4.12 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

Fig. 4.42 Member with eccentric loading.

Fig. 4.43 Internal forces in member with eccentric loading.
\[
\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}
\]

or, recalling Eqs. (1.5) and (4.16):

\[
\sigma_x = \frac{P}{A} + \frac{My}{I}
\]

(4.50)

Fig. 4.44 Stress distribution—eccentric loading.

\[
\frac{20}{y} = \frac{20+140}{30}
\]

Fig. 4.45 Alternative stress distribution—eccentric loading.

\[\text{+ = TENSION} \quad \text{-- = COMPRESSION} \]

\[\text{FINAL MAX} \]
\[ T = \frac{M_y - M_c}{I} = \frac{(6\text{ in})(1.5\text{ in})}{4.5\text{ in}^4} = 2\text{ ksi} \]

\[ I = \frac{(2')(3')^3}{12} = 4.5\text{ in}^4 \]

\[ M = 10\text{ in}^2 \text{ k} \]

\[ T = \frac{M_{yy} - M_{yy}}{I_{yy}} = \frac{M_{yy}(10\text{ in}^2)(1''')}{(3\text{ in})(2\text{ in})^3} = 5\text{ k/\text{in}^2} \]

\[ 6\text{ k/\text{in}^2} \]
Fig. 4.46

The distribution due to the bending couple \( M \) is linear with a maximum stress \( \sigma_m = \frac{Mc}{I} \). We write

\[
\frac{\pi d^4}{64} = I = \frac{1}{4} \pi c^4 = \frac{1}{4} \pi (0.25 \text{ in.})^4 = 3.068 \times 10^{-3} \text{ in}^4
\]

\[
\sigma_m = \frac{Mc}{I} = \frac{M(104 \text{ lb} \cdot \text{ in.})(0.25 \text{ in.})}{3.068 \times 10^{-3} \text{ in}^4} = 8475 \text{ psi}
\]
neutral axis have been assumed to be directed along the \( z \) axis. We recall from Sec. 4.2 that, if we then express that the elementary internal forces \( \sigma_x \, dA \) form a system equivalent to the couple \( \mathbf{M} \), we obtain

\[
\begin{align*}
\sigma_x &= \frac{M}{y} \\
\sigma_z &= \frac{M}{C}
\end{align*}
\]

\[\begin{align*}
\sigma_x &= \frac{M}{y} \\
\sigma_z &= \frac{M}{C}
\end{align*}\]

\[
\begin{align*}
x \text{ components:} & \quad \int \sigma_x \, dA = 0 \quad (4.1) \\
\text{moments about } y \text{ axis:} & \quad \int z \sigma_z \, dA = 0 \quad (4.2) \\
\text{moments about } z \text{ axis:} & \quad \int (-y \sigma_x \, dA) = M \quad (4.3)
\end{align*}
\]

for material, we can substitute \( \sigma_x = -\sigma_m \, y/c \) into Eq. (4.2) and write

\[
\sigma_m \left[ \int z y \left( -\frac{\sigma_m y}{c} \right) \, dA \right] = 0 \quad \text{or} \quad \int y z \, dA = 0 \quad (4.51)
\]

The integral \( \int y z \, dA \) represents the product of inertia \( I_{yz} \) of the cross-section with respect to the \( y \) and \( z \) axes, and will be zero if these...
vector $\mathbf{M}$ representing the forces acting on a given cross section will form the same angle $\theta$ with the horizontal $z$ axis (Fig. 4.55). Resolving the vector $\mathbf{M}$ into component vectors $M_z$ and $M_y$ along the $z$ and $y$ axes, respectively, we write

$$M_z = M \cos \theta \quad M_y = M \sin \theta \quad (4.52)$$

Since the $y$ and $z$ axes are the principal centroidal axes of the cross section, we can use Eq. (4.16) to determine the stresses resulting from the application of either of the couples represented by $M_z$ and $M_y$. The couple $M_z$ acts in a vertical plane and bends the member in that plane (Fig. 4.56). The resulting stresses are

$$\sigma_x = -\frac{M_z y}{I_z} \quad (4.53)$$

where $I_z$ is the moment of inertia of the section about the principal centroidal $z$ axis. The negative sign is due to the fact that we have compression above the $xy$ plane ($y > 0$) and tension below ($y < 0$).
$M_x = 60 \text{ kNm} \cos 63^\circ$

$\sigma = \frac{M_x C_{xx}}{I_{xx}}$

$A = \text{max tension}$

$C = \text{max comp}$
A 1600-lb in. couple is applied to a wooden beam, of rectangular cross section 1.5 by 3.5 in., in a plane forming an angle of 30° with the vertical (Fig. 4.60). Determine (a) the maximum stress in the beam, (b) the angle that the neutral surface forms with the horizontal plane.

Fig. 4.60

Fig. 4.61

Fig. 4.62

Fig. 4.63
4.14 GENERAL CASE OF ECCENTRIC AXIAL LOADING

In Sec. 4.12 you analyzed the stresses produced in a member by an eccentric axial load. Consider a member under an eccentric axial load with a transverse couple. The general case of eccentric axial loading is shown in Fig. 4.64.

Fig. 4.64 Eccentric axial loading.

\[
\sigma_y = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}
\]

(4.58)

\[
\frac{M_z}{I_z} y - \frac{M_y}{I_y} z = \frac{P}{A}
\]
A vertical 4.80-kN load is applied as shown on a wooden post of rectangular cross section, 80 by 120 mm (Fig. 4.65). (a) Determine the stress at points A, B, C, and D. (b) Locate the neutral axis of the cross section.

Fig. 4.65
Pop Quiz

What is value of \( I_{yy} \)?
Pop Quiz

What is the value of $I_{yy}$:

$120$ mm

$80$ mm

$I_{yy} = \frac{1}{12} bh^3$

$\frac{1}{12}(80)(120)^3 = 1.52 \times 10^{-4} m^4$

$\frac{1}{12}(0.08)(0.12)^3 = $