained with larger quakes provided the soil stiffness is not changed. It is possible that the toe quake and toe resistance are larger and that the total resistance should be similarly increased. When the hammer in refusal driving is not able to mobilize the full ultimate soil resistance, dynamic capacity analysis techniques
cannot be expected to result in anything greater than the actual mobilized resistance.

A second CAPWAP analysis was performed using the same soil constants except using a standard 0.1 inch (2.5mm) quake at the pile toe. The force and velocity matches shown in Figure 3 are quite poor at 2L/c. The computed force no longer shows a net tension at 2L/c and the computed velocity is significantly reduced. The toe displacement was equal to the quake at the arrival time of the first input peak at the toe and thus all the available resistance was mobilized. Tension is greatly reduced.

The CAPWAP soil constants were then used in the conventional wave equation analysis program WEAP (7). Two analysis were made; one with the observed quake of 0.5 inch (13mm) and one with the standard quake of 0.1 inch (2.5mm). Both runs used the observed 7.7 foot (2.35m) stroke. As seen in Figure 4, the capacity using a large quake at 20 blows per inch (25mm) is only half the capacity using a small quake. Actually driving beyond 100 blows per foot (300mm) yields little increase in capacity. The tension stresses are equally dramatic. With normal quakes, above 80 blows per foot (300mm) there is no net tension in the pile. With large quakes, the computed tension never is below 0.8 ksi (5.5MPa) and the measured tension was even higher. The large tension stresses in easy driving may be artificially high as the observed stroke was used throughout. WEAP uses a thermodynamic model for the hammer and if allowed to compute its balanced stroke with a normal quake and 200 kips (890kN) resistance, a stroke of 5.9 feet (1.8m) is observed and the maximum tension is then an acceptable 0.3 ksi (2.1MPa).
This pile was later load tested after several weeks to a Davisson failure load of 1150 kips (5122kN). Telltale and strain gage data along the pile length gave excellent correlation with skin friction results from CAPWAP for the first blows at the beginning of this redrive (45 feet or 13.7m penetration). Equally good results were obtained by comparing restrike capacity information on a 16 inch (405mm) dynamically tested pile driven to approximately 60 feet (18.3m) penetration and scaling the shaft friction to account for the different diameters, proving the inherent correctness of the dynamic testing techniques (see Figure 5). It is always recommended that at least some piles on each site be tested during restrike to properly assess the soils static strengths. In this manner setup and relaxation effects are then properly observed.

The large quakes observed dynamically are not in this case reflected in the static load test. It is indeed fortunate that the pore pressure dissipation and soil set-up provided additional capacity. It is perhaps a coincidence that the WEAP prediction with normal quakes matches the observed load test. If this load were used, however, and then a setup factor assumed, the resulting design would not have achieved a proper safety factor.

Case 2 - Mobile

Seven piles were tested dynamically in June 1979. All piles were 24 inch (610mm) square prestressed piles with total lengths below measurements of 122 feet (37.2m). All were prejetted to depths of at least 100 feet (30.5m). Driving was accomplished by a Raymond 80 hammer with a rated energy of 80 kip feet (109kJ).
The dynamic data of two piles is shown in Figure 6. Again a velocity increase is observed, followed by negative (upward or rebound) velocities. In both cases the blow counts were slightly in excess of 200 blows per foot (300mm) and skin friction was minimal. Proportionality between force and velocity for almost the entire first 2L/c indicates no reflections from soil resistance on the skin or pile cross section changes. Observed quakes from CAPWAP were about 0.7 inches (18mm). Although capacities were around 1000 kips (4455kN) in each case, the slow soil response associated with the large quakes produced tension stresses of 0.75 and 1.0 ksi (5.2 and 6.9 MPa) for piles G57A and E56A respectively. Tension stresses in other piles on this site reached maximums of 1.37 ksi (9.5MPa) at 50 blows per foot (300mm).

No trend in setup or relaxation was observed on this site as might be expected in a soil described as dense sand.

Case 3 - Norfolk

Twelve 18 inch (457mm) square prestressed piles were driven and tested dynamically using a Delmag D30 hammer. The pile lengths were 80 feet (24.4m) and the soil was described as a saturated dense fine sand with 30 to 40 percent silt or clay content. Again the piles were prejetted.

Blow counts were erratic at the end of driving, ranging from 20 to over 500 blows per foot (300mm). The example case shown in Figure 7 had 200 blows per foot (300mm). The maximum computed tension was 0.6 ksi (4.1MPa) at the end of driving and as large as 1.3 ksi (9.0MPa) at lower blow counts. Again indicated capacities are low as seen by the large velocity increase at 2L/c.
A CAPWAP analysis was not performed on this pile although analyses of other piles on this site indicated quakes on the order of 0.4 to 0.5 inches (10 to 13mm).

The analysis of the force and velocity traces revealed that one third of the piles had excessive structural damage (8) below grade, a condition not previously recognized due to the erratic blow counts during driving. It is probable that this damage was caused by the excessive tension due to the large quakes.

Large setup factors associated with the fine grained soil and cementation in some layers later provided adequate capacity as determined by re-strike testing. However these strength gains were primarily located below the structural damage. These damaged piles would not have been able to support even the design load and detrimental settlements would have resulted.

Conclusions

The three cases presented here clearly demonstrate the adverse effects of large toe quakes on pile driving. Not only is the driveability and ultimate soil resistance reduced but also increased tension stresses can and do cause structural damage, even in refusal driving conditions.

The only soil condition seemingly common is the presence of saturation. It is felt that excess pore water pressures, caused by displacement piles driven into poorly drained soils and often aggravated by jetting, is the primary cause of these large quakes.
Reliance only on wave equation driving criteria could lead to unsafe foundations although in many cases soil strength gains as pore pressures decrease will compensate for low initial capacities. The only reliable method of determining the actual soil response during driving is by measurements of force and velocity. Subsequent CAPWAP analysis or low Case Method results in near refusal conditions are the best way to detect this behavior. Restrike testing by Case or CAPWAP methods should always be performed, especially sites with saturated soils, to confirm service load capacity. Although these electronic measurement techniques are preferred, accurate restrike blow count for the first few blows and stroke measurements for correlation with a wave equation analysis is the minimum requirement.

If large quakes are found, several corrective actions may be necessary. If jetting is used, it should be reduced or discontinued if possible. Non-displacement pile types could be considered. If concrete piles are long and tension stresses high, the ram stroke may be reduced causing lower compressive input and subsequent reflected tension stresses. Pile cushion thickness may be increased. In these cases, long spliced prestressed piles should be avoided as the prestress disappears at the splice and the splice will thus see the full tension force. Steel piles could be considered as an alternative. In some cases partial driving, waiting for pore pressures to decrease, and then continuing the driving process may be the only other practical solution.

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REFERENCES


Figure 1  Dynamic Records From Seattle
Figure 2  CAPWAP Results With Qt = 0.42 inch
Figure 3  CAPWAP Results With Qt = 0.1 inch
Figure 4  WEAP Wave Equation Results
Figure 5  Resistance Distribution
Figure 6  Dynamic Records From Mobile
Figure 7  Dynamic Records From Norfolk
BACKGROUND OF CAPACITY INTERPRETATION USING DYNAMIC PILE MEASUREMENTS

By Garland E. Likins, Jr.

Measurements taken during impact pile driving have increased dramatically in the last two decades. In many sections of the world, electronic measurements on piling jobs are now the normal procedure rather than the exception. Their use now includes monitoring of pile stresses, hammer energy transfer efficiency, the calculation of pile capacity for determining criteria or quality control, and investigating the structural integrity of piles (1,2,3). While uses other than capacity determination are very important, they are rather straightforward computationally. This paper will therefore be addressed toward the field calculations and interpretations of capacity.

The dynamics of a pile during impact can be completely defined by three interconnected variables, namely, the forces and motions of the pile and the boundary conditions due to the soil. Knowledge of any two of these variables allows the computation of the third; measured pile top force and acceleration allow the closed form solution of soil resistance effects. Research begun at Case Western Reserve University in the early 1960's has led to a reusable system for obtaining analog measurements of force and acceleration (2) which interrupt the driving operation by typically only five minutes per pile.

For interpreting these signals an insight into one-dimensional wave propagation is useful. The general equation of motion of a longitudinal wave in a bar is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$
with a general solution
\[ u = f(x + ct) + f_1(x - ct) \]

implying two waves each travelling in the opposite direction and that superposition is valid. For a compression wave the particle velocity \( v \) is in the same direction as the velocity of wave propagation, but in a tension wave the velocity \( v \) is in the opposite direction. If two waves travelling in opposite directions come together the resultant force is obtained by addition and the resultant particle velocity by subtraction. Two such identical compressive waves travelling in opposite directions result therefore in a doubling of the stress and zero net velocity and hence zero displacement. This condition describes a fixed end. If the stress waves have opposite signs the stresses cancel and the velocities double, the free end condition. These closed form solutions for simple boundary conditions have been available since St. Venant. A series of examples are given to demonstrate the dynamics of pile driving and the reader is encouraged to find proof elsewhere (4) for important results.

In a uniform unsupported elastic pile, a stress wave will travel unchanged through the rod width at a speed which can be calculated from
\[ c = \sqrt{E/\rho} \] (1)

where \( E \) is the material modulus of elasticity and \( \rho \) is the mass density. The particle velocity of some point along the pile length can therefore be computed if it is known at some other point along the pile at a different time. A stress wave suddenly applied at the end of a rod causes the rod to be deformed. This deformation is transmitted along the rod. After time \( \Delta t \) the stress has travelled a distance \( c \Delta t \), and the end of the rod
will have displaced an amount due to the strain \( \varepsilon \)

\[
\delta = \varepsilon(c \Delta t) = \frac{\sigma}{E} c \Delta t
\]

The velocity at the end of the rod is

\[
v = \frac{\delta}{\Delta t} = \frac{\sigma}{E} c
\]

Thus the input force \((F = \sigma A)\) at the pile top is seen to be proportional to the input velocity by

\[
F = \frac{EA}{c} v
\]  \(2\)

as long as no upward travelling waves from resistance or rod end reflections are felt. As a convention throughout the rest of this paper \( v \) will be the particle velocity and \( V \) will be the velocity \( v \) times the pile impedance \( EA/c \).

Let us examine the case of a pile with zero soil resistances. The input compressive wave is seen travelling down the pile in Figure 1a and 1b. When the wave arrives at the bottom (Figure 1c), the force wave reflects in tension, and to maintain the dynamic balance the pile end accelerates again (the velocity wave reflects with the same sign causing a doubling effect). The wave then travels up the pile (Figure 1d). The velocity wave still has a continually downward sign and the force is now a tension wave. In other words, the pile is now being pulled down due to the lack of soil resistance, and this pull is generating a tension force. When the force wave gets back to the pile top, it again reflects now in compression being a free end, causing a net force of zero at the pile top. The velocity again reflects with the same sign and travels back down the pile. The velocity at the pile top is again doubled.

Instead of looking at the stress and velocity distributions in the whole pile, we look at a particular point on the pile; the force and velocity waves
can easily be obtained. The pile top force velocity curves can be computed with time as shown in Figure 1e. Initially force and velocity waves are of the same magnitude. After the initial input, the force will always be zero due to the free ends and zero resistances. The particle velocity at the top at every 2L/c interval (the time necessary for the stress wave to travel from the pile top to the toe and return to the top) will become twice the input velocity magnitude. The force and velocity with time at the pile middle can also be computed, as shown in Figure 1f. There the input force and velocity wave arrive at a time L/2c after the initial impact at the top. Every time the wave passes the midpoint, the velocities are always the same. Every time the wave travels upward, the forces are in tension. When the wave reflects from the top and travels downward the forces are in compression.

For this case with zero pile resistances and free conditions at both the top and pile bottom, the force at a point can be computed if the velocity at that point is known by the equation

\[ FP\!S(t) = V(t) - 2V(t - \frac{2L}{c}) + 2V(t - \frac{4L}{c}) - 2V(t - \frac{6L}{c}) \ldots \]

where L is the length below the known velocity location to the pile bottom. This equation is quickly verified by inspection of Figures 1e and 1f.

Equation 3 is then known as the free pile solution; that is, the force required at a location with known velocities at that location having zero resistances on the pile.

A second analysis can be done assuming a fixed top and a fixed bottom. In this analysis the assumption is that there is resistance at the pile ends. The wave travelling down in Figure 2a arrives at the bottom in Figure 2b,
causing a compressive reflection of force and no pile end movement at the bottom (the velocity reflects with the opposite sign cancelling the input velocity wave). This condition will be satisfied if the resistance at the pile bottom shows a rigid plastic behavior with a resistance at least equal to twice the input force; otherwise the pile will move, violating the fixed condition. Figure 2c shows the wave as it travels up the pile after the first reflection. The force is again in compression, but the velocity is negative; the pile rebounds. At the fixed pile top, the force again reflects in compression and superimposes on the upwards travelling initial reflection wave, and the velocity reflects with a changed sign causing zero net displacement at the pile top (Figure 2d). The pile top and pile mid-section force and velocity waves are shown in Figures 2e and 2f. In this case, the force at 2L/c at the pile top, and every 2L/c thereafter, is twice the input force magnitude and the velocity is zero at the pile top, due to the fixed condition. At the pile middle, each passage of the wave generates a compression force. The velocity is positive if the wave is travelling downward and negative if travelling upward.

Figure 3 shows the results of a resistance force located at the pile mid-point. For demonstration, we assign a rigid plastic resistance force of one-half of the input force magnitude. The arrival of the input stress wave at the pile mid-point will produce resistance waves which travel upward in compression and downward in tension, each with half the magnitude of the resistance force, or in this example, one quarter of the force input. The effect on velocity will be to generate waves of negative sign travelling in both the upward and downward directions. The net force travelling down the
pile (Figure 3b) is reduced due to the superposition of the input wave and
the generated negative resistance wave, and an upward travelling wave is
generated in compression. When the wave gets to the bottom (Figure 3c),
assuming again a free bottom free top condition, the downward input force
reflects in tension. The downward tension resistance force reflects in
compression. At the same time the upward travelling compression wave re-
fects in tension. The velocities reflect with the same sign, the positive
input reflects positively and the resistance velocity waves reflect nega-
atively, and Figure 3d shows the wave's arrival at the pile top. Figure 4a
shows the pile top forces and velocities with time for this assumed re-
sistance case. Note that until a time L/c the force and velocity waves are
unaffected and the solution is similar to the free pile. After that time
the force and velocity waves separate, by a magnitude equal to the resistance
which was applied at the pile mid-point. At time 2L/c the velocities seem
to increase to the original downward input magnitude. After the initial in-
put, the force is always zero. The result of a free pile solution (Equation 3)
for Figure 4a is also shown in Figure 4. The difference between this free
pile solution from Equation 3 and the observed force is a measure of the re-
sistance acting on the pile. The difference, called the measured delta curve,
is the measured force minus the free pile solution. The measured delta curve
for Figure 4a is shown in Figure 4b. As observed, the measured delta curve,
at time L/c rises to a magnitude equal to the applied resistance at the pile
mid-point, and at time 2L/c rises to the magnitude of twice the resistance.
Figure 5a shows an assumed measured force and velocity curve. By obtaining a
free pile solution (Figure 5b) and then a measured delta curve (Figure 5c),
the total capacity of the pile can be obtained as half the value of the
measured delta curve at time 2L/c.
As the delta curve at time $2L/c$ is equal to

$$\Delta(2L/c) = F(2L/c) - FPS(2L/c)$$

where

$$FPS(2L/c) = V(2L/c) - 2V(0)$$

thus since $V$ equals $F$ at impact (time 0)

$$\Delta(2L/c) = F(2L/c) - (V(2L/c) - V(0) - F(0))$$

As seen in Figure 5c, the delta curve at time $2L/c$ is equal to twice the resistance. Substitution of $2RT$ for $\Delta(2L/c)$ yields

$$RT = (F1 + F2 + V1 - V2)/2$$

(4)

where 1 indicates the impact time and 2 the time $2L/c$ later. Equation 4 is the Case Method capacity equation (2). A quick computation with equation 4 on the free pile example shows zero resistance ($(4 + 0 + 4 - 8)/2$ for the top and $(4 - 4 + 4 - 4)/2$ for the middle) as it should. Computation for the fixed end case shows a resistance of twice the input force ($(4 + 8 + 4 - 0)/2$ equals 8 versus input 4). Although the resistance may be more this is all that can be mobilized. Calculation for the case with side resistance gives half the input force ($(4 + 0 + 4 - 4)/2$ equals 2 versus input 4) which is the value we originally assigned to this resistance.

Equation 4 has often erroneously been associated with the assumption that the soil resistances require a rigid plastic model. Such is not the case. Square wave pulses as used here provide the easiest to understand presentations. However, the same conclusions could be made with triangle or any other general curve. In relation to the soil resistance assumed in Figure 3, an elastic plastic soil law could have been used with a general input pulse.
As long as the displacement reached at each point along the pile before the arrival of the peak velocity at that point exceeds the quake (displacement where the soil model goes plastic), Equation 4 will be valid.

The delta curve concept can be used to determine where resistances are located. The value of the measured delta curve at time \( t \) before \( 2L/c \) is equal to the sum of the soil resistances above the location \( x( = ct/2) \) on the pile (see Figure 5c). The soil resistance between two points on a pile is then the difference between the measured delta curve for those points.

An example demonstrating this technique is shown in Figure 6. The pile was a 12 inch (300mm) closed end pipe pile with a cross sectional area of 9.8 square inches (63.2cm²) driven into silty clay. The data is from a restrike after a static load test. Shown with the measured force and velocity curves in Figure 6a are the free pile solution and measured delta curve which has a maximum value of 363 kips (1617kN) implying a total resistance of 181 kips (808kN).

In actuality the resistance in Equation 4 can be either static or damping. Figure 6b shows the results of applying either shear or damping resistances at the pile bottom. In the case of shear, the resistance delta curve rises and retains the static load. For the damper, the load increases to a maximum as the wave arrives at the damper and then decreases as the velocity of the pile decreases. Thus the total resistance can perhaps be separated into static and damping components by investigating what happens to the delta curve after \( 2L/c \).

The delta curve peak at \( 2L/c \) in Figure 6a is then probably due to velocity or damping effects. The residual value may be due to static forces.
Using the damping approach as derived elsewhere (2) with

\[ RS = RT - J(F1 + V1 - RT) \]  

(5)

a value for the Case Method damping constant J can be determined from the delta curve interpretations of RS and RT. Rearranging gives

\[ J_\Delta = \frac{(RT - RS)}{(F1 & V1 - RT)}. \]  

(6)

Comparison of \( J_\Delta \) with empirical J values determined from the soil borings, can be used as a check on Case Method damping constants, thus providing an extra degree of confidence if no further technique is available (i.e. Equation 6 can also be used with static load test results or CAPWAP total capacity, or previous piling experience with similar local soil conditions).

For the presented example, the static resistance is estimated at 75 kips (334kN) based on half the residual value of the delta curve after 2L/c.

Solution of Equation 6 yield a value of 1.12 for \( J_\Delta \) which is high for the assumed silty clay soil where an empirical J of 0.55 had been suggested for this soil type. Use of this empirical J led to a capacity of 129 kips (575kN) whereas the static test failed at 81 kips (361kN). This data set had been the worst case of all static-dynamic correlations obtained to date (2). The static resistance determined from the delta curve is in much better agreement (93%) than the value obtained from the observed soil type (159%). It is possible that the soil type was not classified correctly or that the silt particles were very fine, almost clay particles in size.

A comparison of the skin friction as obtained directly from the measured delta curve and from strain gages along the pile length is shown in Figure 6c. The delta curve presented a direct measure of the RT distribution.
By ratioing RT with RS, the forces in the pile at ultimate load are obtained and are seen to be in excellent agreement with the loads obtained by static testing.

Although the preceding solution technique has definite usage in skin friction distribution, it is not so simple to determine RS. Several factors combine to limit the usefulness of this technique. First the skin friction has a reducing effect on the delta curve after 2L/c as seen in the example presented in Figure 4. The delta curve could be modified to convert skin to equivalent toe resistance by

\[ \Delta_m(t) = \Delta(t) + \Delta(t - 2L/c) \]  

for times \( t \) less than 4 L/c after the initial wave onset at the top. However, this modified delta now contains all damping resistances and the time of pile toe zero velocity (where damping is also zero) must be determined.

The time where

\[ 4V(t - \frac{2L}{c}) = \Delta_m(t) \]  

has been suggested (1). However the correlation of RS determined by this technique is not as good as that obtained by other methods (2).

An additional problem is that in hard driving, the bottom velocity becomes negative causing unloading very early. The shear delta curve then begins to look similar to that for the damper as the resistance begins unloading (Figure 6b) so that distinguishing between the two becomes difficult.

In easy driving, Equation 8 may never be satisfied and damping therefore non zero. The derived RS, even if the minimum delta is chosen, may therefore be too large.

Therefore, the exact interpretation of the delta curve after 2L/c is difficult at best. A more rigorous analysis such as CAPWAP is much better
suited to sorting out static resistances from damping. A minimum RS can be found from the delta curve or maybe even a maximum value from the modified delta curve. Engineering judgements may often be required.

Once RS has been determined, the reduced delta curve defined by
\[ \Delta_r(t) = 2\Delta(t) \cdot \frac{RS}{\Delta(2L/c)} \]  \hspace{1cm} (9)
can be used as shown in Figure 6c to produce very realistic resistance distributions.

**SUMMARY**

The closed form solution to the equation of motion of a wave in an elastic rod has been presented. Several theoretical boundary conditions have been described in detail. The Case Method capacity equation has been informally derived, and ways to separate static and damping resistances have been shown. The skin friction distribution can be estimated from the measured delta curve.

**REFERENCES**


FIGURE 1 PILE WITH ZERO RESISTANCE. FREE TOP AND BOTTOM
Figure 2 Pile with fixed top and bottom
FIGURE 3 PILE WITH FREE TOP AND BOTTOM, RESISTANCE AT MIDPOINT WITH MAGNITUDE EQUAL TO HALF INPUT FORCE PEAK
a) PILE TOP EFFECT FOR FIGURE 3

b) MEASURED DELTA CURVE FOR FIGURE 3

FIGURE 4
FIGURE 5
INTERPRETATION OF DELTA

RT + \frac{(F1 + F2 + V1 - V2)}{2}

- FORCE
- VELOCITY

FREE PILE

\begin{align*}
F1 + V1 &= 2V1 \\
F2 - V2 &= \text{force difference}
\end{align*}
PILE DRIVING ANALYSIS—STATE OF THE ART

SUMMARY REPORT
of
Research Report Number 33-13 (Final)
Study 2-5-62-33

Driving 100-foot long precast, prestressed, concrete piles with Vulcan 014 single acting hammer at Copano Bay causeway at Corpus Christi while TTI researchers record impact stresses in pile penetration in order to correlate with wave theory predictions.

Sponsored by the Texas Highway Department
in Cooperation with the U. S. Department of Transportation
Federal Highway Administration,
Bureau of Public Roads

January, 1969

TEXAS TRANSPORTATION INSTITUTE
Texas A&M University
College Station, Texas
Pile Driving Analysis—State of the Art

by


Introduction
The numerical computer solution of the one dimensional wave equation can be used with reasonable confidence for the analysis of pile driving problems. The wave equation can be used to predict impact stresses in a pile during driving and also be employed to estimate the static soil resistance on a pile at the time of driving from driving records.

By using this method of analysis, the effects of significant parameters such as type and size of pile driving hammer, driving assemblies (capblock, helmet, cushion block, etc.), type and size of pile, and soil condition can be evaluated during the foundation design stage. From such an analysis appropriate piles and driving equipment can be selected to correct or avoid expensive and time consuming construction problems such as excessive driving stresses or pile breakage and inadequate equipment to achieve desired penetration or bearing capacity.

The findings were based on intensive research involving piling behavior which was conducted by the Texas Transportation Institute over a seven-year period for the Texas Highway Department and the Bureau of Public Roads.

Objectives
The broad objective of this study was to develop the computer solution of the wave equation and its use for pile driving analysis, to determine values for the significant parameters involved to enable engineers to predict driving stresses in piling during driving, and to estimate the static soil resistance to penetration on piling at the time of driving from driving resistance records.

Conclusions
Thorough study of the significant parameters involved in pile driving provided a number of conclusions.

The elasticity of the ram was found to have a negligible effect on the solution in the case of steam drop, and other hammers in which steel on steel impact between the ram and anvil is not present. However, in the case of diesel hammers, steel on steel impact does occur, and in this case, if the elasticity of the ram is disregarded,
Conservative solution for driving stresses and permanent set results. When the elasticity of the ram is accounted for, maximum driving stresses and point displacements may be reduced as much as 20%.

Comparisons by the TTI research with the Michigan pile study indicated that a relatively simple yet accurate method of determining the energy output for pile driving hammers can be used. It was determined that for the cases investigated, a simple equation relating energy output for both diesel and steam hammers gave accurate results.

This equation is:

\[ E = (W_s) (h) (e) \]

where:
- \( W_s \) = ram weight
- \( h \) = actual observed total ram stroke (or the equivalent stroke for double acting steam hammers and closed end diesel hammers), and
- \( e \) = efficiency of the hammer in question.

The efficiencies determined during the course of this investigation were 100% for diesel hammers, 87% for double acting steam hammers and 60% for single acting steam hammers. The researchers feel that 60% was unusually low for the single acting hammer and would not recommend it as a typical value. An efficiency of 80% is believed to be more typical for the single acting steam hammer.

Comparisons between field test results and the numerical solution of the wave equation proposed by E. A. L. Smith (Raymond International) were indeed encouraging. To date, the wave equation has been compared with the results of thousands of actual field tests performed throughout the country. Among the more significant were the comparisons with the Michigan pile study, which dealt almost exclusively with extremely long slender steel piles, and a wide variety of prestressed concrete piles driven in the Gulf Coast area for the Texas Highway Department. Extensive correlation and research has and is being conducted by
Three methods were used to determine cushion properties in the research. These included actual full scale cushion tests dynamically loaded between a ram and pile, tests performed using a cushion test stand in which a ram was dropped on the cushion specimen which had been placed on a concrete pedestal atop a large concrete base embedded in the floor, and finally static tests. It was found that the two dynamic testing methods used yielded almost identical results. It was also found that for a given material the dynamic curves during the loading of the specimens were almost identical to the corresponding static curves. Static tests can be used to determine cushion stiffness but not the coefficient of restitution.

It was learned that the stress-strain diagrams for the material used as cushions are not linearly related to compression. Instead, the curve is closely parabolic during the loading phase. However, use of the exact load-deformation curve for the cushion is both time consuming and cumbersome, and its use is relatively impractical.

It was found that the load-deformation diagram of the cushion could be idealized by a straight line having a slope based on the secant modulus of elasticity of the material.

The dynamic coefficient of restitution for the cushion material studied were found to agree generally with commonly recommended values.

When the wave equation was compared with the results of laboratory experiments, the numerical solution to the wave equation proposed by Smith was found to be extremely accurate.

The effect of internal damping in concrete and steel piles was found to be negligible for the cases studied, although if necessary, it can be accurately accounted for by the wave equation.
The effect of pile dimensions on ability to drive the pile varied greatly; in general it was found that the stiffer the pile, the greater soil resistance to penetration it can overcome.

The wave equation can be used to estimate soil resistance on a pile at the time of driving. Before long-term bearing capacity can be extrapolated from this soil resistance at the time of driving, however, engineers must consider the effect of soil "setup" or possible soil "relaxation" which is a function of time, soil type and condition, and size or type of pile, and other time effects which might be of importance.

Recommended driving practices resulting from the research have more or less eliminated breakage of the expensive (approximately $10 per linear foot) precast, prestressed concrete piles on bridge and causeway construction projects in Texas.

**Implementation**

Research Report 33-13F contains a complete listing of the wave equation computer program, data sheets for input information, and sample problems illustrating use of program and interpretation of computer output. Engineers of the Texas Highway Department Bridge Division are currently using this method of analysis on the THD computer in Austin, Texas.

**Bibliography**


PILE DRIVING ANALYSIS—SIMULATION OF HAMMERS, PILE, AND SOILS

SUMMARY REPORT
of
Research Report Number 33-9
Study 2-5-62-33

Sponsored by the Texas Highway Department
in Cooperation with the U. S. Department of Transportation,
Federal Highway Administration,
Bureau of Public Roads

August 1967

TEXAS TRANSPORTATION INSTITUTE
Texas A&M University
College Station, Texas
Pile Driving Analysis—Simulation of Hammers, Pile, and Soils

by


Introduction

The problem of pile driving analysis has been of great interest to engineers for many years. After the first method for predicting the load bearing capacity of a pile was proposed, pile driving became one of the most hotly debated fields in engineering.

In 1960, E. A. L. Smith proposed the first practical solution to pile driving analysis based on sound mathematical methods. His method was based on a numerical solution to the wave equation and was capable of including the effects of any of the parameters known to be involved in pile driving analysis. It was completely general and applicable to tapered, stepped, and composite piles of any material, to nonlinear soil resistance and soil damping, and to pile systems involving several cushions and helmets.

Smith’s method of analysis was of immediate interest to the Texas Highway Department and Bureau of Public Roads who have sponsored extensive and continuous research concerning pile driving analysis by the wave equation.

Objectives

The specific objectives of this research were:

1. To review and summarize Smith’s original method of analysis and to derive a more general solution.
2. To determine the effect of the elasticity of the ram on the solution.
3. To compare results given by the wave equation with those found in laboratory experiments and field tests.
4. To determine the dynamic properties of cushions subjected to impact loads.
5. To study the effects of internal damping in the pile and its significance.
6. To illustrate the significance of the parameters involved, including the internal damping and spring rate of the cushion, ram velocity, material damping in the pile, soil damping and quake, and to determine the quantitative effect of these parameters where possible.
7. To illustrate the use of the wave equation in determining the impact characteristics of the materials involved.

8. To study the effect of hammer energy and if possible to recommend a simple yet accurate method for determining the actual energy output for various hammers.

The Numerical Method of Analysis and Its Application

The numerical solution proposed by Smith is based on idealizing the actual hammer-pile-soil system as a series of concentrated weights connected by weightless springs as shown in the cover illustration. As shown by Smith, the idealized system is readily solved by the use of numerical methods and high speed digital computers.

In order to obtain the objectives in this report, it was necessary to significantly modify Smith's original solution. These modifications are discussed in detail in the report.

Comparison with Field Tests

One of the most important parameters involved in pile driving is the velocity of the ram immediately before impact, since it significantly affects both driving stresses and permanent set of the pile. Although maximum rated energies are suggested by manufacturers of pile driving hammers, these values are arbitrarily reduced to various degrees because of the lack of a consistent method of determining the actual energy output.

In order to study the influence of the driving hammer, experimental solutions reported by the Michigan State Highway Commission were compared with solutions given by the wave equation. Among the variables compared were the permanent set of the pile, displacement of the head of the pile, resistance to penetration at the time of driving, force at the head of the pile, and energy output of the hammer. The effect of the instrumentation used to record experimental data was also studied.

Comparisons with Laboratory Experiments

The properties of cushion materials were determined by using both the wave equation and experimental data obtained for a full scale pile tested under laboratory conditions. The ram and pile were freely suspended horizontally by cables in order to eliminate the effects of soil resistance. Strain gages placed at intervals along the pile were used to record stresses at various points for comparison with stresses predicted by the wave equation, and to determine the dynamic spring rates and coefficients of restitution for various cushion materials. These data were also used to determine the relative significance of material damping in the pile.

Conclusions

1. Pile driving analysis is an extremely complex problem involving a multitude of variables including the kind of hammer, driving appurtenances, cushion types, properties and
dimensions of the pile, and properties of the supporting soil medium. At this time (1968), only the numerical solution to the wave equation proposed by Smith is capable of accounting for all of the parameters involved.

2. When the wave equation is compared with the results of laboratory experiments and field tests, Smith’s method is found to be relatively accurate.

3. The elasticity of the ram was found to have a negligible effect on the solution except in the case of steel-on-steel impact, which occurs in diesel hammers. The problems solved indicate that if the elasticity of the ram of a typical diesel hammer is disregarded, a conservative solution for driving stresses and permanent set of the pile will result; if the elasticity of the ram is accounted for, maximum compressive driving stress and maximum point displacements may be reduced around 20%.

4. It is possible to determine much valuable information using the wave equation even though values for certain key parameters are unknown. For example, several problems can be solved in which the unknown parameter is varied. From the results, the significance of the parameter can be determined, and the range in which the actual solution lies can be established.

5. Comparisons with the Michigan pile study indicate that a relatively simple yet accurate method of determining the energy output for pile driving hammers can be used. It was found that for the cases investigated, a simple equation relating energy output for both diesel and steam hammers could be used. This equation is as follows:

\[ E = (W_h) (h) (e) \]

where \( W_h \) is the ram weight, \( h \) is the observed total ram stroke (or the equivalent stroke for double acting steam hammers and closed end diesel hammers), and \( e \) is the efficiency for the hammer in question. The efficiencies determined during the course of this investigation were 100% for diesel hammers, 87% for double acting steam hammers and 60% for single acting steam hammers. The writers feel that 60% is unusually low for the single acting hammer and do not recommend it as a typical value. An efficiency of 75 to 85% as reported by Chellis is believed to be a more typical value for a single acting steam hammer.

6. Although the most accurate correlations are obtained when the actual dynamic stress-strain curve of the cushion material is used, the nonlinear behavior involved makes this impractical. Further, the use of a linear stress-strain curve having a slope defined by the secant modulus of the material is sufficiently accurate. The dynamic coefficient of restitution for the cushion materials was found to agree with commonly recommended values.

7. The effect of internal damping in concrete and steel piles was negligible for the cases studied, although it can be accurately accounted for using the wave equation.
Increasing the Ability to Drive Long Off-Shore Piles

By

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Abstract

Because offshore pile foundations are being required to support ever-increasing axial loads, two of the most important questions the Design Engineer must answer is whether or not it will be possible to drive these piles to the design penetration, or whether jetting or drilling will be required, and also, what static resistance to penetration will the soil afford at final penetration. Not only are other methods more expensive than driving alone, they also introduce major uncertainties in the load capacity of the pile.

This paper discusses how pile make-up, size and type of hammer, and driving accessories affect the ability to drive long offshore piles, and how the resistance to penetration may be determined. The one-dimensional wave equation was used to determine the significance of changes in parameters such as the cross-sectional area of the pile, cushion stiffness, ram weight and velocity, and others.

References and illustrations at end of paper

Although it might be expected that increasing the ram weight or ram velocity would be the major influences on increasing the ability to drive a given pile, it was found in this study that in many cases, increasing the cross-sectional area of the pile was of far greater importance. Also, the cross-sectional area was always found to be of importance in utilizing any increase of hammer energy.

It was further determined that for a given hammer, an increase in the cushion stiffness or coefficient of restitution could often be used to greatly increase the ability of a given pile to be driven.

Probably the most significant conclusion shown by this study is that by use of the wave equation the Design Engineer can for the first time determine quantitatively, rather than only qualitatively, the effects of any of the numerous variables involved, i.e., how much any variable affects the ability of a pile to be driven.
Introduction

During the past several years, the use of the wave equation to investigate the dynamic behavior of piling during driving has become more and more popular. Widespread interest in the method was begun in 1960 by E.A.L. Smith(1), who proposed a numerical solution to be handled by high-speed digital computers, which permitted the investigation of such factors as ram weight, ram velocity, cushion and pile properties, and dynamic behavior of soils during driving. Since this time, a vast amount of experimental data has been taken to determine accurate input values for this method and a multitude of full-scale pile tests have been correlated which now permits an accurate analysis of the dynamic behavior of piles during driving.

At the present time, a number of the major oil companies and construction firms are using the wave equation to answer such questions as:

1. Can a given hammer drive a pile to the required depth of penetration?

2. What driving stresses will be induced in the pile and hammer?

3. What rate of penetration will the proposed hammer provide, and consequently, how long will the installation require?

4. What changes in the hammer, driving accessories, pile configuration, or installation procedure can be made to improve the situation?

Pile Driving Hammers

One of the most important questions to be answered in the installation of large offshore piles regards the size of hammer necessary to successfully drive the pile to the required penetration. Unfortunately, the driving capability of any hammer is greatly influenced by numerous factors. Many cases have been reported in which the selected hammer was unable to drive the piles to the design penetration. Several examples of this are reported by Daigle(2) and McClelland(3).

The wave equation was used to analyze three offshore pile driving hammers: the Vulcan 020, 040, and 060 hammers. The results of this analysis are shown in Figure 1. In each case, all variables such as the hammer efficiency, cushion stiffness, and other input data required for the wave equation solution were held constant.

As noted in Figure 1, although the energy output of the Vulcan 040 and 060 hammers are respectively 2 and 3 times that of the Vulcan 020 hammer, in no case was the driving capacity of the 020 doubled or tripled by the use of the larger hammers. Actually, the 300 percent increase in energy from a Vulcan 020 to an 060 hammer increased the driving resistance by only 20 percent.

The demands of present offshore construction often require that these piles be driven to resistances of 3,000 tons or more. These requirements cannot usually be achieved with existing hammers, and it is obvious that larger hammers will eventually have to be built. Fortunately, the wave equation can be used to study these hammers.

Figure 2 shows the results of a parametric study of the ability of large drop hammers to drive a typical pile. These curves give the relationship between the rate of penetration and the ram weight and velocity or drop height. For a given ram weight, the rate of penetration increases with an increase in initial ram velocity. For a fixed velocity or drop height, the rate of penetration increases as the ram weight increases. Working conditions, safety, and physical limitations of equipment dictate that the drop height be limited. Existing steam hammers presently utilize a maximum drop height of 3 ft. (14 ft. per sec. initial velocity) and a ram weight of 80 kips, whereas diesel hammers have maximum drops of around 10 ft. (20 ft. per sec. initial velocity) with 10 kip rams. The curves illustrate that only the Vulcan 080 hammer could be used to drive the given pile. In contrast, the figure indicates that by using a 310 kip ram with a 4 ft. drop, the pile could be driven at 25 blows per ft., giving a thirteen-fold increase in driving rate while increasing the hammer energy by a factor of only 3.9 or less than 4.

This limited study indicates that large capacity piles can be driven with the proper equipment.
Cushions

Cushions are normally used to reduce stresses in both the hammer and pile driving. Common cushioning materials are hard and soft woods, plywood, balsa, foam, and micarta and aluminum scs. The most significant properties of cushion include its stiffness and coefficient of restitution.

Figure 3 shows the effect of varying cushion stiffness. For the pile shown, increasing the cushion stiffness increased the ability to drive the pile, especially at high soil resistances.

The effect of variations in coefficient of restitution of the cushion is illustrated in Figure 4. As might be expected, the more efficient cushion increases the ability to drive the pile at all levels of resistance; less energy is absorbed in the cushion.

Driving Accessories

The weight of the pile cap can have a significant influence on the ultimate soil resistance to which a hammer may drive a pile. Figure 5 gives the wave equation results for two hammers driving the same pile with pile cap weight was varied as follows:

In the case of the 020 hammer, increasing the pile cap weight from 5 kips to 25 kips resulted in a 12% decrease in the ultimate distance to which the pile could be driven, an increase to 45 kips resulted in a 2% loss in ultimate resistance. This phenomena can be explained by the fact that energy available to drive the pile was reduced by the work done on the pile cap. Results obtained for the Vulcan 060 are so indicated in Figure 7.

Pile Configuration

One of the more effective methods of increasing the ability of a pile to be driven is to increase its stiffness by increasing its cross-sectional area. Figures 6 and 7 illustrate typical results and by increasing the wall thicknesses of given piles.

As was noted in Figure 1, tripling the energy of a Vulcan 020 hammer increased the ability to drive the pile only 20 percent. Last, as illustrated in Figures 6 and tripling the cross-sectional area of the pile increases the ability of the 020 hammer to drive the pile approximately 60 percent, and increases the ability of the 060 hammer to drive the pile by 100 percent.

It is interesting to note that Figure 6 illustrates that at low soil resistances (less than 1200 kips) the lighter pile drives easier than the heavier pile. This is because at low resistances, the pile moves a large distance per blow, and the increased mass of the heavier pile traveling through this displacement absorbs much of the energy output of the hammer. However, at a higher resistance, the number of blows per foot becomes sufficiently large (i.e., the penetration per blow becomes so small) that the additional capacity of the heavy pile to transmit the inertia wave through the soil far exceeds the inertial effects, thus making it easier to drive.

Conclusions

The use of the one-dimensional wave equation for the solution of problems concerning the driving of offshore piles has been demonstrated. It was used to determine the significance of changes in parameters such as the cross-sectional area of the pile, cushion stiffness, ram weight, etc.

Although it might be expected that increasing the ram weight or ram velocity would be the major influences on increasing the ability to drive a given pile, it was found in this study that in many cases, increasing the cross-sectional area of the pile was of far greater importance. The cross-sectional area was also found to be of importance in utilizing any increase of hammer energy.

It was further determined that for a given hammer, an increase in the cushion stiffness or coefficient of restitution could often be used to increase the ability of a given pile to be driven.

The weight of the pile cap was found to have a significant influence on the resistance to which a pile can be driven. If the pile cap approaches the weight of the hammer's ram a decrease in ultimate soil resistance of approximately 12% can be anticipated.

The feasibility of developing a drop hammer to drive a pile to a capacity of 3000 tons or more was demonstrated.
The most significant conclusion to be drawn from the study is that through the use of the wave equation the design engineer can for the first time determine quantitatively, rather than qualitatively, the effects of any of the numerous variables involved, i.e., how much any variable affects the ability of a pile to be driven.

References


FIG. 1 A TYPICAL COMPARISON OF PILE DRIVING HAMMERS

FIG. 2 RAM WEIGHT VS INITIAL VELOCITY

FIG. 3 EFFECT OF DUSHION STIFFNESS

FIG. 4 EFFECT OF COEFFICIENT OF RESTITUTION
FIG. 5 EFFECT OF HOLE CAP WEIGHT

FIG. 6 EFFECT OF WALL THICKNESS

FIG. 7 EFFECT OF WALL THICKNESS
PILE DRIVING HAMMERS: THEIR OPERATIONAL EVALUATION, AND SELECTION

by

Lee L. Lowery, Jr.

Introduction

The basic requirements of any pile driving hammer have always been as simple to list as they have been difficult to obtain. Basically, the driving hammer must install the pile in an undamaged condition to some predetermined depth of penetration, both as rapidly and economically as possible. Additional requirements are also obvious, for example the hammer should not be highly variable in its nature of operation or in its ability to drive a given pile. Such phenomena as a sudden reduction in final blow count at final penetration (say from 200 blows/ft to 40 blows/ft) is obviously to be avoided, least the engineer in charge suffer undesirable side effects such as palsy, heart failure, etc.

In the past, hammers were commonly selected using a combination of approximately 60% experience, 20% common sense, 10% experimental investigations, and 10% theory (of which 5% was normally wrong and 4% didn't apply). The 10% theory included the use of standard pile driving formulas, and their subsequent use to estimate driving stresses in the pile. Although a multitude of pile driving formulas have been proposed, including both empirical and semi-rational, only the wave equation accurately mimics the actual dynamic behavior of the pile during driving. Although many of the proposed formulas attempt to account for various energy losses, their use is normally restricted to a particular type of soil, pile, and driving conditions for which correlation factors were derived. For example, as piles continually get longer and heavier, especially in the offshore and coastal zones, no dynamic formula yields acceptable results. Such piles generally show much
higher ultimate loads than predicted by pile driving equations. This is increasingly significant since pre-stressed concrete piles of over 12 ft with 52 in. diameters have been driven successfully, and there is presently an intense interest in the ability to drive steel piles with diameters of over 60 inches and lengths of over 1000 feet in the construction of offshore platforms.

Driving stresses are also of major importance in the design of piles, yet no method of predicting tensile stresses during driving was previously available, and the prediction of compressive stresses was commonly determined simply by dividing the ultimate soil resistance (predicted by some pile driving formula) by the cross-sectional area of the pile. Numerous wave equation studies have shown that application of such "theory" can be disastrous. The authors have personally worked on a single job in Louisiana wherein pile breakage due to excessive driving stresses, and the related delays in job completion resulted in well over one million dollars in damages before a wave equation analysis was performed. Corrective measures were immediately instituted, and no additional problems were encountered. Obviously, simply dividing the soil resistance by the cross-sectional area of the pile completely overlooks the true nature of the problem and computed stresses almost never agree with experimental values. Tensile failures of piles have been noted on numerous other occasions and until now the absence of a reliable method of stress analysis has proven to be a serious problem.

Although numerous hammer types exist, they may in general be divided into the following two basic classes:

1. Hammers which generate a relatively long stress wave in the pile, but which has a relatively low maximum or peak stress, and

2. Hammers producing a relatively short, yet high peaked stress wave in the pile.
Although there are exceptions, these two hammer types are generally illustrated by steam and diesel hammers, respectively. The following illustrates the basic difference between these two groups of hammers. The stress waves due to a steam hammer and diesel hammer are quite different. Whereas the steam hammer's stress wave is relatively long with a low peak value, the diesel hammer induces a short, high magnitude stress wave in the pile. This is of great importance if the pile cannot withstand high stresses during driving, such as is commonly encountered in pre-stressed concrete piling. It also explains the basic differences in diesel hammer versus steam hammer curves which will be shown and discussed later. Basically, these differences arise from the fact that at low soil resistances, the peaks of both stress waves will be sufficient to break the pile loose from the adjacent soil. In this case, the long stress wave induced by the steam hammer works on the pile for a much longer time, thereby driving the pile further during each blow of the hammer than will the diesel hammer. On the other hand, at extremely high soil resistances, the peak stress induced by the steam hammer becomes insufficient to overcome the magnitude of the soil resistance, and is unable to break the pile loose from the adjacent soil, at which point the hammer is unable to cause additional penetration, whereas the diesel hammer, producing a higher stress magnitude continues to drive the pile.

Hammer selection based on:

1) Its ability to drive the pile to pre-determined grade or soil resistance.

2) Prevention of excessive stresses induced in pile during driving.

3) Economy (purchase price, driving rate, operation, availability, ancillary equipment requirements, reliability, ease of handling, and other misc. considerations.)
The wave equation is able to answer the first two and most important of these, namely the ability of any hammer to drive a given pile, prediction of driving stresses, and to some extent, the economy of operation.

The following conclusions and comments were derived from the attached figures:

Figure 1. Figure 1 presents the results of a number of wave equation solutions for various hammers driving a 60 ft concrete pile, to illustrate how the solutions are normally presented. Each curve consists of several solutions. For example, the Vulcan 014 curve was generated by first assuming that a static soil resistance of 200 kips was acting on the pile and determining the corresponding blow count using dynamic soil resistance computed by the wave equation. Blow counts corresponding to soil resistances of 400, 600, 800, and 1000 kips were similarly determined and the curve plotted. The other curves were similarly derived.

The second major use of the wave equation is in predicting driving stresses. As noted in Table 1, for the case just solved, the driving stresses for the DE-30 diesel hammer are extremely critical in tension, whereas the 014 stresses are probably well-within acceptable limits.

You may note here that a sort of trend normally seen is shown here - as the soil resistance increases, max. compressive stresses either remain constant or increase slightly, whereas tensile stresses will generally decrease. The apparent increase in tensile stresses for this case was due to reflected compressive waves from the high soil resistance, changing to tensile stresses at the pile head.

Table 2. As a further example of predicting driving stresses, let's assume that the 60 ft concrete pile was to be jetted when it reached 400 kips resistance, down to 300 kips resistance, and re-driven. As seen in Table 2, jetting can cause a severe increase in tensile stresses unless the
hammer fall is reduced until the pile once again reaches a firm point resistance. Although only the Vulcan #1 case is shown here, similar increases of from 2 to 3 are commonly found regardless of hammer type.

Figure 2. Figure 2 also illustrates how various hammers can be compared for a given case. For example, when driving a 42 in. diameter steel pipe pile, 250 ft long, with a wall thickness of 1 in., the Delmag D44 and Vulcan O60 hammers both reach approximately the same ultimate static soil resistance before refusal is obtained. However, as seen from the curves, the Vulcan hammer drives the pile much faster at low resistances than does the Delmag, although the Delmag hammer still seems to have some driving ability even beyond the 300 blows per foot mark.

Figure 3. As shown in Figure 3, the stress waves due to a steam and diesel hammer are quite different. Whereas the steam hammer's stress wave is of relatively low magnitude and long duration, the diesel hammer induces a relatively short but high-magnitude stress wave in the pile. This of course is of great importance if the pile cannot withstand high stresses during driving. It also explains the relative shapes of the curves given in Figure 2. At low soil resistances, the peaks of both stress waves are sufficient to break the pile loose from the adjacent soil; and in this case, the long stress wave induced by the steam hammer will work on the pile for a much longer time thereby driving the pile further each blow than with the Delmag hammer. On the other hand, above 300 blows per foot, the stress due to the steam hammer is insufficient to overcome the soil resistance, and the hammer cannot cause additional penetration; whereas the diesel hammer, producing a higher stress magnitude, continues to drive the pile.

Figure 4. The effect of driving accessories on the dynamic behavior of piling is extremely variable, and it is quite difficult to make generalizations in all but a few cases. The effect of a "stinger" or "follower" is illustrated in Figure 4.
In this case the solutions for five problems were obtained.

1. for no stinger load
2. for a 1" wall welded stinger
3. for a 1" wall loose stinger
4. for a 2" wall loose stinger
5. for a 4" wall loose stinger.

Note that nearly identical solutions were obtained for the no-stinger, 1" welded, and 1" loose stinger cases. However, as the weight per foot of the stinger is increased over that of the driven pile, the ability of the pile to be driven is reduced.

Another solution was obtained for the case of a 1/2 inch wall stinger, but was erroneously omitted from Figure 4. This curve plotted slightly below the 2" wall curve, indicating that stiffness of the stinger should probably not be less than that of the driven pile.

**Conclusions**

1) Stinger stiffness should be close to that of the driven pile for best results.

2) The stinger, in general, need not be welded or rigidly attached to the pile head.

**Restrictions**

Cases bearing out above conclusions have been run only for steel piles driven by Vulcan and Daiming hammers. Whether or not they are accurate for concrete piles in general is not yet known.

**Table 3.** One of the most common driving accessories is the cushion, and it is commonly accepted as being necessary. However, it is one of the least inspected and most often overlooked point in the driving set-up - a fact which has lead to innumerable pile failures. In our experience since 1960, poor design or mis-use of the cushions has accounted for more pile failures.
than any other single factor.

As illustrated in Table 3, the effect of increasing the cushion stiffness, say by failure to replace the cushion at regular intervals, causes ever-increasing compressive and tensile stresses in the pile. Note that in this case the effect of cushion stiffness on blow count is variable. Actually, it is often possible to find an "optimum" cushion stiffness (in this case, around $6 \times 10^6$ lbs/inch).

**Figure 5.** Large offshore piles, on the other hand, don't seem to have any "optimum" cushions - rather, the pile drives better with increasing cushion stiffness, right up to and including removal of the cushion. However, as before, induced stresses both in the hammer and pile may lead to failure.

**Figure 6.** A second cushion property, its coefficient of restitution, is one of the few variables whose effect is consistent and always reasonable. As seen in Figure 6, the greater energy absorbed in the cushion, the more difficult it becomes to drive the pile. Obviously, cushioning material with high values of $e$ are preferable.

**Figure 7.** The necessity of a pile cap to adapt the hammer to the driven pile is of course obvious. Fortunately, it has little effect on the solution when its weight is small as is usually the case for land-based piles. However, for offshore piles, when it may be necessary to adapt the hammer to as large as a 60" pile, the pile cap weight becomes significant, as seen in Figure 7. In general, it is always desirable to keep the pile cap weight to an absolute minimum, regardless of the type or size of pile being driven.

**Figure 8.** One of the most valuable assets of the wave equation is its ability to determine to what extent a change in a given variable affects the solution. For example, most engineers recognize that an increase in cross-sectional area of a pile increases its ability to be driven, but only by use of the wave equation can the extent of this increase be determined.

Furthermore, the wave equation can be used to indicate the relative merit
of several possible means of increasing the ability of the pile to be driven.

For example, assume that a 42" pipe pile with 0.5" wall thickness were to be driven to a resistance of 2000 kips with a Vulcan 020 hammer. Obviously, as seen from Figure 8, this hammer-pile combination will be inadequate for the job, since the maximum indicated resistance is only around 1600 kips.

It might now seem probable that because the 020 could drive the pile to 1600 kips resistance, then an 040 or surely an 060 would be adequate to attain the 2000 kip. required. However, as illustrated in Figure 8, this is not true, and not even the 060 will be adequate.

Why so small an increase in resistance capacity, when the energy output of the hammer has been tripled? Simply because the pile itself was too flexible to transmit the increased energy - e.g. it acted as an energy storer rather than an energy transmitter.

Therefore, what effect would increasing the pile stiffness have? Obviously, even the 060 can now drive the pile to the required resistance, as seen in Figure 8.

Furthermore, note that the larger hammers are able to be used much more efficiently on the 1" wall pile.

Conclusion

The stiffer the pile the easier it will be to drive with a given hammer (there is usually some limiting value involved).

Figure 9. Figure 9 illustrates the effect of the ram weight on stresses, holding energy output constant. Note that in general the heaviest-ram will cause the lowest driving stresses in the pile.

Figures 10 & 11. Figures 10 and 11 give sets of curves for use by the design engineer in estimating the probable ability of various Vulcan hammers to overcome various soil resistances. For example, as seen in Figure 10, a Vulcan 020 hammer should be able to drive the typical offshore
pile shown to a static soil resistance of around 2400 kips, as long as the pile has a cross-sectional area of at least 200 sq. in.

Figure 12. Figure 12 demonstrates that regardless of present beliefs, extremely large piles should be able to be driven to high resistances using present-day hammers.

Figure 13. Figure 13 illustrates the effect of pile length on one particular case, and illustrates that the length of the pile has little influence on the ability of the pile to be driven. It is therefore concluded that piles of extreme length could also be able to be driven with existing hammers.
THE VARIABILITY OF NATURAL SOILS

PETER LUMB

ABSTRACT

The variations in properties of four typical natural soils are shown to be random variations about a mean or linear trend, related to the "normal" or "Gaussian" statistical distribution. Examples are given of soil properties following the normal, log-normal, and bi-normal distributions. Properties studied include Atterberg limits, grading, and, for undisturbed samples, strength and compressibility characteristics.

A rational basis for the choice of a design parameter, such as strength or compressibility, is the probability that the parameter could be less than the design value. For any particular probability the design parameter can be determined using the normal distribution.

In the case of bearing capacity estimates, an analysis of the conventional factor of safety suggests that a suitable value of probability or "risk" of failure for design is of the order of $10^{-4}$ to $10^{-2}$ per cent.

In the case of settlement estimates, upper and lower bounds to the magnitude and rate of settlement can be associated with a particular probability or risk.

All natural soils show variations in properties from point to point in the ground because of inherent variations in composition and consistency during formation.

The object of this paper is to show that most soil properties can be regarded as random variables conforming to the "normal" or "Gaussian" theoretical distribution. Consequently established statistical methods based on the normal distribution may safely be applied in estimating design parameters and in other problems.

Four natural soils of differing type will be discussed: a soft marine clay deposited in shallow coastal waters, an alluvial sandy clay, a residual silty sand, and a residual clayey silt.

After showing that the composition and properties of undisturbed samples of these soils can be taken as following the normal distribution or a simple transform of the normal distribution, some implications with regard to design will be discussed, with particular reference to estimation of bearing capacity, and the conventional factor of safety will be interpreted in terms of the risk of failure or probability of failure.

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Soinsare

Il est démontré que les variations dans les propriétés de quatre sols naturels typiques présentent une dispersion autour d’une ligne moyenne à tendance linéaire et qui est en relation avec la distribution statistique normale ou de Gauss. Des exemples sont donnés de propriétés de sols qui suivent des distributions normales, normales logarithmiques ou bi-normales. Les propriétés étudiées incluent les limites d’Atterberg, la granulométrie et, pour les échantillons non-remaniés, les caractéristiques de la force de cisaillement et de la compressibilité.

Une base rationnelle pour le choix de paramètres à employer dans le calcul, tels que les paramètres de la force de cisaillement et de la compressibilité, est la probabilité que la valeur du paramètre puisse être inférieure à celle adoptée pour le calcul. Pour s’assurer quelle probabilité, le paramètre employé dans le calcul peut être déterminé en employant la distribution normale.

Dans le cas des estimations de la capacité portante, une analyse du facteur de sécurité conventionnel montre que pour le calcul, une valeur convenable de probabilité ou « risque » de rupture est de l’ordre de $10^{-2}$ à $10^{-4}$ pour cent.

Dans le cas des estimations de tassement, les limites supérieures et inférieures de la grandeur et du taux de tassement peuvent être reliées à une probabilité ou « risque » particulier.
The four soils considered are from Hong Kong but are not peculiar to Hong Kong alone. The marine clay is a typical soft normally consolidated clay, the sandy clay is typical of coastal alluvium, and the residual silty sand and clayey silt, although residual soils, can be taken as typical frictional and cohesive-frictional soils. The conclusions drawn from the data presented here should also apply to soils in other parts of the world.

**Random Variables**

In general any soil property such as voids ratio, cohesion, and so on, will not be constant at all points in a soil mass but will depend on the location of the point. For the results to be described later the properties can be taken to be independent of lateral position and at most linearly dependent on the depth $z$ below the soil surface, and the value of a certain property $v$ at a point $k$ can be written as

$$\nu_k = \alpha + \beta z_k + \delta_k$$

where $\alpha$ and $\beta$ are constants and $\delta$ is a random variable of mean value zero.

It will occasionally be more convenient to write this random variable in the form $\delta = \sigma u$ where $\sigma$, the standard deviation, may itself be a function of the depth $z$. The new variable $u$ is now a *standardized random variable* of mean value zero and standard deviation unity, and can be calculated for all data once $\alpha$, $\beta$, and $\sigma$ have been estimated.

The probability that the standardized random variable $u$ is less than or equal to a particular value $u_1$ will be written as $P(u_1) = P(u \leq u_1)$ and can be defined in terms of a probability density function $g(u)$ as

$$P(u_1) = \int_{-\infty}^{u_1} g(u) du$$

For the important case of the normal or Gaussian distribution the density function is

$$g(u) = \left(2\pi\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}u^2\right).$$

The value of the standardized random variable $u$, associated with a particular probability $P$ will be given the notation $\xi_P$ if the variable follows the normal distribution and will be referred to as the *standardized normal variate*. Values of $\xi$ are given in published tables, for example Pearson and Hartley, 1936.

If a random variable can be regarded as arising from the *sum* of a series of deviations from the mean such that positive and negative deviations are equally likely, small deviations are more likely than large, and very large deviations do not occur, then it can be shown that the variable follows the normal distribution, although the proof is not rigorous.

Similarly, if the variable can be regarded as arising from the *product* of a series of normal deviations, then the variable follows the log-normal distribution $P(u)$ with $\log u = \xi$ and $g(\xi) = \left(2\pi\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \xi^2\right)$.

When two standardized normal variates $x$ and $y$, say, are linearly related with a correlation coefficient $\rho$ then the joint probability $P(x \leq x_1, y \leq y_1)$ is given by the bi-normal distribution.
\[ P(x, y, \rho) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y, \rho) \, dx \, dy \]

where
\[ g(x, y, \rho) = \left[2\pi (1 - \rho^2)^{1/2}\right]^{-1} \exp \left\{ - \frac{x^2 - 2\rho xy + y^2}{2(1 - \rho^2)} \right\}. \]

From the form of the density function it will be obvious that contours of equal probability in the \( x, y \) plane will be ellipses. By a simple linear transform and rotation of axes the probability contours can be changed to circles, making the calculation of probabilities much easier. If \( x = (v - \mu_1)/\sigma_1, y = (w - \mu_2)/\sigma_2 \) where \( \mu_1, \mu_2, \sigma_1, \sigma_2 \) are the means and standard deviations, the necessary transform is
\[
\begin{align*}
\xi & = x\sigma_1 \cos \theta + y\sigma_2 \cos \theta \\
\eta & = \sigma_1 \cos \theta + y\sigma_2 \sin \theta
\end{align*}
\]

where
\[
\tan 2\theta = \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}, \quad (\sigma_1 \pm \sigma_2)^2 = \sigma_1^2 + \sigma_2^2 \pm 2\sigma_1\sigma_2(1 - \rho^2)
\]

and
\[
P(\xi, \eta) = \int_{-\infty}^{\xi} \int_{-\infty}^{\eta} g(\xi, \eta) \, d\xi \, d\eta, \quad g(\xi, \eta) = (2\pi)^{-1} \exp \left\{ - \frac{1}{2} (\xi^2 + \eta^2) \right\}.
\]

**Comparison of Observed and Theoretical Distributions**

For a particular set of measurements of a variable \( v \) there will be \( N \) results which can be arranged in order of increasing rank \( v_1, v_2, \ldots, v_i, \ldots, v_N \). These results can be represented graphically by plotting the measured value against the corresponding theoretical standardized variate \( u \), where \( P \), the probability, is taken as the relative rank \( i/(N + 1) \). If the observed results agree with the theoretical distribution then the plotted points should fall on a straight line.

A less subjective test than this graphical comparison is given by Pearson’s \( \chi^2 \) test (see, e.g. Kendall and Stuart, 1961) in which the \( N \) values are divided into \( k \) classes, the \( i \)th class containing \( n_i \) results lying between \( v_i \) and \( v_{i+1} \), say. The theoretical expected number \( m_i \) lying between the same limits can be calculated from the theoretical distribution for each class, and \( \chi^2 \) is then determined from the equation
\[
\chi^2 = \sum_{i=1}^{k} \frac{n_i^2 - m_i^2}{m_i}. \]

For consistency, the class divisions \( v_i, v_{i+1}, \ldots \) will be taken such that there is equal theoretical probability \( 1/k \) for each class and hence \( m_i = N/k \) is constant and \( \chi^2 \) reduces to
\[
\chi^2 = (k/N) \sum_{i=1}^{k} n_i^2 - N.
\]

A numerical example of the calculations is given in the Appendix. The calculated value of \( \chi^2 \) is now compared with the value which could arise by chance with a probability \( P_r(\chi^2) \) if the variable did in fact follow the theoretical distribution, and if this probability is greater than about 5 per cent
there is reasonable confirmation of agreement. This theoretical probability $P_*(x^2)$, which can be found from published tables (e.g., Pearson and Hartley, 1956), depends only on $v$, the number of degrees of freedom, given by $v = k - r$ where $r$ is the number of parameters needed to calculate $m$. For the simplest case of a normal distribution $x = \mu + \sigma^2$, three parameters $\mu$, $\sigma$, and $x$ are required to calculate $m$, and $v = k - 3$.

**Description of the Soils**

Detailed descriptions of the soils to be dealt with can be found elsewhere (Holt, 1962; Lamb, 1962a, b, and 1965) and the following brief descriptions summarize their characteristics for present purposes.

**Marine Clay**

This soil consists of normally consolidated soft clay about 30 to 40 ft. thick deposited in a narrow cove at a depth of about 20 to 40 ft. below sea level. The soil is relatively uniform both horizontally and vertically over the top 20 to 30 ft. of thickness but shows a sudden change in nature in the bottom 10 ft., the liquid limit dropping from about 120 to 150 per cent in the upper layer to 30 to 90 per cent near the base. Only the upper layer will be considered here. The soil is cohesive, is of low strength and high compressibility, and is sensitive to remoulding.

**Alluvial Sandy Clay**

This soil was deposited at an earlier geological period than the marine clay when the relative sea level was about 100 ft. below the present level. It is more heterogeneous than the marine clay and varies in texture from sand to clay but is typically a sandy clay. From the nature of the soil it is probable that it was deposited on the inter-tidal zones, the source material being residual soil washed down from inland areas by streams and storm water run-off. The soil is of medium strength and compressibility and of medium to low sensitivity.

**Residual Silty Sand**

This soil is formed by the in situ decomposition of medium-grained granite, and is essentially a coarse to medium sand with a variable content of fine sand, silt, and clay. Below a thin oxidized surface layer which will not be considered here the soil grading shows no great change with depth, lateral variations being as large as vertical variations. Both grading and natural voids ratio can be regarded for present purposes as independent of depth. The soil is insensitive to remoulding and its strength is principally frictional.

**Residual Clayey Silt**

This soil is formed by the in situ decomposition of fine-grained acid volcanic rocks and is essentially a coarse to medium silt with a variable content of clay and sand. As with the silty sand the grading and natural voids ratio can be regarded as independent of depth. The soil is insensitive, and its strength is partly cohesive and partly frictional, since it is generally unsaturated in the natural state.
VARIATIONS IN COMPOSITION AND CONSISTENCY

For the clay and the sandy clay the composition of the soils is taken to be characterized by the Atterberg limits. Figures 1 and 2a show the variation with depth of these limits, and Figure 3a shows the relation between plasticity index and liquid limit. The results for the clay show a slight but significant decrease with depth while the results for the sandy clay are independent of depth.

![Graph showing water content, liquidity index, and depth for marine clay](image)

**Figure 1.** Atterberg limits and liquidity index against depth for marine clay

Tables I and II, giving the results of the $\chi^2$ test, show that both liquid and plastic limits follow the normal distribution. The plasticity index, being the difference of two normal random variables, should also follow the normal distribution and Tables I and II show that this is so for both soils.

Since the plasticity index and the liquid limit are both normal random variables the relation between the two would be expected to follow the bimodal distribution. Figure 3b shows the results replotted in terms of the transformed variables $\xi$ and $\eta$, and the $\chi^2$-test again shows good agreement. The circular contours of Figure 3b are the theoretical bi-normal contours enclosing 25, 50, 75, and 90 per cent of all points.

The composition of both soils can thus be taken as a normal random variable.

The consistency of the clay can be taken to be characterized by the liquidity index and Figure 1 shows a mean trend decreasing with depth, as would be
expected since the soil is normally consolidated. The actual results are plotted against the standardized normal variate $Z$ in Figure 4, and this and Table I show good agreement with the normal distribution. The Atterberg limit tests on the sandy clay were performed on the finer fraction of the soil passing the

**TABLE I**

<table>
<thead>
<tr>
<th>Property</th>
<th>$N$</th>
<th>$x^2$</th>
<th>$v$</th>
<th>$P(x^2)/%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid limit</td>
<td>120</td>
<td>17.5</td>
<td>17</td>
<td>42.1</td>
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<tr>
<td>Plastic limit</td>
<td>120</td>
<td>7.0</td>
<td>7</td>
<td>42.0</td>
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<tr>
<td>Plasticity index</td>
<td>120</td>
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<td>17</td>
<td>54.6</td>
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<tr>
<td>Liquidity index</td>
<td>120</td>
<td>24.4</td>
<td>17</td>
<td>10.9</td>
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<tr>
<td>Plasticity index v. liquid limit</td>
<td>120</td>
<td>19.0</td>
<td>14</td>
<td>16.5</td>
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<tr>
<td>Cohesion</td>
<td>114</td>
<td>19.0</td>
<td>16</td>
<td>26.9</td>
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**TABLE II**

<table>
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<tr>
<th>Property</th>
<th>$N$</th>
<th>$x^2$</th>
<th>$v$</th>
<th>$P(x^2)/%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid limit</td>
<td>83</td>
<td>6.3</td>
<td>7</td>
<td>54.0</td>
</tr>
<tr>
<td>Plastic limit</td>
<td>83</td>
<td>1.9</td>
<td>7</td>
<td>96.5</td>
</tr>
<tr>
<td>Plasticity index</td>
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<td>11.8</td>
<td>7</td>
<td>10.7</td>
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<tr>
<td>Plasticity index v. liquid limit</td>
<td>83</td>
<td>7.7</td>
<td>4</td>
<td>10.3</td>
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<tr>
<td>Compression index</td>
<td>66</td>
<td>10.7</td>
<td>7</td>
<td>15.3</td>
</tr>
<tr>
<td>Coefficient of consolidation</td>
<td>53</td>
<td>58.1</td>
<td>7</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Logarithm of coefficient of consolidation</td>
<td>53</td>
<td>7.2</td>
<td>7</td>
<td>41.0</td>
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</table>
FIGURE 3. (a) Plasticity index versus liquid limit for marine clay and sandy clay. (b) Transformed variables $x$ and $y$ for marine clay and sandy clay.

**TABLE III**

<table>
<thead>
<tr>
<th>Property</th>
<th>$N$</th>
<th>$x^3$</th>
<th>$y$</th>
<th>$P_x(x^3)%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>114</td>
<td>22.1</td>
<td>17</td>
<td>18.1</td>
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<tr>
<td>Deviation</td>
<td>114</td>
<td>11.3</td>
<td>17</td>
<td>84.0</td>
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<tr>
<td>Skewness</td>
<td>114</td>
<td>16.5</td>
<td>17</td>
<td>45.9</td>
</tr>
<tr>
<td>Voids ratio</td>
<td>106</td>
<td>23.1</td>
<td>17</td>
<td>14.6</td>
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<tr>
<td>$c_0$</td>
<td>82</td>
<td>6.5</td>
<td>7</td>
<td>48.3</td>
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<tr>
<td>$c_1$</td>
<td>82</td>
<td>5.3</td>
<td>7</td>
<td>62.4</td>
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</table>

BS 36 sieve and consequently liquidity indexes cannot be determined for this soil.

For the coarser silty sand and clayey silt soils Atterberg limits are of little value as a measure of composition and three parameters derived from the
grading curves will be used to characterize the composition. These parameters, based on the percentile diameters $d_y$ are defined as:

- **Median** $Md = \log_{10}d_{50}$,
- **Deviation** $Dv = \frac{1}{2}\log_{10}\left(\frac{d_{75}}{d_{25}}\right)$,
- **Skewness** $Sk = \frac{1}{2}\log_{10}\left(\frac{d_{75}/d_{25}}{d_{50}/d_{25}}\right)$.

![Graph](image)

**Figure 4.** Liquidity index of marine clay versus standardized normal variate

**Table IV**

<table>
<thead>
<tr>
<th>Property</th>
<th>$N$</th>
<th>$x^2$</th>
<th>$v$</th>
<th>$Pr(x^2)$%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>51</td>
<td>12.3</td>
<td>7</td>
<td>9.1</td>
</tr>
<tr>
<td>Deviation</td>
<td>51</td>
<td>5.3</td>
<td>7</td>
<td>62.3</td>
</tr>
<tr>
<td>Skewness</td>
<td>51</td>
<td>8.8</td>
<td>7</td>
<td>26.7</td>
</tr>
<tr>
<td>Voids ratio</td>
<td>49</td>
<td>11.1</td>
<td>7</td>
<td>13.8</td>
</tr>
<tr>
<td>$\phi$</td>
<td>49</td>
<td>4.7</td>
<td>7</td>
<td>70.0</td>
</tr>
<tr>
<td>$\tan \phi$</td>
<td>49</td>
<td>3.0</td>
<td>7</td>
<td>88.1</td>
</tr>
<tr>
<td>Cohesion Site A</td>
<td>37</td>
<td>7.3</td>
<td>3</td>
<td>6.3</td>
</tr>
<tr>
<td>Site B</td>
<td>27</td>
<td>3.9</td>
<td>1</td>
<td>4.8</td>
</tr>
</tbody>
</table>

As shown in Figure 5 and Tables III and IV the three parameters again follow the normal distribution for both soils.

The consistency, as measured by the natural voids ratio $\epsilon_0$ also follows the normal distribution, as shown by Figure 6 and Tables III and IV.

Summing up, the composition and consistency of these widely differing soil types can be regarded as normal random variables in all cases.
The results of a series of consolidation tests on undisturbed samples of the sandy clay are shown in the form of compression index and coefficient of consolidation (for equal load increments of from \( \frac{1}{2} \) to 1 ton/sq. ft.) against depth in Figures 2b and 2c. The compression index results show a slight but
Figure 6. Natural voids ratio versus standardized normal variate:
(a) silty sand, (b) clayey silt.

Figure 7. Compression index and logarithm of coefficient of consolidation for sandy clay
versus standardized normal variate.
significant decrease with depth but the coefficient of consolidation is independent of depth.

It is generally found that the compression index of a soil is roughly proportional to the liquid limit, and since the liquid limit is a normal random variable the compression index should also be a normal random variable. This is found to be the case for the sandy clay as can be seen from Figure 7 and Table II. On the other hand, the coefficient of consolidation is related to the ratio of volume compressibility to permeability, both of which can be taken to depend on voids ratio and liquid limit. Consequently the coefficient of consolidation is of the form of a product of random variables and would be expected to follow a log-normal distribution rather than the simple normal distribution. Table II and Figure 7 show that the present results do in fact agree satisfactorily with the log-normal distribution, \( P_r(x^2) \) being 11 per cent for the log-normal case whereas \( P_r(x^2) \) is less than 0.001 per cent for the normal case.

Figure 8. (a) Undrained cohesive strength against depth for marine clay; (b) Standard deviation of strength against depth for marine clay.
The standard deviations of both compression index and coefficient of consolidation are independent of depth. This is to be expected since the liquid limit for this soil is itself independent of depth.

**Variations in Strength**

Strength characteristics were determined for the marine clay, the residual silty sand, and the residual clayey silt, all in an undisturbed state. As mentioned earlier these soils can be considered as cohesive, frictional, and cohesive-frictional respectively, and the results will be discussed under these three headings.

**Cohesive Soil**

The variation of strength with depth of the clay is shown in Figure 8a, which gives the results of in situ vane tests and also unconfined compression tests on specimens obtained by very careful sampling using a Swedish foil type sampler. There is no significant difference between vane or unconfined compression results and it will be assumed that the results of Figure 8a are completely representative of the in situ soil strength.

Skempton (1957) has shown that the undrained cohesive strength \( c_u \) of a normally consolidated clay is related to the plasticity index \( w \) and the effective overburden pressure. For present purposes the relation between strength and depth \( z \) below the surface is more important and Skempton's equation will be written as

\[
c_u = az(1 + bw) \quad \text{where} \quad a \quad \text{and} \quad b \quad \text{are constants.}
\]

The results of Figure 8a show a small but significantly non-zero strength at the soil surface, presumably due to inter-particle attraction, and the equation must be modified to read

\[
c_u = az'(1 + bw)
\]

where \( z' = z + z_0 \) and \( z_0 \) is an empirical term, in this case 0 ft.

Now \( w \), the plasticity index, is a normal random variable which can be written as \( w_k = w_0 + \sigma\xi_k \), ignoring the slight trend with depth. Thus the strength should also be a random variable given by

\[
(c_u)_k = az'_k(1 + bw_0) + ab\sigma\xi_k z_k
\]

or

\[
(c_u)_k = (c_u)_0 + \sigma_c\xi_k
\]

where

\[
(c_u)_0 = az'(1 + bw_0) = dz'
\]

and

\[
\sigma_c = abz'\sigma_u = fs'.
\]

Thus the standard deviation of the strength variate ought to be proportional to the modified depth \( z' \) and Figure 8b shows that for the marine clay this is in fact the case. Figure 9 shows the standardized variable \( u_k = [(c_u)_k - (c_u)_0]/\sigma_c \) plotted against the standardized normal variate \( \xi_k \) and indicates the good
agreement with the normal distribution. Table I shows that the $\chi^2$ test gives $P_{10}(\chi^2) = 26.9$ per cent, confirming the agreement.

The fact that the standard deviation of the strength variate increases with depth has implications on the estimation of design parameters, as will be shown later. Additional confirmation of this increase with depth is given by tests on the heavily over-consolidated London clay, as shown in Figures 10a and 10b. These results, from unconfined compression tests on samples carved from undisturbed blocks, were reported by Ward et al. (1950), but unfortunately insufficient data were given to test the normality of the distribution.

**Frictional Soil**

The silty sand is free-draining and its strength best characterized by drained triaxial compression tests. When unsaturated the drained cohesion can be appreciably large, but this cohesion is strongly dependent on the degree of saturation. Below a water table the cohesion can be ignored, and above a water table the cohesion varies seasonally due to seasonal variations in rainwater infiltration. The cohesion of this soil will not be considered here and the soil treated as purely frictional.

It has been shown elsewhere (Lumb, 1965) that the drained angle of shearing resistance $\phi_d$ is statistically independent of grading parameters, voids ratio, and degree of saturation, and Table III shows that either $\phi_d$ itself or $\tan \phi_d$ can be taken as a normal random variable. Since $\chi^2$ is rather smaller for the case
Figure 10. (a) Undrained cohesive strength against depth for London clay. (b) Standard deviation of strength against depth for London clay

of tan $\phi_d$ than for $\phi_d$ it may be preferable to regard the variable as tan $\phi_d$ and not $\phi_d$. The values of tan $\phi_d$ are plotted on a normal probability scale in Figure 11.

Figure 11. Tangent of drained angle of shearing resistance versus standardized normal variate; (a) silty sand, (b) clayey silt
Cohesive-Frictional Soil

The clayey silt is generally unsaturated in its natural state and has a high cohesion and angle of shearing resistance. Below the upper 10 to 20 ft. the soil is more or less unaffected by seasonal variations in degree of saturation and thus behaves as a \( c - \phi \) soil. The drained strength parameters again give the best measure of its strength.

As with the silty sand, the drained angle of shearing resistance is statistically independent of the grading parameters, voids ratio, and degree of saturation, and Table IV shows that both \( \phi_d \) and \( \tan \phi_d \) follow the normal distribution.
but that a slightly better fit is obtained with tan \( \phi_e \). Figure 11 shows the values of tan \( \phi_e \) plotted to a normal probability scale.

The cohesion is generally more variable than the angle of shearing resistance, as is illustrated by Figure 12 which shows the results obtained on two small sites. It is of interest to consider whether the variations in \( c_d \) and \( \phi_e \) may be regarded as variations in either \( c_d \) or \( \phi_e \) alone, with the other parameter constant. Figure 13 shows the triaxial compression test results for the two sites in the form of major effective principal stress at failure \( \sigma_1 \) plotted against minor effective principal stress \( \sigma_3 \). The deviations of these results from the mean trend line are only slightly dependent on \( \sigma_3 \), as is shown by the standard deviations in Figure 13, and hence it might well be considered that all the variation is confined to the cohesion term and the angle of shearing resistance is constant, for a small site. On this basis of constant \( \phi_e \) the deviations from the mean trend generally follow the normal distribution, as is shown by Table IV and Figure 14. The number of test results is small, 37 and 27 for the two sites, so the statistical test is somewhat dubious, but on present evidence it seems reasonable to take the soil strength on a small site as given by

\[
\phi_e = \text{constant, and } c_d = c_0 + \sigma \xi, \quad \text{where } c_0 \text{ and } \sigma \text{ are constant and } \xi \text{ is the standardized normal variate.}
\]

![Figure 14](image)

**Figure 14.** Drained cohesion of clayey silt. Standardized variable \( n \) versus standardized normal variate

**Application to Design**

The problem of engineering design is complicated by three types of uncertainty: uncertainty in the value of the soil properties, uncertainty in the value
of the applied loads, and uncertainty in the accuracy of the design method. This paper is mainly concerned with the first type of uncertainty but some discussion of the second and third types is warranted. In order to fix ideas the case of bearing capacity of foundations will be considered.

**Accuracy of Design Method**

Suppose for the moment that the exact manner of variation in strength of the soil and the exact loads to be applied to a foundation are known. An exact relationship between strength and load can be found only for very simple cases, using assumptions that the soil is an ideal elastic or rigid-plastic material, that the foundation is perfectly flexible or perfectly rigid, and so on. Confirmation of theoretical solutions has been given only for soils of uniform strength, either purely cohesive or purely frictional, and even here there is some ambiguity in the choice of numerical bearing capacity factors. If the soil strength is not uniform but varies in some way with depth then in general there are no known rigorous solutions even for a simple surface footing, apart from Kuznetsov's (1958) solution for a cohesive soil whose strength increases linearly with depth. This solution can be written in the form

$$q = 5.14c_0 + 0.99bB$$

where $q$ is the bearing capacity of a long footing of width $B$ on soil of cohesive strength $c = c_0 + bz$, where $z$ is the depth.

For a cohesive-frictional soil, no equivalent to Kuznetsov's equation is available, and for discussion purposes Prandtl's (1920) solution for a shallow narrow long footing will be taken as a typical equation. This can be written as

$$q = N_o(p_0 + c_0 \tan \phi_0) \exp(\pi \tan \phi_0)$$

where $c_0$ and $\phi_0$ are the constant strength parameters, $p_0$ is the overburden pressure, and $N_o = \tan^2(\pi/4 + \phi_0/2)$.

The presence of pockets of soil where the strength is substantially greater than or less than $c_0$, $\phi_0$, or $c_0 \pm b$ will change the bearing capacity above or below Prandtl's or Kuznetsov's solution by an indeterminate amount. For variable soils perhaps the best approach at present is the very conservative one of assuming that the bearing capacity is given by the theoretical solution using parameters $c_0$, $\phi_0$, $b$, such that the actual soil strength is everywhere greater than or equal to these values, and ignoring the increase in bearing capacity due to stronger zones.

**Value of Applied Load**

The actual load that will be transmitted to the soil is rarely known accurately, but upper and lower limits can usually be set. Estimates of dead load $q_1$ and dead plus live load $q_2$ are usually governed by conservative building regulations, and the probability of the actual load being less than $q_1$ or greater than $q_2$ can be taken as zero. For want of an accurate estimate of the true probability $R(q)$ of the load being greater than or equal to $q$, it may often be sufficient to assume a simple rectangular distribution.
$R(q) = \frac{q_2 - q}{q_1 - q_1}$

If the probability $P_1(q)$, say, of the bearing capacity being less than or equal to $q$ were known, the influence of the variability of the loads could be taken into account to give the joint probability of failure $P_2(q)$, say, where

$$P_2(q) = \int P_1(q) \frac{dR}{dq} dq.$$  

**Value of Soil Properties**

A rational choice of a parameter for design can be made from the mean value or mean trend line provided that the form of the variance is known, and the probability $P$ or "risk" that the parameter is anywhere less than a certain value can easily be calculated if the parameter follows the normal distribution. Three cases of differing form of the variance will be discussed, corresponding to the cases shown in Figure 15.

**Case 1.** Parameter $v$ normally distributed about mean value $\mu$, standard deviation $\sigma$ independent of depth

$$v = \mu + \sigma \xi = \mu(1 + V_1 \xi), \quad V_1 = \sigma / \mu.$$  

Example: tan $\phi$ for silty sand.

**Case 2.** Parameter $v$ normally distributed about linear trend $a + bz$; standard deviation $\sigma$ independent of depth.

$$v = a + bz + \sigma \xi = a(1 + V_2 \xi) + bz, \quad V_2 = \sigma / a.$$  

Example: Compression index for sandy clay.

**Case 3.** Parameter $v$ normally distributed about linear trend $a + bz = b \xi'$. Standard deviation increasing with depth $\sigma = dz'$

$$v = b \xi' + dz' \xi = b(1 + V_3 \xi') \xi', \quad V_3 = d / b.$$  

Example: Cohesion of marine clay.

$V_1, V_2, V_3$ are coefficients of variation and $\xi$ is the standardized normal variate.

Table V gives selected values of $\xi$ corresponding to specific risks.$P$, and for
the various soils and properties discussed earlier the values of means, standard deviations, and coefficients of variation are given in Table VI.

### TABLE V
Risk and Standard Normal Variate

<table>
<thead>
<tr>
<th>Risk, per cent</th>
<th>50</th>
<th>10</th>
<th>1</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standardized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>normal variate</td>
<td>0</td>
<td>-1.28</td>
<td>-2.33</td>
<td>-3.09</td>
<td>-3.72</td>
<td>-4.26</td>
<td>-4.75</td>
<td>-5.20</td>
</tr>
</tbody>
</table>

As an illustration, the design value of the drained angle of shearing resistance of the silty sand would be given by $\tan \phi_d = 0.692 + 0.0958 z$ and for different risks could be taken as at least equal to the following values.

- **Risk %**: 50, 0.1, $10^{-4}$
- **$\tan \phi_d$**: 0.692, 0.396, 0.227
- **$\phi_d$**: 34.7, 21.6, 13.4

### TABLE VI
Means, Deviations, and Coefficients of Variation

<table>
<thead>
<tr>
<th>Soil</th>
<th>Property</th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marine clay</td>
<td>Cohesion lb./sq. ft.</td>
<td>6.39$(z + 6)*$</td>
<td>1.21$(z + 6)$</td>
<td>0.184 (3)</td>
</tr>
<tr>
<td>London clay</td>
<td>Cohesion lb./sq. ft.</td>
<td>141z</td>
<td>22.8z</td>
<td>0.162 (3)</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>Compression index</td>
<td>0.139-7.5 $\times 10^{-4}z$</td>
<td>0.0354</td>
<td>0.255 (2)</td>
</tr>
<tr>
<td></td>
<td>$\log_6(c, 10^{-4} \text{sq.in./min.})$</td>
<td>1.185</td>
<td>0.405</td>
<td>0.342 (1)</td>
</tr>
<tr>
<td>Silty sand</td>
<td>$\tan \phi_d$</td>
<td>0.692</td>
<td>0.0958</td>
<td>0.138 (1)</td>
</tr>
<tr>
<td>Clayey silt</td>
<td>$\tan \phi_d$</td>
<td>0.597</td>
<td>0.0881</td>
<td>0.148 (1)</td>
</tr>
<tr>
<td>Site A</td>
<td>Cohesion lb./sq. ft.</td>
<td>1820</td>
<td>576</td>
<td>0.316 (1)</td>
</tr>
<tr>
<td>Site B</td>
<td>Cohesion lb./sq. ft.</td>
<td>1620</td>
<td>435</td>
<td>0.239 (1)</td>
</tr>
</tbody>
</table>

* $z$ is the depth below ground surface in feet.

†(1), (2), and (3), represent the form of variance. See Figure 15.

**Factor of Safety and Associated Risk**

The conventional approach to bearing capacity estimates is to apply a factor of safety to the theoretical ultimate bearing capacity. This factor of safety can be defined in two ways, either as a factor applied to the load or as a factor applied to the strength, and in general the two definitions do not agree.

Suppose the theoretical ultimate bearing capacity $q_u = f(s)$, where $f(s)$ is a known function of the strength $s$, the actual load is $q$, and the mean soil strength is $s_e$. The factor of safety on load $F_q$ is then given by

$$F_q q = q_u = f(s_e).$$
while the factor of safety on strength \( F_2 \) is given by

\[
q = f(s_0/F_2)
\]

and the two factors are not the same unless \( f(s) \) is linear in \( s \).

For a purely cohesive soil whose mean strength and standard deviation are both either independent or linearly proportional to the depth (cases 1 and 3), then \( F_1 \) and \( F_2 \) are identical and the factor of safety can be related to the risk \( P \) through the equation

\[
F = (1 + V\xi)^{-1}
\]

where \( V \) is the associated coefficient of variation \( V_1 \) or \( V_2 \).

For a purely frictional soil whose mean strength and standard deviation are independent of depth, the factors of safety, based on Prandtl's solution, are

\[
F_1 = N_1 \exp(\pi\sigma)
F_2 = \tan\phi_0/[\tan\phi - (1/\pi\log N_2)].
\]

where

\[
\tan\phi = \tan\phi_0 + \sigma\xi
N_1 = [\tan(\pi/4 + \phi_0/2)/\tan(\pi/4 + \phi/2)]^2
N_2 = [\tan[\pi/4 + \frac{1}{2}\tan^{-1}(\tan\phi_0/F_2)]/\tan(\pi/4 + \phi/2)]^2.
\]

For a cohesive-frictional soil whose strength may be taken as

\[
s = c_0(1 + V\xi) + \rho \tan\phi_0,
\]

that is, the variance is confined to the cohesion term, the factors of safety, again based on Prandtl's solution, are

\[
F_1 = \frac{\rho_0 + c_0 \cot\phi_0}{\rho_0 + c_0(1 + V\xi) \cot\phi_0}
F_2 = (1 + V\xi)^{-1}
\]

and \( F_1 \) only equals \( F_2 \) for a surface foundation, when \( \rho_0 = 0 \).

For a given value of \( F \) the associated risk determined from the equations above will be the maximum risk, since the influence of non-linear strength variations on the form of \( f(s) \) is ignored. Furthermore, the load \( q \) is generally taken to be the maximum load \( q_2 \).

For equal numerical values of \( F_1 \) and \( F_2 \) the risk is greater for the factor on load than on strength (unless the two are identical); consequently it would seem preferable to use a factor on strength in design.

Some general conclusions can be drawn from Figure 16, which shows the relation between risk and safety factor for the various soils considered. The usual safety factor employed in foundation design is between 3 and 4, and since this is found to be satisfactory in practice it follows that the actual risk of failure is sufficiently small. For the marine clay and London clay this range in safety factor is equivalent to a risk of \( 10^{-2} \) to \( 10^{-3} \) per cent, and consequently a design risk of this order can be regarded as quite adequate.
It is common knowledge that foundations on sandy soils with a factor of safety of 3 to 4 never fail through inadequate bearing capacity, and hence the risk must be very small indeed. This is confirmed by the results for silty sand (based on a factor on strength) where the risk is of the order of $10^{-4}$ to $10^{-6}$ per cent. For a reasonable risk the factor of safety could quite well be reduced to 2 to 3.

For the clayey silt the risk at usual safety factors is very high, 2 to 0.1 per cent. However, no bearing capacity failures are known to have occurred on this soil, which implies that the calculated risk is too conservative. This may be due to inaccuracy of the assumption that the variance is confined to the cohesions, the assumption of normality, or to inaccuracy in the use of Prandtl’s solution, but is more likely to be due to the fact that in actual design the safety factor is much larger than the nominal 3 to 4. At Sites A and B the ultimate bearing capacity for a surface foundation is about 25 tons/sq. ft. and the usual building loads are only of the order of 2 to 3 tons/sq. ft.; consequently the true safety factor is of the order of 10.

**Estimates of Settlement Parameters**

A rather simpler approach can be made with regard to settlement estimates where the concept of factor of safety does not apply. Perhaps the best method is to calculate the probability that the settlements will lie within a certain range. If the mean settlement $h_0$ is calculated using the mean value of the compression index, the probability $Q$ that the actual settlement lies between
\( h_0 (1 \pm 1.645) = Q = 1 - 2P \); where \( P \) and \( \zeta \) have the usual meaning, and \( V \) is the coefficient of variation of the compression index.

For the case of the sandy clay the following results are obtained:

<table>
<thead>
<tr>
<th>Q per cent</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>95</th>
<th>99</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>0.675</td>
<td>1.150</td>
<td>1.645</td>
<td>1.960</td>
<td>2.576</td>
</tr>
<tr>
<td>( V )</td>
<td>0.172</td>
<td>0.294</td>
<td>0.420</td>
<td>0.500</td>
<td>0.636</td>
</tr>
</tbody>
</table>

Thus, there is a 90 per cent probability that the settlement will be between 0.58\( h_0 \) and 1.42\( h_0 \).

Similarly, for the rate of settlement, upper and lower bounds can be placed on the coefficient of consolidation, bearing in mind that this parameter follows the log-normal distribution. Thus, again for the sandy clay, there is a 90 per cent probability that the coefficient of consolidation lies between \( 7.1 \times 10^{-4} \) and \( 3.3 \times 10^{-4} \) sq. in./min.

**Acknowledgments**

Acknowledgments are gratefully made to the Research Grants Committee of the University of Hong Kong for financial assistance, and to the Director of Public Works, Hong Kong, Messrs. Binnie & Partners, and Messrs. Scott & Wilson, Kirkpatrick & Partners, for providing the test results for the marine clay and the alluvial sandy clay.

**Appendix: Numerical Example of \( \chi^2 \) Test**

As an illustration the actual distribution of the liquidity index of the marine clay will be compared with the normal distribution. The liquidity index results are listed in order of rank in Table A.I.

**Table A.I**

<table>
<thead>
<tr>
<th>1.020</th>
<th>.936</th>
<th>.918</th>
<th>.885</th>
<th>.870</th>
<th>.854</th>
<th>.834</th>
<th>.821</th>
<th>.800</th>
<th>.753</th>
</tr>
</thead>
<tbody>
<tr>
<td>980</td>
<td>.934</td>
<td>.909</td>
<td>.882</td>
<td>.870</td>
<td>.854</td>
<td>.832</td>
<td>.816</td>
<td>.787</td>
<td>.751</td>
</tr>
<tr>
<td>981</td>
<td>.934</td>
<td>.909</td>
<td>.882</td>
<td>.870</td>
<td>.850</td>
<td>.831</td>
<td>.815</td>
<td>.781</td>
<td>.740</td>
</tr>
<tr>
<td>980</td>
<td>.932</td>
<td>.904</td>
<td>.879</td>
<td>.869</td>
<td>.848</td>
<td>.831</td>
<td>.814</td>
<td>.779</td>
<td>.745</td>
</tr>
<tr>
<td>966</td>
<td>.922</td>
<td>.890</td>
<td>.879</td>
<td>.866</td>
<td>.845</td>
<td>.829</td>
<td>.814</td>
<td>.778</td>
<td>.731</td>
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<td>.922</td>
<td>.897</td>
<td>.878</td>
<td>.865</td>
<td>.846</td>
<td>.830</td>
<td>.814</td>
<td>.769</td>
<td>.731</td>
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<td>950</td>
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<td>.811</td>
<td>.767</td>
<td>.729</td>
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<tr>
<td>950</td>
<td>.929</td>
<td>.893</td>
<td>.875</td>
<td>.861</td>
<td>.840</td>
<td>.826</td>
<td>.810</td>
<td>.766</td>
<td>.724</td>
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The mean and standard deviation of this set of 120 results are calculated as \( \mu = .832 \), and \( \sigma = .0690 \).

The theoretical divisions of the ranges for the \( \chi^2 \) test are found from \( \nu = \mu + \sigma \) where \( \zeta \) is the standardized normal variate. Taking \( k = 20 \) and equal theoretical probabilities of 0.05 for each range, Table A.II below gives the values of \( P, \zeta, \nu \), and the actual observed number \( n \), for each range.
TABLE A.11

<table>
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<tr>
<th>$P$</th>
<th>$\gamma$</th>
<th>$\nu$</th>
<th>$n_i$</th>
<th>$n_i^2$</th>
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<td>−1.645</td>
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<td>64</td>
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<td>42½</td>
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<td>.924</td>
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<tr>
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</tbody>
</table>

Sum 120 866½

$$\chi^2 = \sum_{i=1}^{k} (n_i^2 - m_i^2) / m_i = (k/N) \sum_{i=1}^{k} n_i^2 - N$$

$$= \frac{866\frac{1}{2}}{6.0} - 120 = 24.4$$

In calculating $\chi^2$ three parameters $N$, $\mu$, and $\sigma$ are required, hence $\nu = k - r = 20 - 3 = 17$.

From published tables the probability $P(\chi^2)$ that $\chi^2$ is less than or equal to 24.4 with 17 degrees of freedom is 10.9 per cent.

References


