Problem 1) Determine the magnitude of the resulting force and the angle at which it acts from the z axis, for the forces shown below. Use \( F_1 = 200\, \text{kN} \), and \( F_2 = 500\, \text{kN} \).

\[
\begin{align*}
\cos^2(60^\circ) + \cos^2(90^\circ) + \cos^2(30^\circ) &= 1 \\
\cos^2 \theta &= 0 \\
\cos \theta &= 0 \\
\Theta_1 &= 90^\circ \\
\end{align*}
\]

\[
\begin{align*}
F_{1x} &= (200\, \text{kN})(\frac{y}{4.105}) = -87.287\, \text{kN} \\
F_{1y} &= (200\, \text{kN})(\frac{y}{4.105}) = 43.644\, \text{kN} \\
F_{1z} &= (200\, \text{kN})(\frac{8}{4.105}) = 174.577\, \text{kN} \\
\vec{F}_1 &= \langle -87.287, 43.644, 174.577 \rangle \\
F_{2x} &= -(500\, \text{kN}) \cos(60^\circ) = -250\, \text{kN} \\
F_{2y} &= 500\, \text{kN} \cos(90^\circ) = 0\, \text{kN} \\
F_{2z} &= -(500\, \text{kN}) \cos(30^\circ) = -433.013\, \text{kN} \\
\vec{F}_2 &= \langle -250, 0, -433.013 \rangle \\
\vec{R} &= \vec{F}_1 + \vec{F}_2 = \langle -87.287-250, 43.644+0, 174.577-433.013 \rangle \\
\vec{R} &= \langle -337.29, 43.64, -258.44 \rangle \, \text{kN}
\]

\[
\begin{align*}
|\vec{R}| &= 427.15\, \text{kN} \\
\cos \Theta_2 &= \frac{\vec{R}_2}{|\vec{R}|} = \frac{-258.44}{427.15} \\
\cos \Theta_2 &= -0.605 \\
\Theta_2 &= 127.23^\circ
\end{align*}
\]
Problem 2) Two circular rods (10 feet long) are stacked in a ditch as shown. Rod A is steel (490 #/cu ft), with a 10" diameter. Rod B is brass (170 #/cu ft), with a 16" diameter. The ditch is 22 inches wide and the sides are smooth (no friction on the rods.) Determine all forces acting on the two rods and show these forces on a free body for each rod.

\[ W_A = \left( \frac{490 \text{ #}}{\text{ft}^3} \right) \left( 10 \text{ ft} \right) \left( \pi \left( \frac{1}{16} \right)^2 \right) \]
\[ W_A = 2672.535 \text{ #} \]
\[ W_B = \left( \frac{170 \text{ #}}{\text{ft}^3} \right) \left( 10 \text{ ft} \right) \left( \pi \left( \frac{1}{16} \right)^2 \right) \]
\[ W_B = 2373.648 \text{ #} \]

\[ \theta = 46.19^\circ \]

**FB-A**

\[ F_x = N_2 - N_3 \cos(46.19^\circ) = 0 \]
\[ N_2 = 3389.24 \text{ #} \]
\[ N_2 = 2277.04 \text{ #} \]

\[ F_y = N_1 - W_A - N_3 \sin(46.19^\circ) \]
\[ N_1 = 2672.535 + 3389.24 \sin(46.19^\circ) \]
\[ N_1 = 5046.18 \text{ #} \]

**FB-B**

\[ F_x = N_3 \cos(46.19^\circ) - N_4 = 0 \]
\[ N_4 = 3389.24 \cos(46.19^\circ) \]
\[ N_4 = 2277.04 \text{ #} \]

\[ F_y = N_3 \sin(46.19^\circ) - W_B = 0 \]
\[ N_3 \sin(46.19^\circ) = 2373.648 \text{ #} \]
\[ N_3 = 3389.24 \text{ #} \]
Problem 3) Replace all forces and moments shown on the structure with a single equivalent force and a couple at the wall. Use: \( P_1 = 5k \), \( P_2 = 10k \), \( \alpha = 30^\circ \), \( M = 40 \text{ k ft} \), \( w = 2 \text{ k/ft} \), \( d_1 = 3 \text{ ft} \), \( d_2 = 2 \text{ ft} \), \( d_3 = 4 \text{ ft} \), \( d_4 = 1 \text{ ft} \), \( d_5 = 2 \text{ ft} \).

\[
\sum M_A = -5k(3) - 10k \sin(30^\circ)(1) - 10 \cos(30^\circ)(9) - \frac{1}{2}(3 \cdot 2)(1) - 40 \text{ k ft} \\
= -141 \text{ k ft}
\]

\[
\sum F_x = -\frac{1}{2}(3 \cdot 2) + 10k \sin(30^\circ) = 2k
\]

\[
\sum F_y = -5k - 10k \cos(30^\circ) = -13.7 \text{ k}
\]

\[
|\Sigma F| = \sqrt{(2)^2 + (13.7)^2} = 13.8 \text{ k}
\]

\[
\theta = \tan^{-1}\left(-\frac{13.7}{2}\right) = -81.7^\circ
\]
Problem 3) For the structure loaded as shown below, use vector forces and position vectors to determine the scalar component of the moment caused by $F_1$ about the shaft $AB$. Use $F_1 = 1400 \text{ N}$. You do not have to solve for the final answer, but can rather express your answer in determinant and dot products. However, you must fully express any determinants and dot product operations required for me to get a final answer. You must also put a box around any force or position vectors to be used in your solution.

\[
\vec{F} = 433 \hat{x} - 433 \hat{y} + 433 \hat{z}
\]

\[
\begin{align*}
\vec{R}_{C/A} &= 0 \hat{x} + 450 \hat{y} + 200 \hat{z} \\
\vec{R}_{C/B} &= 0 \hat{x} - 200 \hat{y} + 200 \hat{z} \\
\vec{R}_{C/D} &= 0 \hat{x} + 0 \hat{y} + 200 \hat{z}
\end{align*}
\]

\[
\vec{e}_{AB} = 0 \hat{x} + 1 \hat{y} + 0 \hat{z}
\]

\[
M_{AB} = \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
\vec{r}_x & \vec{r}_y & \vec{r}_z \\
\vec{F}_x & \vec{F}_y & \vec{F}_z
\end{vmatrix} \cdot \vec{e}_{AB}
\]

\[
\begin{vmatrix}
0 & 1 & 0 \\
\vec{r}_x & \vec{r}_y & \vec{r}_z \\
\vec{F}_x & \vec{F}_y & \vec{F}_z
\end{vmatrix}
\]
Problem 1) Replace all forces and moments shown on the structure with a single equivalent force and a couple at the floor. Use: P1 = 5k, P2 = 10k, alpha = 20 degrees, M = 40 k ft, w = 2k/ft, d1 = 6 ft, d2 = 4 ft, d3 = 8 ft, d4 = 2 ft, d5 = 4 ft.

\[ \sum F_x = -5k \sin 20^\circ - 6k = -7.71k = R_y \checkmark \]
\[ \sum F_y = -5k \cos 20^\circ + 10k = 5.30k = R_x \checkmark \]
\[ \sum M_A = 5k \sin (20^\circ)(2') + 5k \cos (20^\circ)(18') - 10k(6') - 40k \text{ ft} = -12.01k \text{ ft} \checkmark \]

\[ R = \sqrt{R_x^2 + R_y^2} = 9.36k \checkmark \]
\[ \theta = \tan^{-1} \left( \frac{|R_y|}{R_x} \right) = 55.4^\circ \checkmark \]

\[ C = M_A = -12.01k \text{ ft} \checkmark \]
Problem 2) Determine the magnitude of the resulting force and the angle at which it acts from the z axis, for the forces shown below. Use $F_1 = 500 \text{kN}$, and $F_2 = 200 \text{kN}$.

$$\overrightarrow{OA} = \langle -y, z, 0 \rangle$$

$$|\overrightarrow{OA}| = \sqrt{y^2 + z^2} = 9.17$$

$$F_{2y} = F_z \left( \frac{z}{9.17} \right) = -97.24 \text{ kN}$$

$$F_{2y} = F_z \left( \frac{z}{9.17} \right) = 43.62 \text{ kN}$$

$$F_{2z} = F_z \left( \frac{z}{9.17} \right) = 174.49 \text{ kN}$$

$$F_{1y} = -F_1 \cos(60^\circ) = -250 \text{ kN}$$

$$F_{1y} = F_1 \cos(90^\circ) = 0$$

$$F_{1z} = -F_1 \cos(30^\circ) = -433.01 \text{ kN}$$

$$R_x = F_{1y} + F_{2x} = -337.24 \text{ kN}$$

$$R_y = F_{1y} + F_{2y} = 43.62 \text{ kN}$$

$$R_z = F_{1z} + F_{2z} = -259.53 \text{ kN}$$

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 427.16 \text{ kN}$$

$$\Theta_{R_z} = \cos^{-1} \left( \frac{R_z}{R} \right) = 52.75^\circ$$

$$R = 427.16 \text{ kN}$$

$$\Theta_{R_z} = 127.25^\circ$$
Problem 3) For the structure loaded as shown below, use vector forces and position vectors to determine the scalar component of the moment caused by $F_1$ about the shaft $AB$. Use $F_1 = 1400 \text{ N}$. You do not have to solve for the final answer, but can rather express your answer in determinant and dot products. However, you must fully express any determinants and dot product operations required for me to get a final answer. You must also put a box around any force or position vectors to be used in your solution.

$$\vec{F} = 808 \hat{i} - 808 \hat{j} + 808 \hat{k}$$

$$\vec{R}_{CA} = 0 \hat{i} + 450 \hat{j} + 200 \hat{k}$$

$$\vec{R}_{CB} = 0 \hat{i} - 200 \hat{j} + 200 \hat{k}$$

$$\vec{R}_{CD} = 0 \hat{i} + 0 \hat{j} + 200 \hat{k}$$

$$\vec{E}_{AB} = 0 \hat{i} + 1 \hat{j} + 0 \hat{k}$$

$$\vec{M}_{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ Ry & r_{3y} & r_{3z} \\ F_x & F_y & F_z \end{vmatrix} \cdot \vec{E}_{AB}$$

$$\sigma \vec{M}_{AB} = \begin{vmatrix} 0 & 1 & 0 \\ Ry & r_{3y} & r_{3z} \\ F_x & F_y & F_z \end{vmatrix}$$
Problem 4) Two circular rods, 8 feet long, are stacked in a ditch as shown. Rod A is steel (490 #/cu ft), with a 20" diameter. Rod B is brass (120 #/cu ft), with a 16" diameter. The ditch is 30 inches wide and the sides are smooth (no friction on the rods.) Determine all forces acting on the two rods and show these forces on a free body for each rod.

\[ \theta = \cos^{-1} \left(\frac{12}{18}\right) \approx 48.190^\circ \]

\[ W_B = \left[\pi \left(\frac{3}{12}\right)^2\right] (8') (120 #/ft^3) \]
\[ = 1340.41 \# \]

\[ \sum F_x = 0 = F_{WB} - F_{Ball} \cos(48.190^\circ) \]
\[ F_{WB} = 7649.24 \# \]

\[ \sum F_y = 0 = F_{A} - W_B - F_{Ball} \sin(48.190^\circ) \]
\[ F_{A} = 9892.53 \# \]

\[ W_A = \left[\pi \left(\frac{10}{12}\right)^2\right] (8') (490 #/ft^3) \]
\[ = 8552.11 \# \]

\[ \sum F_x = 0 = F_{Ball} \sin(48.190^\circ) - 8552.11 \# \]
\[ F_{Ball} = 11473.9 \# \]

\[ \sum F_y = 0 = F_{Ball} \cos(48.190^\circ) - F_{WA} \]
\[ F_{WA} = 7649.24 \# \]