Application of the wave equation analysis to friction piles in sand

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Based on the information gathered during a design pile testing program and the subsequent construction of a large pile foundation in Quebec City, the potential and limitation of the wave equation method for analyzing pile driving behaviour are demonstrated.

The application of the wave equation method to instrumented precast concrete piles driven under well-controlled conditions leads to excellent predictions both in terms of driving stresses and of ultimate bearing capacities.

The same high quality of prediction is obtained for a steel H-pile and a precast concrete pile driven incrementally to a depth of 53 ft (16.2 m) in a uniform sand deposit. However, proper simulation of the joints is required for the precast concrete pile.

On the contrary, a very poor correlation between computed and observed ultimate capacities is obtained for the production piles. This is attributed to the poor reliability of the blow counts used as input data in the classical wave equation method of analysis.

A partir des informations recueillies lors d'un programme d'essais de design et lors de la construction d'une fondation sur pieux à Québec, on démontre le potentiel et les limites de la méthode de la 'wave equation' pour analyser le comportement en battage des pieux.

L'application de la méthode de la 'wave equation' à l'analyse de pieux de béton instrumentés battus dans des conditions bien contrôlées conduit à des prédictions excellentes des contraintes de battage aussi bien que des forces portantes.

La même bonne qualité de prédiction est obtenue pour un pieu H en acier et un pieu de béton préfabriqué en étales jusqu'à 53 pieds (16.2 m) dans un dépôt de sable uniforme. Toutefois, une simulation adéquate des joints est nécessaire dans le cas du pieu de béton préfabriqué.

Au contraire, une corrélation de très mauvaise qualité est obtenue entre les capacités portantes prédites et observées des pieux de production. Ceci est attribué à la fiabilité très faible des nombres de coups de battage utilisés comme donnée de base dans l'analyse classique par 'wave equation'.


Introduction

Following the study of pile driving impact by Smith (1950), the use of the theory of wave propagation for the analysis of pile driving has gradually developed to result in the formal proposition of the so-called wave equation method for analyzing the driving behaviour of piles and for predicting their bearing capacity (Smith 1960). Extensive research was carried out in the period of 1960 to 1970 to develop the method and in particular to establish the techniques for modeling the driving system, the pile, and the soil. Since 1970, the wave equation method has found an increased application in the United States for both onshore and offshore projects. However, and for unclear reasons, the method is used only on a very limited scale in Canada. While the fundamental aspects of the wave equation method have been the object of numerous papers, relatively little data have been published on the application of this method to production piles so that the potential and limitations of the wave equation when applied to real field situations are not clearly established. It is the purpose of the present study to provide such information in the case of precast concrete piles driven in sand by means of free-fall hammers.

The construction of two 12 500 ft (3800 m) long retaining walls on both sides of the Saint-Charles River in downtown Quebec City implied the design and construction of a major
pile foundation in a sand deposit of medium density. The design of this foundation was based on the results of an extensive pile driving and test loading program, in which the variation of the driving resistance and the static bearing capacity with the depth of embedment in sand were determined for four types of piles and in particular for a 12 BP steel H-pile and for an H-800 Herkules precast concrete pile. Details on the soil properties at the site, on the pile testing program and on the results obtained have been presented by Tavenas (1971).

The hexagonal precast concrete piles were finally selected for the foundation and their length was specified in accordance with the results of the test program. In order to ensure that the necessary quality of the production piles, the contract documents required the recording of the driving resistance on all piles as well as the test loading of 39 piles, selected at random by the engineer. As a result, extensive information was gathered on the driving behaviour of the piles as well as on eventual correlations between the length of pile, the final driving resistance and the static bearing capacity of precast concrete piles driven by free-fall hammer in medium dense sand. These results were used to determine the limitations of the usual pile driving formulas by Tavenas and Audy (1972). The same set of data can now be used to investigate the capabilities and limitations of the wave equation method of pile analysis.

The investigation reported herein has been divided into three sections:

1. Using the observations made on four production piles which had been instrumented for stress measurement (Tavenas and Audy 1972), a brief parametric study was carried out to evaluate the influence of the input parameters on the driving stresses and the bearing capacity. The 'most representative' values were selected as those producing the best fit between observed and computed driving stresses and bearing capacities. The values were then used in the remainder of the investigation.

2. The wave equation method was applied to a steel H-pile and a Herkules precast concrete pile driven and tested under well controlled conditions during the design testing program. In addition to the calibrated model resulting from the first step of the study, an analysis was also made assuming typical limit values of the input parameters to estimate the possible accuracy of bearing capacity predictions in cases where no detailed study of such parameters could be made.

3. Finally, the quality of the bearing capacity predictions for 39 production piles test loaded to failure was evaluated by comparing the wave equation results to the observations made on the production piles. This case is thought to be more representative of standard field conditions where driving conditions as well as quality controls were less uniform.

Characteristics of the Project

Geotechnical Conditions

The geotechnical conditions prevailing on the Saint-Charles River site are very uniform. They have been described in detail by Tavenas et al. (1970), Tavenas (1971), and Tavenas and Audy (1972). A typical soil profile is shown in Fig. 1. The sand layer which is the main feature of this deposit has been proven uniform over the entire project area, both in terms of gradation and of density. The natural backfill shown on Fig. 1 was excavated prior to piling and was replaced by a 15 ft (4.5 m) thick layer of uncompacted 0-10 in. (0-10 cm) crushed stone. This uniformity of the soil conditions first made it possible to design the foundation assuming that the results obtained during the test program carried out at a central location were applicable to piles at any location.
tion on the site; it further allows us to consider the observations made on the production piles over the entire project area as a homogeneous set of data for the purpose of the present investigation.

Test Program During the Design Stage

The pile driving and test loading program carried out in 1968 during the design stage has been described by Tavenas (1971). It was devised so as to yield the following design information:

1. The relative efficiency of four types of piles, 12 in. (30 cm) diameter timber pile, 12 in. (30 cm) nominal diameter cast-in-place Franki pile, 12 BP steel H-pile and H-800 Herkules precast concrete pile.

2. The variations of the bearing capacity and the driving resistance with the length of pile in sand for the timber pile, the steel H-pile, and the precast concrete pile.

For these two latter types of piles the investigation was very comprehensive; each pile was successively driven to and test loaded at depths of 3, 13, 23, 33, 43, and 53 ft (0.9, 4.0, 7.0, 10.0, 13.1, and 16.2 m) in the sand deposit.

The driving and test loading conditions were kept uniform during the entire program. The piles were driven with a free-fall hammer, the characteristics of the hammer and the driving cap were determined prior to driving, the cap-block and the cushion were checked prior to each driving sequence and were changed whenever necessary. The driving was carried out at a slow rate and a constant control was kept on the drop height and the set. Consequently, the measured blow counts were representative of the energy delivered to the piles throughout the driving operation. All load tests were started within 12 h after the end of driving and were conducted up to failure following a uniform procedure. The results were interpreted by the same person using the same criterion to determine the failure load of all piles. Thus the uniformity in test conditions was fully ensured.

Instrumented Piles During Construction

The piles selected for the construction were hexagonal precast concrete piles similar to the Herkules H-800 pile in both geometry and mechanical properties. The piles, with lengths varying from 28 to 44 ft (8.5 to 13.4 m), were produced at an industrial plant in Quebec City.

To further study the behaviour of the piles during driving and test loading, four piles were instrumented. The details of the instrumentation are given by Tavenas and Audy (1972). At various stages during the driving of these piles, measurements of the distribution of driving forces along the pile were made. The piles were test loaded up to failure about 10 days after driving.

Production Piles

All production piles were driven by means of free-fall hammers. Two to four driving rigs were in operation on the site. While the characteristics of the hammers and caps were known, the properties of the cap-blocks and cushions were certainly variable with time and from pile to pile, as is generally the case on a piling job. The hammers were dropped by the usual method of releasing the clutch on the driving machine so that the actual drop heights can be expected to have varied from blow to blow. The blow counts were taken on a continuous basis but full records were established only for 5% of the piles. Thirty-nine tests were carried out on piles selected at random by the engineer. The same test loading method, as used during the initial test program, was applied to the production piles, and the results were interpreted in the same manner. However, the tests were carried out after delays which varied from 3 to 56 days after the end of driving.

The information gathered during the construction of this piled foundation can be considered typical of any normal field situation and can be used to evaluate the applicability of any design method to usual projects.

The Wave Equation Method

History

In the early stages of development of the piling practice, it was admitted that the laws of the Newtonian impact would apply to pile driving, and the so-called pile driving formulae were established on the assumption that the energy delivered by the hammer would be immediately transmitted to the tip of the pile at impact. Experience proved the results yielded by these formulas to be very variable in quality and numerous attempts were made
to establish 'improved' formulas of increasing complexity. However, all these attempts remained unsuccessful due to the failure to recognize the fallacy of the basic assumption of a Newtonian impact used in all cases. Isanove (1931) was the first to point out that the energy transmission from the hammer to the pile tip was not instantaneous at impact, but rather that a wave action occurred in the hammer, after the impact, so that the behavior of the hammer-pile-soil system during driving would have to be analyzed by applying the theory of wave propagation. While the mathematics of this theory was well known, the solution of the pile driving problem remained impracticable until 1950 when the development of computers made the numerical solution of the wave equation possible within an acceptable amount of time and effort. It is unfortunate that, during this 20 year period much effort continued to be put in the development of the pile driving formulas in spite of their fundamental limitation.

When a hammer strikes a pile, an elastic wave is generated at the pile head. With time, this strain progresses longitudinally through the pile down to its tip where the soil is displaced to produce the pile penetration or set. The propagation of the strain wave is governed by the one-dimensional wave equation:

$$\frac{\partial^2 D}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 D}{\partial x^2} + R$$

where $\rho$ and $E$ are the mass density and the Young's modulus of the pile material, $D$ the displacement of the pile in the longitudinal direction $x$, and $R$ the external resistance to the motion of the pile. Smith (1959) presented a first numerical solution of this equation, using a simple model for the hammer-pile system, to analyze the pile displacements under a hammer blow. The techniques for simulating the driving cap and the soil resistance were relatively crude at that stage. In a second paper Smith (1955) applied the method to the analysis of impacts between elongated structural elements and considering the problem of simulating cushions. In 1957 Smith presented the results of wave equation analysis for predicting the variations of pile bearing capacities with driving resistance; he concluded that "the acquired ability to solve the wave equation as applied to pile driving offers a means of obtaining a truly mathematical solution". Finally Smith (1960) presented a comprehensive treatment of the application of the wave equation method to the analysis of pile driving. This paper, which is still the basic reference on the problem, presents not only the fundamentals of the method but also a discussion of the simulation of the hammer, the driving cap with capblock and cushion, the pile, and the soil. In particular Smith defines in this paper all the parameters involved in the simulation and gives typical values resulting from his experience in applying the wave equation method to a variety of pile driving case histories. Most of the material presented by Smith (1960) can be used directly in the solution of practical problems. The reader is referred to these references for a detailed description of the wave equation method.

Many investigations on the use of the wave equation method have been carried out since then. They were generally aimed at the analysis of the parameters involved in the simulation of hammers, capblocks, cushions, and soils. Forehand and Reese (1964) have presented a good summary on the simulation of the soil behavior; they show how the soil quake and damping vary with different soil types and densities and how these variations affect the results of the analysis. In a comprehensive study carried out at Texas A & M University, all aspects of the problem have been considered, and the resulting state of the art report by Lowery et al. (1969) may be used as a basic reference.

Recently, Rempe (1975) and Goble and Rausche (1976) have solved the problem of accurate modeling of the explosive force and steel on steel impact occurring in diesel hammers.

**Computer Program**

The computer program used in the present analysis is that proposed by Edwards (1967). The original program includes a provision for simulating the effect of discontinuities such as joints or tension cracks in the pile on the transmission of tensile forces. As discussed later, this part of the program was modified to also take into account the effect of such discontinuities on the transmission of compressive forces, in order to simulate properly the be-
haviour of mechanical joints in precast concrete piles.

**Modeling of the Driving System**

The driving system used throughout this investigation consisted of a free-fall hammer and a driving cap with cap-block and cushion.

The hammers used were short so that their own elasticity could be neglected. They were therefore simulated as a mass \( M(1) \) falling from a height \( H \) to produce the driving energy with an efficiency, \( e \). In the absence of any direct measurement, the usual value of \( e = 75\% \), proposed for free-fall hammers in the literature, was first used. However, the analysis of the instrumented piles indicated that an efficiency of \( 85\% \) was more representative of the system, and this second value was used for the entire investigation.

The driving cap consisted of a steel helmet weighing about 1500 lb (6.67 kN), carrying a cap-block of dry hardwood. The cap-block had an initial height of 6 in. (15 cm) and an area of about 124 in.\(^2\) (800 cm\(^2\)). To compute the stiffness \( K(1) = A b/L \) of the cap-block, the common modulus value of 45,000 psi (315 kPa) proposed by Lowery et al. (1969) was first used, to obtain \( K(1) = 930,000 \) lb/in. (170,000 kN/m). The stiffness \( K(1) \) was then varied as part of a limited parametric analysis on the instrumented piles, and a higher value of \( K(1) = 1,500,000 \) lb/in. (260,000 kN/m) was finally selected as more representative. The coefficient of restitution of the cap-block was taken equal to 0.5 as proposed by Lowery et al. (1969).

A cushion was placed between the helmet and the pile head when driving precast concrete piles. This cushion consisted of masonite plates with an area of 124 in.\(^2\) (800 cm\(^2\)) and a total initial thickness of 1 in. (2.5 cm). The properties of masonite were assumed similar to those of asbestos discs and were taken from Lowery et al. (1969). Again here, the resulting stiffness of the cushion was varied in a parametric analysis, and a lower value was finally retained. This resulted in a combined stiffness \( K(2) \) of the cushion and the first pile element \( 2,000,000 \) lb/in. (350,000 kN/m). The coefficient of restitution of the cushion was chosen as 0.3.

**Modeling the Pile–Soil System**

In any wave equation analysis the pile is divided into segments and is simulated as a series of masses, representing the weight of the segments, connected by springs, representing the stiffness of the segments. If the pile is jointed, a slack, corresponding to the possibility of looseness of the joint, is specified at the juncture between two segments on each side of the joint. The restraint offered by the soil at the periphery of each pile segment or hole, the tip of the pile, is modeled as an elastic-plastic model. The yield displacement is called the quake \( Q \), while the energy dissipated in the soil is represented by a damping constant \( J \). The amounts of point resistance and friction mobilized are specified in terms of percentages of the total driving resistance.

Figure 2 shows the model of one of the piles considered in the present study.

The piles were generally divided into 5 or 6 (1.5 m) long segments. The precast concrete pile weight and stiffness were determined from direct measurements on 3 ft (0.9 m) long test specimens, cast, cured, and stored in the same...
way as the actual piles. The Young's modulus of the precast concrete piles averaged 40,000,000 psi (28,000 kPa).

The soil quake and damping are difficult to measure directly. Fortunately their influence on the results of the analysis was shown to be very limited by various investigators. It is thus the accepted practice to take the values of quake and damping from the tables published by Smith (1960) or by Forehand and Reese (1964). For piles driven in sand, it is generally agreed to use a quake of 0.1 to 0.15 in. (0.25 to 0.4 cm) for both the point and side resistance. Similarly the point damping constant $J$ is commonly taken equal to 0.15 s/ft (0.4 s/m) and the side damping constant $J'$ as 1.3 of $J$. The ratio between point resistance and skin friction can be evaluated from the classical bearing capacity formulas; the influence of this parameter on the computed driving resistance will be shown during the analysis of the test piles.

The final values of the simulation parameters used throughout the present study are summarized in Table 1.

**Table 1. Parameters used in the wave equation analysis**

<table>
<thead>
<tr>
<th>Drop hammers</th>
<th>Driving up</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight $W(1) = 5580$ to $7800$ lb (24.8 to 34.7 kN)</td>
<td>Weight $W(2) = 1500$ lb (6.67 kN)</td>
<td>Point quake $Q = 0.15$ in. (0.40 cm)</td>
</tr>
<tr>
<td>Drop height $H = 1.25$ to $3.5$ ft (0.38 to 1.08 m)</td>
<td>Capblock stiffness $K(1) = 1500000$ lb/ft. (260,000 kN/m)</td>
<td>Side quake $Q' = 0.15$ in. (0.40 cm)</td>
</tr>
<tr>
<td>Efficiency $e = 85%$</td>
<td>Capblock restitution $ERES(1) = 0.5$</td>
<td>Point damping $J = 0.15$ s/ft (0.44 s/m)</td>
</tr>
<tr>
<td>Cushion stiffness $K(2c) = 3000000$ lb/ft. (250,000 kN/m) (to be combined with the stiffness $(K(2) - p)$ of the first pile element).</td>
<td>Cushion restitution $ERES(2) = 0.3$</td>
<td>Side damping $J' = 0.05$ s/ft (0.15 s/m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Point resistance $R_{op} = 20%$ to $80%$ of $R_{moen}$ (most representative value $= 40%)$</td>
</tr>
</tbody>
</table>

The agreement between predictions and observations is remarkable. While such agreement was to be expected for pile A2 for which the wave equation model was calibrated, it is equally good for piles C2 and C3 in spite of the differences in pile length, depth in sand and hammer weights, thus demonstrating the quality of the analysis.

It is also interesting to note that the computed bearing capacities increase linearly with depth down to a depth of about 20 ft (6 m) in the sand deposit corresponding to 20 pile diameters. Below this depth, the computed driving resistances remain more or less constant. Thus it would appear that the concept
of critical depth proposed by Vesic (1967) and supported by the results presented by Tavenas (1971) governs not only the static bearing capacity of piles in sand but also their driving resistance, the critical depth being the same in both cases.

Driving Stress Predictions

Figure 4 shows the distributions of driving stresses computed at two stages of penetration of pile A2. Also shown on the figure are the corresponding stresses observed in the pile. The agreement in both magnitude and distribution is good but some scattering can be observed, which may be possibly attributed to the accuracy of the stress measuring system. Similar results were obtained for all instrumented piles.

The driving stresses computed at the different stages of penetration of piles A2, C1, and C3 are compared to the corresponding stress observations on Fig. 5; the data for pile C2 were not used here because of the 4/1 batter of this pile which resulted in erratic stress measurements. As can be seen, practically all data are within ±30% of a perfect correlation.

Considering the difficulty associated with dynamic stress measurements in a composite pile section, the quality of the stress prediction from the wave equation analysis can be considered very good. The tight control of the driving conditions has certainly contributed to the quality of the stress predictions; under normal field conditions where such tight controls cannot be maintained, more important deviations between the computed and the observed stresses should be expected.

Analysis of the Test Piles II and J

As already mentioned the pile driving and test loading program carried out as part of the design included the analysis of a steel H-pile (referred to as pile H) and a Hercules precast concrete pile type H-800 (referred to as pile J), driven and test loaded in succession to depths of 3, 13, 23, 33, 43, and 53 ft (0.9, 4.0, 7.0, 10.0, 13.1, and 16.2 m) into the sand deposit. The details of this program were presented by Tavenas (1971). The two piles have been analyzed at all stages of penetration by means of the wave equation method.

Model and Input Parameters

Both piles consisted of a first 20 ft (6 m) long section and of additional 10 ft (3 m) long sections. For the purpose of the analysis the piles were divided into 5 ft (1.5 m) segments so that the joints in the piles could be easily simulated.

The drop hammer used to drive the test piles had a weight of 6300 lb (28 kN), and the drop height was varied from 1.25 ft (0.4 m) for the driving of the first two sections of the concrete pile, to 2 ft (0.6 m) for the driving of the last concrete pile section and to 2.33 ft (0.7 m) for the steel H-pile. The driving cap had a weight of 1300 lb (5.8 kN) and carried a 6 in. (15 cm) high capblock of hardwood. A 1 in. (2.5 cm) thick masonite cushion was used when driving the precast concrete pile. The input parameters for the driving system were taken equal to those resulting from the first analysis and given in Table 1.

In order to evaluate the potential of the wave equation method for the prediction of pile capacities in the common cases where no preliminary information on the soil parameters
Based on the actual measurements made during the driving and loading tests, assign quake values varying from 0.07 to 0.20 in. (0.18 to 0.51 cm), and point resistance contribution values decreasing with the increasing depth of pile in sand. The damping constants were maintained at the standard values.

The first two sets of input parameters should give the limits and the last set the most probable value of the bearing capacity of the considered piles.

**Steel H-Pile**

The steel pile used in the program was a 12 BP 74. Its properties were taken from the mill specifications. Since the successive pile sections were connected by welding, no joints were simulated in the analysis.

The load test results had indicated that the relative importance of point resistance and skin friction was more or less constant at all depths and the following input parameters were selected for the precise analysis:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Depth in sand (ft)</th>
<th>Quake (in.)</th>
<th>Point resistance contribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-1</td>
<td>2</td>
<td>0.07</td>
<td>40</td>
</tr>
<tr>
<td>H-2</td>
<td>12</td>
<td>0.19</td>
<td>40</td>
</tr>
<tr>
<td>H-3</td>
<td>22</td>
<td>0.13</td>
<td>45</td>
</tr>
<tr>
<td>H-4</td>
<td>32</td>
<td>0.15</td>
<td>46</td>
</tr>
<tr>
<td>H-5</td>
<td>42</td>
<td>0.18</td>
<td>41</td>
</tr>
<tr>
<td>H-6</td>
<td>52</td>
<td>0.20</td>
<td>37</td>
</tr>
</tbody>
</table>

would be available, the following three sets of input soils parameters were considered:

Assume for the quake and damping the values proposed by Smith (1960) and assign to the point resistance contribution the minimum value of 20% of the total resistance.

With the same quake and damping assign to the point resistance contribution the maximum of 80%.
Figure 6 shows a typical series of results in terms of the variations of computed ultimate bearing capacities $RU$ with the driving resistance in blows per inch ($BP$), for the three sets of assumptions. By plotting the observed driving resistance on these curves one obtains the predicted ultimate bearing capacity of the pile. In the case presented on Fig. 6, the minimum and maximum capacities are equal to 72 and 85 t (641 and 783 kN) respectively, resulting in an average variation of $\pm 10\%$. The capacity obtained from the precise input parameters is equal to 82 t (730 kN), i.e. nearly identical to the measured capacity of 80 t (712 kN).

The same type and quality of results has been obtained for the first four tests, as shown on Fig. 7. The variation between the minimum and maximum computed capacities is in the order of $\pm 10\%$, and the capacities obtained from the precise input parameters are within 5 t (45 kN) of the observed capacities. Larger deviations between the computed and observed capacities can be noted for the tests at greater depth in the sand, the computed capacity being too high by 11 t (98 kN) or 10% for test H-5, and too low by 39 t (347 kN) or 27% for test H-6. A change in the behaviour of pile H had been observed during the interpretation of the load tests and Tavenas (1977) has suggested the following explanation: in the first four stages of driving the soil within the pile flanges was carried down with the plug which was also effective during the static load tests; at a depth in excess of 40 ft (10.5 to 12 m) this plug was dislocated during driving, thus causing the pile to act as a non-displacement pile, probably during driving and possibly also during the static load tests. The present results would support this explanation, in particular for test H-6: the low predicted capacity would correspond to a non-displacement pile with a small point surface resistance, while the measured high capacity would result from the formation of a new plug causing the mobilization of a high point resistance.

In spite of this local problem, the bearing capacity predictions obtained from the wave equation analysis are excellent for the steel H-pile.

**Herkules Precast Concrete Pile H**

The Herkules H-800 precast concrete pile consisted of a first 20 ft (6 m) long section and of additional 10 ft (3 m) long sections, the connections being made with the standard Herkules mechanical joint. As already noted, the properties of the pile were determined by direct measurement in the laboratory.

The load test results had indicated a regular decrease of the point resistance contribution with the depth of pile in sand, and the following input parameters were selected for the precise analysis:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Depth in sand (ft)</th>
<th>Quake (in.)</th>
<th>Point resistance contribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-1</td>
<td>3</td>
<td>0.07</td>
<td>80</td>
</tr>
<tr>
<td>J-2</td>
<td>13</td>
<td>0.10</td>
<td>59</td>
</tr>
<tr>
<td>J-3</td>
<td>23</td>
<td>0.13</td>
<td>51</td>
</tr>
<tr>
<td>J-4</td>
<td>33</td>
<td>0.15</td>
<td>43</td>
</tr>
<tr>
<td>J-5</td>
<td>43</td>
<td>0.18</td>
<td>38</td>
</tr>
<tr>
<td>J-6</td>
<td>53</td>
<td>0.20</td>
<td>33</td>
</tr>
</tbody>
</table>

In a first analysis, the mechanical joints in the pile were not simulated. The predicted minimum, maximum, and probable bearing capacities for the six tests are shown on Fig. 8. While the predictions agree with the observation for the first pile section, the measured
ultimate bearing capacities are consistently lower than the predictions for all other cases, and the difference between predictions and observations increases with the length of pile in sand. Since the pile tested in the first test was the only one without joint, the poor agreement between predictions and observations for all tests but the first was thought to be the effect of the joints on the driving behaviour of the pile. As a next step, the joints were taken into account by assuming that they would allow immediate transmission of the compression force wave, but that the tensile forces would be transmitted from one pile section to the other only after a given slack would have been taken in the joint. The original computer program had a provision for the simulation of this type of joint effect. The magnitude of the slack was taken equal to the specified construction tolerance of the Herkules joint at the time of the test, i.e. 0.03 in. (0.75 mm). The bearing capacities computed from this new analysis were practically identical to those of the first analysis without joint simulation. This simple joint simulation was thus not representative of the field behaviour.

The Herkules piles used in the test program were equipped with a central inspection pile.
In order to keep the pile clean and dry, the joints were fitted with a rubber O-ring, so that the two faces of the pile joint were not in solid contact initially. As a consequence it is most likely that, during driving, the initial gap in the joints had first to be closed before any transmission of the compressive force wave to the lower pile element could occur. The computer program was thus modified to allow for a slack effect in both tension and compression in each joint, called here a bidirectional slack.

In order to evaluate the effect of a joint with a bidirectional slack, a detailed analysis of a pile with one joint was carried out. The joint was located in the middle of the pile and varying amounts of slack were specified. Figure 9thb shows the effect of the slack joint on the compression wave propagation as compared to the wave propagation with no joint (Fig. 9thb). The development of a secondary impact in the joint can be observed, which results in a significant disturbance of the shape of the force wave. The resulting effect of this disturbance on the drivability of the pile can be seen on Fig. 10; the reduction in ultimate capacity at a given driving resistance is more or less linear with the magnitude of the slack and amounts to about 9% per 0.01 in (0.25 mm).

In order to correctly simulate the field behaviour, the magnitude of bidirectional slack in the test piles was taken equal to half the construction tolerance of the Herkules joint. With this new model, the wave equation analysis resulted in a significant reduction of the predicted capacities of the piles. Figure 11 shows the effect of the joints on the ultimate capacity vs. BPI curve for test number J-5. The presence of four joints with bidirectional slack results in a decrease of the predicted ultimate capacity in the order of 20%, and the capacity predicted from the jointed model is remarkably close to the result of the load test. This effect of mechanical joints on the drivability of precast concrete piles is consistent with the experience reported by the Swedish Pile Commission (1964). The minimum, maximum, and probable bearing capacities predicted from the jointed model for the six test piles are shown on Fig. 12 along with the results of the load tests. The agreement between the predictions and observations is excellent, the maximum deviation being in the order of 5 t (45 kN) or 10%. Again here, the possible range of variation of the predicted
Climate capacity using common values of the parameters is in the order of $\pm 10\%$, i.e., practically acceptable for any practical case.

Analysis of the Production Piles

Introduction

In the first two phases of this investigation, the reliability of the wave equation method for analyzing piles driven under tightly controlled field conditions has been established. However, the driving operations carried out during normal piling jobs are governed not only by quality requirements but also by economic considerations and in particular a search for productivity. As a result, the driving conditions are generally less tightly controlled for production piles and large variations are unavoidable in such important parameters as the quality, thickness, and rigidity of the capblocks and cushions, in the actual drop heights, in the rhythm of driving, and in the resulting efficiency of each hammer blow. Therefore, driving resistance measurements done on production piles by the usual method of counting the hammer blows per unit length of penetration are likely to be affected by a random error equivalent to the random variation in the actual energy delivered to the pile by each hammer blow.

The driving records of about 500 precast concrete piles driven in the homogeneous sand deposit at the Saint-Charles River site have supported this assumption. As reported by Tavenas and Audibert (1972), the observed variations of the driving energies are extremely wide with the computed standard deviations ranging from 23 to 45% of the average values. Since the wave equation method of predicting pile capacities is based on the assumption that each hammer blow delivers the rated energy, and since it makes use of the standard blow count, it seems likely that the observed variability of the driving energies should be reflected on the results of the wave equation analysis.

Results

In order to verify the reliability of the wave equation for the prediction of the ultimate capacity of production piles, the method was applied to 35 piles driven on the Saint-Charles River site, and for which full driving records as well as load test results were available. The precast concrete piles were similar to the Her-
kules H-800 piles described earlier; they were driven by drop hammers, using essentially the same equipment as in the test program. Consequently, the same input parameters used in the analysis of the instrumented piles (see Table 1) were applied here. The characteristics of the piles are given in Table 2. Also given on this table are the ultimate bearing capacities \( Q_{tu} \) obtained from the load test carried out and interpreted by means of the same technique as that used during the design testing program. Finally, the ultimate pile capacities \( Q_{tu} \) obtained from the wave equation analysis and the measured driving resistances are also given on Table 2. The results are presented in the form of the relation between the computed \( Q_{tu} \) and the measured \( Q_{tu} \) ultimate bearing capacities on Fig. 13. The wide scatter of the data is obvious: 16 out of 39 data points are outside of the \( \pm 30\% \) limits, with a marked tendency for the wave equation to underestimate the pile capacity. By means of a least square adjustment method, the regression lines \( Q_{tu} = a \times Q_{tu} + b \) and \( Q_{tu} = a' \times Q_{tu} + b' \) have been defined as shown on Fig. 13; they confirm the poor correlation between the computed and measured ultimate capacities indicating that, in fact, no correlation exists between the two sets of data, in the mathematical sense of the term. The result of this analysis thus confirms the earlier suggestion that the random variability of the driving energy measurements used as input would directly affect the quality of the wave equation analysis.

To further evaluate the true potential of the wave equation analysis when applied to production piles, it is interesting to compare the results of the present study with those presented by Tavenas and Audy (1972), in an
## TABLE 2. Characteristics of tested piles

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<th>$L$ (ft)</th>
<th>$W_{pi}$ (kip)</th>
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* $Q_{lm}$ estimated.

Investigation of the quality of some of the usual pile driving formulas. Using the same set of data, Tavenas and Audibert have established the same lack of correlation between the observed ultimate capacities and the capacities $Q_{lm}$ predicted from the ENR, Hiley, Danish, Gates, and modified Gates formulas. Table 3 summarizes the correlation characteristics between $Q_{lm}$, $Q_{lu}$, and $Q_{lm}$. It can be seen that the wave equation predictions are no better than any of the predictions made by the pile driving formulas. However, this does not mean that the wave equation method is not reliable, but simply that the energy input is so variable that it destroys any value of the method. On the other hand, a careful examination of Fig. 14 indicates that the correlations using the pile driving formulas were poor not only for the
39 production piles but also for the six test piles. This is an important finding indicating that, given the proper reliable input data, the wave equation method leads to correct estimates of the ultimate pile capacity while the usual pile driving formulas do not.

In their paper Tavenas and Audy (1972) had indicated that time effect may have had an influence on the ultimate capacities measured in the load tests, and they had indicated the trend of variation of the pile capacity with time reproduced on Fig. 15. In order to check whether this time effect could affect the correlation between observed and computed capacities, the variations of the ratio of \( Q_{tu} \) to \( Q_{tu} \) with the time lag between driving and testing were plotted on Fig. 16. No trend can be detected, thus suggesting that if any time effects were present, they were not sufficiently great to obliterate the random variations due to the poor reliability of the energy input data.

**Summary and Conclusions**

The experimental data gathered during the design and construction of a large pile foundation in the homogeneous sand deposits of the Saint-Charles River site in Quebec City, provided an excellent basis for the evaluation of the potential and limitations of the wave equation method of pile capacity prediction, both under the well controlled conditions of a detailed test program and the normal conditions of production piling.

Using for the various input parameters the common values presented in the literature (Lowery et al. 1969), the analysis of the driving behaviour of instrumented precast concrete piles has shown that driving stresses can be predicted within \( \pm 30\% \), and ultimate capacities within \( \pm 10\% \), under well controlled driving conditions.

The application of the same method to the steel H-pile and the precast concrete pile driven and tested during the design testing program, confirmed the high potential of the wave equation to predict the pile capacities when all parameters of the driving system are well known and kept constant. For all but the last test on the steel H-pile, the capacity prediction was within \( \pm 10\% \) of the observation. For the precast concrete pile the effect of mechanical joints in the pile was put in evidence, and, with a proper simulation of the joints, the capacity predictions were also within \( \pm 10\% \) of the observations.

The quality of the predictions was quite different in the case of the 39 production piles analyzed in this study. One third of the predictions were beyond \( \pm 30\% \) of the observations, and the coefficient of correlation resulting from a regression analysis between observed and predicted ultimate capacities was as low as 0.141. A comparison with the predictions obtained from five common pile driving formulas showed that the wave equation predictions were not any better. This poor performance was attributed not to the method itself but rather to the highly variable energy input.

The present investigation thus demonstrates that the wave equation method of pile analysis is a very reliable tool for the prediction of pile capacities provided the pile driving system is kept under very tight and continuous control to ensure that the measured blow counts are truly representative of the driving energy imparted to the pile. However, under the usual conditions of a piling job, a high variability of
Fig. 14. Comparison of ultimate capacities observed and computed from pile driving formulas—39 production piles.
the properties of the driving system and of the energies delivered to the pile by successive blows is unavoidable and the blow counts are not representative of the true driving energies. The pile capacity predictions obtained from this variable input data through the wave equation method are no more reliable than the input data itself. In order for the wave equation method to be used to its full potential in production piling it appears necessary to resort to direct measurements of the driving energies; the method proposed by Goble and his co-workers is based on such direct measurements and should lead to predictions of acceptable quality. In the absence of such direct measurement, the wave equation method appears from a practical point of view, to be no more than a complicated and expensive pile driving formula, at least as far as the reliability of production pile capacity predictions is concerned.

Acknowledgements
This investigation was initiated while the second author was a visiting research associate at Laval University. The financial support of the Ministry of Education of Quebec and the National Research Council of Canada, grant number A-7724 is acknowledged. The analysis of the production piles was carried out as part of an undergraduate project by Messrs. Normand Toussaint and Michel Cantin.
TAVENAS AND AUDIBERT


Limitations of the Driving Formulas for Predicting the Bearing Capacities of Piles in Sand

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It has often been shown by previous investigators that the existing pile driving formulas yield bearing capacities which usually do not stand in good correlation to the actual bearing capacities of piles in sand. These poor correlations have always been attributed to the formulation of the equations.

Based on the observations made on a very large foundation built in an homogeneous sand deposit it is shown in the present paper, that the poor quality of the usual pile driving formulas originates essentially in the estimate of the driving energy; while it is assumed that each blow delivers a constant energy equal to \( W \times H \) for drop hammers, it appears actually that the energy delivered by a given equipment varies systematically from blow to blow. This conclusion, drawn from a statistical analysis of 478 driving records, is confirmed by the driving tests made on four instrumented piles.

The results of 45 load tests also confirm this conclusion in showing no correlation between the actual bearing capacities and the estimated driving energy or the bearing capacities computed from five different formulas. The observations also show a possible time effect on the bearing capacities of concrete piles in sand.

Il a été démontré antérieurement par divers chercheurs que les formules actuelles de battage de pieux conduisent à la prédiction de capacités portantes qui ne concordent pas avec les capacités portantes réelles des pieux dans le sable. Ce manque de concordance a toujours été attribué à un défaut dans la formulation des équations.

Le présent article fait état des observations faites lors de l'étude d'une fondation importante construite sur un dépôt homogène de sable. Ces observations démontrent que le défaut des formules usuelles de battage provient dans l'estimation de l'énergie de battage; alors que l'on admet généralement que chaque coup d'un marteau pilon fournit une énergie constante égale à \( W \times H \), il appert en fait que l'énergie fournie par un équipement donné varie systématiquement d'un coup à l'autre. Cette conclusion tirée d'une analyse statistique de 478 rapports de battage, est confirmée par des essais de battage exécutés sur quatre pieux instrumentés.

Les résultats de 45 essais de chargement confirment également cette conclusion en démontrant le manque de corrélation entre les capacités portantes réelles et celles estimées en partant de l'énergie de battage au moyen de cinq formules différentes. Les observations démontrent aussi qu'il existe possiblement un effet de temps sur les capacités portantes des pieux de béton dans le sable.

Introduction

The most usual method of construction of piled foundations being by dynamic driving, the idea of measuring the energy necessary to bring the pile to its final depth in the soil and of using this energy to predict the bearing capacity of the pile has been logically put forward very early in the history of piling.

The first mathematical formulation of a correlation between the static bearing capacity and the dynamic driving resistance of a pile is due to Sanders (1851) and was of the most simple form, i.e.,

\[
Q_1 = \frac{(W \times H)}{S}
\]

where \( W \) is the weight of the ram in tons, \( H \) the drop height in inches, and \( S \) the set per blow in inches. Although the simplest possible, this formula is no more mentioned or used, so that Wellington's formula, also called the Engineering News Formula (Wellington 1893) is the oldest one to be well known and used. It is still simple but a constant \( C \) has already been added to take in account the energy losses occurring at the impact:

\[
Q_1 = \frac{(H \times H)}{(S + C)}
\]

In the search for a greater "accuracy" or for more mathematically looking formulas,
a large number of sophisticated expressions have been proposed since that time. As a matter of fact, Dean (1935) listed 27 different formulas, while Chells (1951) referred to 36 expressions; actually the real number of existing formulas may be somewhat from 50 to 100, and possibly more. However it is of great interest to note that the only differences between the various formulas lie in the introduction of a variable quantity of empirical constants, while the basic parameters remain exactly the same for all expressions, i.e. the weight of the hammer, the height of the drop and the penetration of the pile, also called the set under the effect of one blow. This has already been noted by Cummings (1942) who stated: "there are only five basic types of dynamic pile driving formulas, ..., and all of them can be represented by the formula:

\[ W \times H = R_q S + Q \]

where \( R_q \) is the dynamic resistance of the pile and \( Q \) the energy losses that occur during impact".

With the great development of pile driving operations, the various formulas have been increasingly used and at the same time their validity has been investigated and merely questioned. Many references in the literature are dealing with the validity of one or the other formula or with the fundamental aspects of pile driving formulas; important papers have been presented by Cummings (1940), ASCE (1941) with related discussions by Mohr (1942), Terzaghi (1942), Peck (1942) and Cummings (1942), Sorensen and Hansen (1957), Agerschou (1962), Housel (1966), and recently, Olson and Flaat (1967).

Typically, many of these papers are concerned with the validity of the different coefficients involved in the analyzed formulas and are based on actual field loading tests performed on various sites. They show the evidence of a very large scattering in the results given by any formula and eventually try to reduce it by some kind of statistical adjustment of the coefficient or by introducing new empirical parameters. For example, Olson and Flaat (1967) analyze seven formulas and show that the coefficient of correlation, between the observed failure loads and the ultimate loads of 93 piles as computed by these formulas, vary from 0.29 to 0.81 while it should be equal to one for a perfect correlation; by a statistical adjustment of the Gates formula they establish a new formula which seems to be slightly better as the coefficient of correlation is equal to 0.85.

Surprisingly enough, while the empirical coefficients involved in the various formulas have been the object of numerous investigations, only one good study has been carried out on the basic parameter involved in all formulas, namely the energy delivered to the pile during usual driving operations. The Michigan study on pile driving hammers reported by Housel (1965) was concerned essentially with the efficiency of a number of diesel and steam hammers but succeeded in giving valuable information on the general behavior of pile driving systems and on the variability of the energy delivered to a pile from blow to blow. In particular, for the piles driven in cohesionless soils at the Muskegon site, Housel showed that the energy per blow, described by the \( R \)-value (ratio of the number of blows of a given hammer to drive a pile on a given distance to the average number of blows of all hammers to drive this type of pile on that distance), is widely scattered between limits of 0.2 and 1.5 for example. He concluded that energy losses occurring at impact were the cause of the large scattering. While this study showed that the actual energy delivered by a diesel or steam hammer was systematically different from its "rated energy" and was much variable, therefore making the use of that rated energy in driving formulas very questionable, the problem of the evaluation of the energy delivered by a free fall hammer is still to be solved. It could be formulated as follows: what is the accuracy of the driving energy's estimate obtained by multiplying the weight of the hammer \( W \) by the height of drop \( H \)? Also, does each blow of a given driving system deliver exactly the same energy, equal to \( W \times H \), to the pile, and if this is not the case, what is the variation from blow to blow? To evaluate these phenomena, one should analyze a large quantity of driving records of identical piles driven by the same equipment in at homogeneous soil deposit.
Such conditions have been encountered at the St. Charles River site in Quebec City, Canada, where a large pile foundation consisting of three parallel rows of precast concrete piles was built in a fairly homogeneous deposit of fine sand. In this paper the properties of the deposit, the characteristics of the foundation and the analysis of the pile driving observations are presented.

Geotechnical Properties of the St. Charles River Site

The foundation under consideration is part of two 12,500 ft (3800 m) long retaining walls in construction on both sides of the St. Charles River in downtown Quebec City. The soil investigation consisted of 32 standard borings with split spoon sampling and standard penetration tests and of 10 special borings with undisturbed sampling of the sand. The technique used and the results obtained have been published by Tavenas et al (1970).

Over a length of about 9000 ft (2700 m) on both sides of the river the soils conditions are very uniform. A typical soil profile is shown on fig. 1. The sand layer has been investigated by means of more than 300 split spoon samples and standard penetration tests, and 78 undisturbed piston samples. One hundred and thirty-seven grain size distributions have been determined on samples originating from locations distributed all over the site; the results, presented in fig. 2, show that the sand is a uniform medium sand (note that particles larger than 1 mm were found only on 5% of the samples in a limited superficial zone). The statistical distribution of the N values is shown in fig. 3. The standard penetration index N increases slightly with depth with a mean value of 23 for the entire deposit. As this average value applies to any zone of the site this medium dense sand deposit

![Image of figure 2: Grain size distribution of St. Charles River sand.](image2)

![Image of figure 3: Statistical distribution of N and \( \gamma_g \).](image3)

![Image of figure 1: Typical soil profile at St. Charles River site.](image1)
may be considered as very homogeneous. This is confirmed by the results obtained on the undisturbed samples: the in situ dry unit weight varies from 93.5 to 102 lb/ft$^3$ (1380 to 1571 kg/m$^3$) around an average of 97.5 lb/ft$^3$ (1501.5 kg/m$^3$) and the relative density measured according to ASTM D-2043-64T is equal to $58\%\pm15\%$.

Due to this homogeneity this site is perfectly suited for any large scale investigation on the behaviour of foundations, in particular of piles.

**Design and Characteristics of the Pile Foundation**

Due to structural and architectural considerations, it was proposed to rest the walls on two or three rows of piles driven in the sand; typical plans and sections are shown in fig. 4. As the bearing capacity of friction piles in sand is practically impossible to evaluate with a satisfactory accuracy from any of the existing bearing capacity formulas, and taking advantage of the uniformity of the sand deposit, the design of the foundation was based on the results of an extensive full-scale pile driving and testing program.

The testing program was planned in such a way as, first to reproduce as exactly as possible the working conditions of the piles in the foundation; second to allow a comparison between different pile materials, i.e., steel, precast concrete, and timber; third to give information not only on the variations of the ultimate bearing capacities with the length of the piles in the sand and with the driving resistances but also on the variations of the unit skin friction and point resistance with the depth in sand. The technique and the results have been reported by Tavenas (1969, 1971).

For design purposes the three following relationships were developed:

1) The relationship between the ultimate bearing capacity $Q_t$ and the length of piles in sand (fig. 5), which was used to determine

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**FIG. 4.** Typical implantations of the piles in the foundation.
2) The relationship between \( Q_l \) and the energy necessary to drive the last foot of each pile (fig. 6), which was intended to be used as a control of the quality of each pile driven in the actual foundation, and of the uniformity of the piling work.

3) The relationship between \( Q_l \) and the total energy necessary to drive the piles in the sand (fig. 7), which could possibly be used as a complement to the second correlation.

According to the results obtained, five categories of precast concrete piles were designed with working capacities of 20, 22, 25, 27.5, and 32.5 tons (18.0, 19.8, 22.5, 24.8, and 29.3 t) and corresponding lengths of 15, 18, 22, 25, and 32 ft (4.6, 5.5, 6.7, 7.6, and 9.7 m) in the sand. The specifications called for the piles being driven by means of a free fall hammer weighing about 7000 lb (3214 kg) operated by clutch release at a drop height of no more than 30 in. (76.2 cm) the driving energy had to be controlled for the last 2 ft (0.6 m) of each pile and for the entire length of one every 20 piles, and the energy necessary to drive the last foot of each pile was specified in order to comply with the values given in fig. 6. Finally load bearing tests to failure were specified on one of every 150 piles of each category.

A total of 4160 piles on the north shore and 4520 piles on the south shore had to be driven. Approximately 70% of the work is now completed; 478 full driving records have been compiled on the south shore, and 45 driving and load bearing tests have been made on both shores. These results obtained on identical piles driven with the same equipment in an homogeneous sand deposit will be further analyzed.

Analysis of the Observed Driving Energies

Very early in the control of the driving operations it appeared that the observed energies varied much from pile to pile. At the same time, load bearing tests carried out on piles, for which the energy in the last foot was much below the required limit, showed that the ultimate bearing capacity was in excess of the required value. It appeared therefore, that the relationship shown in fig. 6 was not unique and could not be used
to control the quality of the individual piles, and that an analysis of the observed driving energies would be of great interest.

A statistical analysis of the observations on the driving energy has been made for 478 piles from the south shore. The energies necessary to drive the 1st, 5th, 10th, 20th, 25th, and 30th ft of pile in the sand have been analyzed and the following parameters determined: the minimum, maximum and average of all observations; the statistical distribution of the observed values in form of cumulated frequency curves; the standard deviation and the coefficient of dispersion, equal to the ratio of the standard deviation to the average.

Fig. 8 gives the variations of the minimum, maximum and average energy per linear foot as a function of the depth of pile in sand. While the average energy increases linearly with depth, this figure shows that the individual observations are very large scattered around this average. The cumulated frequency curves shown on fig. 9 for each depth level are more or less linear between cumulated frequencies of 15% and 85%, thus indicating that 70% of the observations are randomly distributed between the different intervals. As shown in Table 1, the standard deviation increases with an increasing energy and a decreasing number of observations from 86 kips ft ft (38.7 t m/m) at a depth of 1 ft (.3 m) to 173 kips ft ft (77.9 t m/m) depth of 30 ft (9.1 m). On fig. 8, the curves representing the variation with the depth of the average energy plus or minus the standard deviation are shown. Their form confirms that the observed variability of the energy is not related to the behaviour of the soil in which the pile is driven but is due to a phenomenon independent of the total energy. The computed coefficients of dispersion vary from 0.450 to 0.225, a value of 0.25 being possibly typical. Such values have to be compared to the possible limits of the coefficient of dispersion, equal to 0 for a perfectly reproducible phenomenon and to about 0.5 for an absolutely non-reproducible or randomly scattered phenomenon. The actual values indicate a phenomenon of low reproducibility or large scattering.

Fig. 8. Driving energy vs. depth: minimum, maximum, and average.
If we now consider that the soil deposit is fairly homogeneous and that the piles are identical and driven in rows (see fig. 4), thus eliminating any possible group effect, so that the driving energies should actually be quite uniform and statistically distributed according to a Gaussian law with a very low coefficient of dispersion, the observed random distribution has to be attributed to the procedure for evaluating the energy, and more precisely to the assumption that the energy generated by each blow is equal to the product of the weight of the hammer by a constant height of drop, and therefore is a constant. On the contrary, it becomes evident that the energy delivered to the pile varies widely from blow to blow due to the many variable parameters involved: the actual height of drop which is determined by hand of the operator of the rig and is essentially variable, the drag exerted on the hammer along the lead during the drop, the relative position of the hammer, the driving cap and the pile at impact, the state of the wood cushion on top of the driving cap, which certainly varies from blow to blow, the state of the fiber cushion between the cap and the pile head, the rhythm of blows application, the elastic behaviour of the pile, etc. Therefore, the assumption that the energy delivered to the pile is identical from blow to blow and equal to \( W \times H \) must be considered as a large over-simplification of the phenomenon; the fact that the energies so computed are widely scattered, as shown in fig. 8 and 9, is no at all surprising.

**Driving Tests on Instrumented Piles**

In order to further investigate the driving behaviour of the piles and to confirm the previous observations, four driving tests have been carried out on instrumented piles of various lengths. The piles were equipped with a central vertical pipe on which elastic strain gauges were glued at five different levels, in particular at 1 ft (.3 m) above the pile tip and at 8 ft (.4 m) below the pile head.

The strain gauges were glued on the outside of a 1/4 in. extra strong steel pipe, in groups of four, with two active and two reference gauges connected in full bridge. The gauges were protected with a waterproof coating and two layers of electric tape.

The instrumented piles, equipped with dowels spaced at 12 in. (30.5 cm) symmetrically to the gauge levels, were attached in the axis of the reinforcement cages made up of six 5/8 with 5/3 spirals, spaced at 6 in. (15.2 cm) (fig. 10). After correct positioning of the cages, the piles were cast in the standard hexagonal molds with a concrete having a 7500 p.s.i. (52 700 kg/m²) minimum strength. In the same mold a reference piece of pile was casted with the same concrete; the reference
piece had a length of 36 in. (91.44 cm) and was equipped with an instrument pipe bearing a single strain gauges group glued at mid-height: this reference piece was used to measure in the laboratory the modulus of elasticity of the finished pile on the same day as the tests were made and to check the readings given by the strain gauges.

For the tests, after balancing the different bridges with the pile in horizontal position, the standard bridge measuring unit was connected to an oscillograph and the sensitivity of the oscillograph set so as to give 250 μ in. per 1 in. (.63 cm) division on the cathode screen. At different states of driving, dynamic stress measurements were made for series of three consecutive blows, photos being taken of the traces on the oscillograph. For the same blows the deformation of the piles were measured at ground level. Fig. 11 reproduce examples of the results obtained.

A total of 20 series of measurements have been made.

From the peak driving force \( F_h \) observed at pile head, and the maximum displacement \( S_h \) at the same level under the impact of a given blow, the actual energy at pile head \( E_h = F_h \times S_h \) has been computed. From the peak driving force \( F_i \) at pile tip and making the assumption that the final set \( S \) at ground level represented to tip displacement under the impact of a given blow, the actual energy at pile tip \( E_i = F_i \times S \) was also computed.

The computed energies \( E_h \) and \( E_i \) have been compared to the energy theoretically delivered by the blow, i.e. \( E = W \times H \).
statistical distribution of the computed ratios $E_i/E$ and $E_r/E$ is given in fig. 12. If the product $W \times H$ would accurately represent the energy delivered to the pile and if each blow would be nearly identical, the $E_i/E$ curve should be nearly a vertical line passing by the abscissa 1; as shown in fig. 9, this is by far not the case; the observed values of $E_i/E$ being scattered between 0.6 and 2.2 around an average of 1.1. This result therefore confirms the previous conclusions: the actual energy delivered by a free fall hammer during normal driving operations differs greatly from blow to blow and is generally not equal to the product $W \times H$.

Validity of the Driving Formulas

The analysis of the observations on the assumed driving energy has led to the conclusion that the usual energy estimate was not satisfactory, and when used in pile driving formulas, would have a detrimental effect in inducing an important scattering of the results. This conclusion can be verified on the observations made on 45 piles which were submitted to driving and loading tests.

The loading tests have been performed on piles, driven at various locations on both shores of the river, and which were usually selected on the basis of a final driving resistance below average. The final sets observed on these piles varied from 0.22 in. to 0.66 in. per blow. The test load was applied by means of the same jack for all piles, the necessary reaction being provided either by a loaded platform or by pull-out reaction on four neighbouring piles. The loads were applied in 12.5 tons (11.3 kN) increments sustained for 60 min. up to a maximum load equal to four times the design bearing capacity of the piles. A typical load-settlement curve is given in fig. 13, which shows a clear definition of the failure load; however, on 25% of the piles no failure was reached, the ultimate bearing capacity being then extrapolated from the form of the load-settlement curve. Table 3 gives the complete basic information on the 45 tested piles.

Beside the energy necessary to drive the last foot of each pile, five pile driving formulas have been considered for comparison with the observed failure loads (Table 2).

![Graph showing energy distribution](image)
In all these formulas, the constants were selected as follows:

\[ e_h = \text{efficiency of the hammer} = 0.8 \]

\[ C = \text{constant of the hammer} = 1 \]

\[ C_1 = 0.15, \quad C_2 = 0.50, \quad C_3 = 0.10 \]

\[ n = 0.25 \] according to Chellis (1951)

\[ A = \text{Cross sectional area of the pile} \quad 125 \text{ in.}^2 \quad (806.3 \text{ cm}^2) \]

\[ E = \text{Young's modulus of the pile} = 6 \times 10^6 \text{ lb/in.}^2 \quad (4218.6 \times 10^6 \text{ kg/m}^2) \]

The parameters are defined as follows:

\[ W = \text{weight of the hammer in tons} \]

\[ H = \text{height of drop in inches} \]

\[ S = \text{set of the pile per blow in inches} \]

\[ W_p = \text{weight of the pile in tons} \]

\[ Q_c = \text{ultimate bearing capacity in tons} \]

The values of these parameters for each pile are given in Table 3.

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**Table 1.** Average, standard deviation and coefficient of dispersion of the energy.

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<th>Depth ft</th>
<th>Number of observations</th>
<th>E_{\text{minimum}} k-ft/ft</th>
<th>E_{\text{maximum}} k-ft/ft</th>
<th>E_{\text{average}} k-ft/ft</th>
<th>Standard deviation k-ft/ft</th>
<th>Coefficient of dispersion</th>
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**Table 2.** Pile driving formulas considered in this investigation

\[ Q_{c1} = \frac{e_h \times W \times H}{S + C} \]

\[ Q_{c2} = \frac{e_h \times W \times H}{S + 1} \left( C_1 + \frac{C_2 + C_3}{C_1} \right) \frac{W + n^2 W_p}{W + W_p} \]

\[ Q_{c3} = \frac{e_h \times W \times H}{S + \left( \frac{e_h \times W \times H}{2 \times A \times E} \right)^{\frac{1}{3}}} \]

\[ Q_{c4} = 5.6 \times \left( e_h \times W \times H \right)^{\frac{1}{3}} \times \log_{10} \frac{S}{S} \]

\[ Q_{c5} = \left[ 9 \times \left( e_h \times W \times H \right)^{\frac{1}{3}} \times \log_{10} \frac{S}{S} \right]^{\frac{1}{3}} \]

**Note:** None of the presented formulas contains a factor of safety.
TAVERAS AND AUDY: LIMITATIONS OF DRIVING FORMULAS

Table 3. Characteristics of tested piles

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<th>Test No.</th>
<th>Pile No.</th>
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<th>$L$ Foot</th>
<th>$W_{II}$ Kips</th>
<th>$H$ Foot</th>
<th>Blows/foot</th>
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</table>

*Estimated.

The computed values of $Q_{im}$ and the energy delivered to drive the last foot of pile have been compared to the measured ultimate bearing capacity $Q_{im}$. The results obtained are given in Fig. 14, where the light symbols refer to the piles tested during the construction of the foundation and the dark symbols to the piles tested during the design testing program. The sole observation of Fig. 14 shows that no valuable correlation exists between any of the computed bearing capacities and $Q_{im}$. By means of a least square adjustment method the regression lines $Q_{lc} = a Q_{im} + b$ and $Q_{im} = a' Q_{lc} + b'$ have been defined as shown on Fig. 14: the values of the coefficients $a$, $b$, $a'$, and $b'$ as well as...
Fig. 14. Comparison between $Q_e$ and $Q_{sm}$ for five formulas.
of the coefficients of correlation which have been computed are given in Table 4. All these results confirm that no correlation can be found between \( Q_e \) and \( Q_{in} \) for the piles tested, and that all five formulas are practically similar in their inefficiency as the coefficient of correlation vary from 0.13 for the Hiley formula to 0.27 for the Danish formula.

These results are in concordance with the observed scattering of the driving energy on which the formulas are based. Therefore, it can be concluded that for the type of piles tested the usual driving formulas are of no practical value, not because of an erroneous formulation but at least partly because the basic parameters which are used, i.e. the driving energy and the set per blow are not accurately controlled and are therefore affected by a very large scattering which directly reflects on the results yielded by the formulas.

The Effect of Time on the Bearing Capacity

The influence of time on the behaviour of piles driving in cohesionless soils has often been recognized as a potential difficulty related to the use of pile driving formulas for predicting the static bearing capacity of such piles.

Parsons (1966) has presented case histories from the New York City area which show that the penetration resistance of piles in granular soils may vary with time. As reported the observed penetration resistances at the beginning of re-driving of steel piles in submerged sand were much lower than at the end of the initial driving which had been performed weeks or months before; also, the measured static bearing capacities determined by load testing were on some occasions much smaller than expected from driving formulas. Parsons called this phenomenon of the reduction of the soil resistance around the pile, “relaxation”, but failed to propose valuable explanations for it.

Yang (1970) has recently analyzed very extensively this relaxation phenomenon of piles in sand and showed that pore-water pressures as well as changes in the soil structure after driving could lead to variations of the driving resistance. Investigating the influences of these two parameters, he showed that, for piles in fine sand, the permeability is low enough to allow the build up of pore pressures during driving and therefore the dissipation of some energy in the pore water; consequently, a high driving resistance is measured which however is not significant for the long-term behaviour of these piles. Yang also considers the effect of the soil structure and suggests that for piles driven in dense sand a low driving resistance has to be expected at redriving while for loose sand the driving resistance will increase with time; however, he failed to show an evidence of this last statement on case histories.

During the initial testing program, as reported by Tavenas (1971) the piles were driven in 10 ft (3.0 m) increments and load tested after each driving sequence. Delays of at least 12 h between the end of driving and the beginning of the test, and about 24 h between the end of the test and the start of the next driving sequence were provided. No significant differences were observed in the driving resistances at the end of driving and at the beginning of redriving despite a delay of at least 48 h between these operations. Thus, as shown on fig. 14, the correlation between \( Q_e \) and the driving energy is the same whichever energy is considered.

If we now consider the results of the test piles in the actual foundation, fig. 14 shows clearly that their behaviour was sensibly different, their static bearing capacity being usually much larger for the same final driving energy. As these piles were perfectly identical to the reference test piles with respect to material, geometry, driving equipment, spac-

<table>
<thead>
<tr>
<th>Pile driving formula</th>
<th>Coefficient of correlation</th>
<th>Regression line</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a  b  a' b'</td>
</tr>
<tr>
<td>ENR</td>
<td>0.131</td>
<td>0.052 44.0 0.810 60.8</td>
</tr>
<tr>
<td>Hiley</td>
<td>0.131</td>
<td>0.030 46.5 0.284 86.6</td>
</tr>
<tr>
<td>Danish</td>
<td>0.272</td>
<td>0.234 100.5 0.247 52.7</td>
</tr>
<tr>
<td>Gates</td>
<td>0.225</td>
<td>0.281 59.0 1.169 27.2</td>
</tr>
<tr>
<td>Gates modified by Olson and Flaat</td>
<td>0.253</td>
<td>0.130 67.9 0.750 41.8</td>
</tr>
</tbody>
</table>
ing, and surrounding soil, the systematic increase of the ultimate bearing capacity has to be explained by the influence of the only variable parameters, i.e., the time lag between driving and testing of each pile. Fig. 15 shows the relationship between the time lag and the ratio of the ultimate bearing capacity of each pile to the ultimate bearing capacity of the corresponding test pile for which the time lag was 12 h. Despite an important scattering of the results, a definite tendency can be observed: the ultimate bearing capacity increasing during the first 15 to 20 days to reach a constant value 70\% higher than that observed at 12 h. As observed by Tavenas (1970) in an analysis of the first 15 tests performed at the St. Charles River Site, this phenomenon cannot be explained by the influence of pore pressure, as the sand is so pervious (K = 10⁻² cm/sec) that complete pore pressure dissipation can be expected to occur within a few hours; the increase in bearing capacity has therefore to be related to changes occurring in the sand structure around the pile.

Conclusions

Very often in the past the validity of the existing pile driving formulas has been investigated and questioned. However, these analyses were usually based on observations made on various types of piles driven at different sites so that not real uniformity of the basic information was ensured. Thus, the conclusions of these investigations could always be related, not only to the basic phenomenon under investigation, but also to variations in the site conditions, in the type of piles as well as in the testing techniques, and were therefore not definite.

On the contrary, the investigation presented in this paper has been carried out on a very large pile foundation where a good uniformity in all the parameters was ensured: a unique type of piles, hexagonal precast concrete piles with an equivalent diameter of 12 in (30.5 cm), was driven by the same equipment (drop hammer) in a sand deposit which was proven to be uniform in its physical and mechanical properties. Therefore, ideal conditions were provided for an extensive investigation of the validity of the usual controls of quality based on the driving energy.

The statistical analysis of 478 complete driving record shows that the driving energy, as evaluated by multiplying the weight of the hammer by the drop height, is very widely scattered. The limits of the range of variation
are related to the average value $E_{ave}$ by $E_{ave} \pm 70\%$, and the coefficient of variation, defined as the ratio of the standard deviation to $E_{ave}$, varies from 0.225 to 0.450 despite the very large number of observations. As the soils conditions are uniform this scattering must be attributed, not to variations in the driving behaviour of the pile soil system, but to the erroneous evaluation of the driving energy delivered to the pile, and mainly on the assumption that each blow generates an identical energy equal to $W \times H$.

Driving tests carried out on instrumented piles show that the actual energy delivered by different blows varies much and is usually not equal to $W \times H$.

The usual energy estimate being proved erroneous, it is possible to conclude, first that any pile driving formulas in which this estimate will be used will also be erroneous, second that the control of the driving energy cannot even be used to verify the uniformity in the bearing capacities of the piles and to extrapolate to them the observations made on a few tests piles. Actually the driving energy control was absolutely useless on the reported job.

The analysis of the results of 45 load tests confirms this conclusion; it is shown that the bearing capacity computed by any of the five following formulas, Wellington, Hiley, Danish, Gates, Gates modified by Olson, and Flaate, had no useful correlation to the actual bearing capacity, the highest coefficient of correlation being 0.27.

As this poor quality of the results, obtained by pile driving formulas, is at least partly, related to the erroneous estimate of the driving energy, efforts should be made now develop a technique for measuring as accurately as possible the actual energy delivered to the pile by each blow; this could probably be achieved by developing an instrumented driving cap. In the meantime the validity of the usual pile driving formulas is best defined by a statement made by Peck (1942): "It can be demonstrated by a purely statistical approach that the chances of guessing the bearing capacity of a pile are better than of computing it by a pile driving formula".

Finally, it is shown that, even if a method of measuring the driving energy would be developed, the use of good pile formulas would not necessarily lead to a valuable estimate of the static bearing capacities of piles, due to possible time effects. For the sand and the concrete piles under consideration, the bearing capacity shows a trend to increase by about 70\%, in the first two to three weeks after driving, the long-term behaviour of these piles being therefore not related to the driving resistance.

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It is known that vibrating hammers are highly effective for driving pipes. However, their wide use is prevented by their insufficient durability and high energy-consumption, which are determined mainly by the high velocity of the hammer at the moment of impact against the driven element.

The results of the investigations described in this article indicated that the driving capacity of vibrating hammers in soils of medium density can be improved without increasing the impact velocity, by increasing the length of the path traveled jointly by the hammer and the pipe after each blow. When the vibrating hammer operates under longitudinal action, the above-mentioned effect can be obtained by adjusting the force in the springs, in case of large negative gaps. High driving capacity is possessed also by vibrating hammers operating under longitudinal-rotary action, in which the path traveled jointly by the hammer and the pipe is increased as a result of torsional vibrations of the driven pipe.

The experimental investigations conducted in order to justify the analytical model indicated that during vibration-impact driving of pipes open at the bottom, the blow can be regarded as inelastic and instantaneous, while the penetration resistance can be regarded as plastic.

A longitudinal-action free spring vibrating hammer and a longitudinal-rotary action vibrating hammer were investigated.

In the first case (Fig. 1), the springs, which were not connected to the driving element, were compressed by applying an external force. For the study of this analytical model, the following assumptions were made:

1. The vibrating hammer and the pipe are absolutely rigid bodies.
2. The disturbing force created during rotation of the eccentric weights acts according to the law
   \[ P = P_0 \sin \omega t. \]
3. The impact of the vibrating hammer against the pipe is instantaneous and inelastic.
4. The forces which resist the motion are constant.

The investigators considered a steady motion with a period \( 2 \pi/\omega \), in which driving of the pipe starts at the moment of the impact caused by the ram of the vibrating hammer and ends before the following impact.

The motion of the vibrating hammer for \( 0 < t < \tau_1 \) is described by the equation
\[
M_1 \dot{z}_1 = P_0 \sin (\omega t + \alpha) - Q_1 - c \dot{z}_2 + M_1 \dot{z}_1. \tag{1}
\]
in which \( M_1 \) and \( Q_1 \) are the mass and weight of the vibrating hammer;
\( z_1 \) is the coordinate of the vibrating hammer with respect to the driven element;
\( z_2 \) is the coordinate of the driven element;
\( z_p \) is the initial force in the springs which compress the vibrating hammer against the driven pipe (considered to be constant);
\( c \) is the coefficient of rigidity of the springs;

Fig. 1. Analytical scheme of free spring vibrating hammer.

Fig. 2. Graphs representing the variation of the impact velocities (dashed lines) of the vibrating hammer and the penetration (full lines) after each blow for different spring compressions and penetration resistances.

\[ t \text{ is the time reckoned from the moment at which the vibrating hammer is separated from the driven element;} \]
\[ t_0 \text{ is the time at which the pipe stops moving;} \]
\[ P_0 \text{ is the amplitude of the disturbing force;} \]
\[ \omega \text{ is the angular frequency of the disturbing force;} \]
\[ \alpha \text{ is the initial phase angle.} \]

Equation (1) describes the general case of travel of the vibrating hammer, which can start when the pipe is already moving.

For a fixed pipe, the travel of the vibrating hammer for \( t_0 < t < t_c \) is expressed by the equation

\[ M_1 z_1 = P_0 \sin (\omega t + \alpha) - Q_1 - c_1 (t_1 + t_2), \]  
(1, a)

in which \( t_c \) is the time of completion of the travel of the vibrating hammer.

When the motion of the vibrating hammer takes a time \( 0 < t < t_0 \), the driving of the pipe is described by the equation

\[ M_1 z_1 = Q_2 - (R + F), \]
(2)

for joint motion of the vibrating hammer and the pipe after the blow, and for \( t_c < t < 0 \)

\[ (M_1 + M_2) \dot{z}_2 = P_0 \sin (\omega t + \alpha) + (Q_1 + Q_2 + c_2 t_2) - (R + F), \]  
(β)

in which \( M_2 \) and \( Q_2 \) are the mass and weight of the driven element; and \( R \) and \( F \) are the frontal and lateral penetration resistances.
The above results were confirmed in a practical manner by constructing an industrial model of the VHS-1 free swinging vertical hammer (Fig. 4), which operated with precision and exhibited good results.

The hammer's vertical impact was controlled by an angle of 150-110°. The impact velocity was determined by the displacement of the hammer's hammer and the hammer's impact velocity at the time of impact. The hammer's vertical impact was controlled by the hammer's vertical impact velocity, which was equal to the hammer's vertical impact velocity at the time of impact.

The hammer's vertical impact velocity at the time of impact was equal to the hammer's vertical impact velocity at the time of impact.

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According to the adopted assumptions, the velocity of the hammer at the time of impact is equal to

\[ v = \frac{w}{w} \]

Fig. 4. The hammer's vertical impact velocity at the time of impact.
The vibrating hammer can be used for driving and extracting casing pipes in 100-m deep wells by means of UKS-22 and UKS-30 rigs. It has the following indices:

- Torque of eccentric weights of vibrator, kg-cm: 2500
- Speed of shafts of eccentric weights, rpm: 600
- Ratio of period of blows to period of rotation of shafts of eccentric weights: 1
- Capacity of electric motor, kW: 22
- Compressive force of operating springs, kg: 9600
- Rigidity of operating springs, kg/cm: 800
- Mass of vibrating hammer, kg: 2400

The vibrating hammer was used for driving pipes 325, 426, 539, and 630 mm in diameter, and 20 to 90 m long, through saturated and dense soils, at a rate of 0.5 to 2.0 m/min. When the winch for tightening the operating springs was connected, it became possible to drive the casing pipes 50–60 m into the soil.

Depending upon the nature of the soil and the degree of filling of the internal cavity with earth during the process of driving of a pipe, the operating springs were adjusted so as to obtain a force between 4 and 8 tons; this ensured a maximum driving rate for stable operation of the vibrating hammer in the regimen of "blow after return."

The use of the BVS-1 vibrating hammer made it possible to increase the rate of driving of the pipes by a factor of 2 and even more, to increase the penetration of the columns into the soil by 30%, and to reduce the number of changes from studge pumping to driving by a factor of about 3.

The tightening of the springs by a hauling rope wound around a winch installed separately or on a drilling rig (movable unit) is not always technically convenient; it should correspond to the specific characteristics of the given type of work.

Another method considered for increasing the driving capacity of the vibrating hammers without increasing the impact velocity was the use of longitudinal-rotary vibrating hammers, the analytical scheme of which is shown in Fig. 4. In this case an additional factor taken into account is the rotary motion of the hammer–pipe system under the action of the torsional moment Pdr cos ωt.

During vibration-impact driving, the following resistance is overcome on the lateral surfaces:

\[ F_z = F \frac{r_1^2}{\sqrt{(r_1^2 + \varphi^2)^2}} \text{ along } z \text{ axis,} \]

\[ F_r = F \frac{\varphi r}{\sqrt{(r_1^2 + \varphi^2)^2}} \text{ along } \varphi \text{ axis} \]

in which \( r \) is the pipe radius, \( r_1 \) is the radius of application of the disturbing force, and \( \varphi \) is the angular coordinate of the hammer–pipe system.

The equations of motion for this scheme are presented in [1].

An analysis of several recorded vibration-impact driving regimens indicated that in the presence of a rotary component, the velocity increases on the average by 10–15% after the blow.

For experimental work carried out on an area covered by clay soils having different consistencies, use was made of an experimental vibrating hammer, which had been set for longitudinal and longitudinal-rotary impact regimens, for driving pipes 168 mm in diameter and 10 m long [4].

A comparison of the effectiveness of the longitudinal and longitudinal-rotary vibration hammers indicated that for the same vibration parameters the limiting driving depth of the latter type of vibrating hammer was greater in all the tests. The value of the capacity per unit depth of driving for longitudinal blows \( W_L = N_L / L \) is 1.5–4.5 times greater than the value of the capacity for longitudinal-rotary blow \( W_{LR} = N_{LR} / L \).

The substantial advantages of the longitudinal-rotary vibrating hammers were the bases for the development of an experimental model of a Type-PYVN-1 vibrating hammer, for construction of bored piles up to 550 mm in diameter and 18 m long (Fig. 5), which has the following indices:

- Rated capacity, kW: 60
- Torque of eccentric weights, kg-cm: 7000
Amplitude of torsional moment with respect to the maximum force, ton-cm
Frequency of vibrations, min
Total rigidity of springs, kg/cm
Mass of ram, tons
Mass of entire installation, tons

The tests on a PVN-1 vibrating hammer used for driving 426-mm diameter pipes in different soils ranging from soft-plastic clays to hard sandy loams, indicated its high driving capacity. (The mean rate of driving to a depth of 18 m was 0.8-1.5 m/min.)

Thus, the investigations indicated the possibility of a substantial increase of the driving capacity of vibrating hammers when used for pipes, by applying regulating tensions on the hammer springs, for large negative gaps and rotary vibrations of the driven pipes.

LITERATURE CITED