SAMPLE PROBLEM 4.9

A horizontal load $P$ is applied as shown to a short section of an S10 × 25.4 rolled-steel member. Knowing that the compressive stress in the member is not to exceed 12 ksi, determine the largest permissible load $P$.

$$P = P$$

$$M_{xx} = P \frac{C_{xx}}{C_{xx}} = P \left(4.75''\right)$$

$$\sigma_{xx} = \frac{M_{xx}}{C_{xx}} = \frac{P \left(4.75''\right)}{C_{xx}} = 24.7 \text{ ksi}$$

$$I_{xx} = \frac{1}{2} \pi \left(R^4 - r^4\right)$$

$$\sigma_{yy} = \frac{M_{yy}}{S_{yy}} = \frac{P \left(1.5''\right)}{2.91 \text{ ksi}}$$
Example

\[ \tau_A = \tau_B = \tau_C = \tau_D = \frac{P}{A} = \frac{60k}{(6\text{ in})(2\text{ in})} \]

\[ = -5.0 \text{ ksi COMP} \]

\[ I_{xx} = \frac{(2')(6\text{ in})^3}{12} = 36\text{ in}^4 \]

\[ M_{xx} = 60k(1.33') = 80\text{ in k} \]

\[ \sigma = \frac{M_{xx}}{I} = \frac{M_{xx} C_{xx}}{I_{xx}} = \frac{80\text{ in k}(3\text{ in})}{36\text{ in}^4} = \pm 6.67\text{ psi} \]

\[ \sigma_A = 6.67\text{ psi Tension} \]

\[ \sigma_B = 6.67\text{ psi Compression} \]

\[ \sigma_C = 6.67\text{ psi Comp} \]

\[ \tau_D = 6.66\text{ psi Tens} \]
\[ M_{yy} = 60 \times (0.333) = 20 \text{ in.k} \]
\[ I_{yy} = \frac{(6\text{ in})(2\text{ in})^3}{12} = \frac{4}{4\text{ in}^4} \]
\[ \sigma = \frac{M_{yy}C_{yy}}{I_{yy}} = \frac{20 \text{ in.k} \times 1\text{ in}}{4\text{ in}^4} = \pm 5 \text{ k/in}^2 \]

If put everything on there at once:

Worst stress \( \sigma_c = -5 \text{ ksi} - 6.67 - 5 \text{ ksi} = -16.67 \text{ ksi} \text{ (compression)} \)
Chapter 5 Analysis and Design of Beams for Bending

5.1 INTRODUCTION

Fig. 5.1 Transversely loaded beams.

Fig. 5.2 Common beam support configurations.
Fig. 5.3 Beams connected by hinges.

(a) Transversely-loaded beam

(b) Free-body diagram to find support reactions

(c) Free-body diagram to find internal forces at C

Fig. 5.4 Analysis of a simply supported beam.
\[ \sigma_m = \frac{|M|}{S} = \frac{M/C}{I} \] (5.3)
\[ \Sigma F_y = 0 = -V_0 + 8 \text{ kN} \]
\[ V_0 = +8 \text{ kN} \]
\[ \Sigma M_{cut} = 0 = M_0 - 8 \text{ kN} \times (0.000001) \]
\[ M_0 = 0 \]

\[ \Sigma F_v = 0 = +8 \text{ kN} - (2 \text{ kN/m})x \]
\[ V_x = +8 - 2x \]

\[ W = 2 \text{ kN/m} \]

\[ \Sigma M_{cut} = 0 = -(8 \text{ kN})(x) + (2 \text{ kN/m})(x)(x) + M_x \]
\[ M_x = 8x - x^2 \]
\[ = x(8-x) \]
\[ = 1(7) @ 1 \text{ m} \]
\[ = 2(6) @ 2 \text{ m} \]
\[ = 3(5) @ 3 \text{ m} \]
\[ = 4(4) @ 4 \text{ m} \]
\[ \Sigma F_V = 0 = +8 - 12 - V_X \]
\[ V_X = -4 \text{kN} \]
\[ \Sigma M_{\text{cut}} = 0 = M_X + 12(X - 3) - (8)(4) \]
\[ M_X = 8X - 12(X - 3) \]
Pop Quiz 6/21/12

4' 9 FT 18 K

9 K

6 k/ft

3/8 d

V at 4' = ?
These conventions can be more easily remembered if we note that

1. The shear at any given point of a beam is positive when the external forces (loads and reactions) acting on the beam tend to shear off the beam at that point as indicated in Fig. 5.6b.

![Diagram](image)

**Fig. 5.5** Determination of $V$ and $M$.

**Fig. 5.6** Sign convention for shear and bending moment.

*Note* that this convention is the same that we used earlier in Sec. 4.2.
### Constructing Shear and Moment Diagrams

#### Areas and Centroids

<table>
<thead>
<tr>
<th>Curve</th>
<th>Equation</th>
<th>Shape</th>
<th>Centroid (From Fat End of Figure)</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Nothing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>$ax^0$</td>
<td>Straight Horizontal line</td>
<td>b/2</td>
<td>bh</td>
</tr>
<tr>
<td>h</td>
<td>$ax^1$</td>
<td>Straight Sloping Line</td>
<td>b/3</td>
<td>bh/2</td>
</tr>
<tr>
<td>h</td>
<td>$ax^2$</td>
<td>Parabola</td>
<td>b/4</td>
<td>bh/3</td>
</tr>
<tr>
<td>h</td>
<td>$ax^3$</td>
<td>Cubic</td>
<td>b/5</td>
<td>bh/4</td>
</tr>
</tbody>
</table>

$b$ is the length of the member

#### To Solve For The Reactions:

1) Draw a free body of the beam showing any actual distributed loads.
2) Draw a second free body, replacing any distributed loads with their equivalent concentrated loads. The magnitude of the loads can be computed from their areas and placed at their centroids, as listed in the table above. Note that the areas shown are for the equations listed only. Thus the area for $3x^2 + 2x$ is not listed! Also, the zero ends of the parabolas, cubics, etc. are vertices (i.e. the shape starts with zero slope.) The areas are not listed for any other conditions.
3) Sum moments about the left reaction to determine the right reaction. Then sum moments about the right reaction to determine the left reaction. Then sum forces vertically to check the results.
4) Erase the second load diagram with the distributed loads replaced by concentrated loads. This diagram is used ONLY to solve for the reactions.

#### To Construct A Shear Diagram

1) Under the first load diagram, drop vertical lines at every concentrated load, at every concentrated moment, and at both ends of every distributed load.
2) Starting at the left end of the figure, do whatever the loads tell you to do. If you cross a zero width load (a concentrated load) going DOWN, the area under that load (it’s magnitude) will drive the shear diagram DOWN by the magnitude of that load, over the zero width distance. (Replace DOWN with UP when appropriate.) Thus after passing a concentrated load, the value of the shear diagram should instantaneously change by the magnitude of the load, and in the direction that the load is pointing.

3) If you cross a distributed load going DOWN, the magnitude under that distributed load (it’s area) will drive the shear diagram DOWN by that amount, over the base dimension of the distributed load. (Replace DOWN with UP when appropriate.) Thus after you finish passing over the width of a distributed load, the value of the shear diagram will have changed by the magnitude of the distributed load, and in the direction that load is pointing. Distributed loads that point down drive the shear diagram down, and vise versa.

4) The shape of the load diagram will determine the shape of the shear diagram directly below. The shape of the load diagram always turns into the next shape shown in the “Areas and Centroids” table above. Thus if the load is a straight horizontal line, the shape of the shear diagram will be a straight sloping line. If the load diagram is a parabola, the shear diagram will be a cubic.

5) You can tell if a triangular load diagram should “turn into” a “skinny” parabola or a “fat” parabola by using the calculus: The value at any point on any diagram “turns into” (integrates into) the slope of the next diagram. Thus if you see a zero magnitude load anywhere on a beam, you should see a zero magnitude slope on the shear diagram at this same point. If you see small loads, they should “turn into” shear diagrams with “small” slopes. If you see big loads, they should “turn into” big slopes on the shear diagram.

6) Since a concentrated moment has no “up and down” force, it does not cause any change in the magnitude of the shear diagram at its point of application. That does not mean that they do not influence the shear diagram, because they do. They influence it by changing the reactions, which in turn influences the shear diagram. Thus you will see no change in the shear diagram at the point of application of a concentrated moment.

7) To determine where the shear diagram crosses the x-axis: $X_{bar} = \text{Starting Shear} \div \text{Load Rate}$

**To Construct A Moment Diagram**

1) Under the shear diagram, drop vertical lines at every point of interest including every time the shear diagram crosses the axis, and at concentrated moments.

2) Starting at the left end of the figure, do whatever the shears tell you to do. If you cross a distributed shear going DOWN, the magnitude under that distributed shear (it’s area) will drive the moment diagram DOWN by that amount, over the base dimension of the distributed shear. (Replace DOWN with UP when appropriate.) Thus after you finish passing over the width of a distributed shear, the value of the moment diagram will have changed by the magnitude of the distributed shear, and in the direction that the shear tells you. Since the shear areas will not have little arrows pointing up or down, as did the load diagrams, use shear areas above the axis as positive (pushes the moment diagram up) and shear areas below the axis as negative (pushes the moment diagram down.)

3) The shape of the shear diagram will predict the shape of the moment diagram directly below. The shape of the shear diagram always turns into the next shape shown in the “Areas and Centroids” table above. Thus if the shear is a straight sloping line, the shape of the shear
diagram will be a parabola. If the shear diagram is a parabola, the moment diagram will be a cubic.

4) You can tell if a triangular shear diagram should “turn into” a “skinny” parabola or a “fat” parabola by using the calculus: The value at any point on any diagram “turns into” (integrates into) the slope of the next diagram. Thus if you see a zero magnitude shear anywhere on a beam, you should see a zero magnitude slope on the moment diagram at this same point. If you see small shears, they should “turn into” moment diagrams with “small” slopes. If you see big shears, they should “turn into” big slopes on the moment diagram.

5) Concentrated moments cause the magnitude of the moment diagram to “jump” at their points of application. Clockwise external moments applied to a beam cause the internal moment in the beam, to the right of the application point, to go positive (or more positive than if the moment were not applied.) Thus, clockwise external moments applied to a beam cause the moment diagram to instantly “jump up” from its current value. The amount that the moment diagram “jumps up” is the magnitude of the applied moment. Counterclockwise moments cause the moment diagram to “jump down.” Thus you get the interesting effect that “positive” externally applied moments (using your statics sign convention) cause “negative” jumps in the moment diagram (using your beam design sign convention.) Again, you must be drawing your diagrams from left to right for these rules to apply.
For $L_a = 4$ ft, $L_b = 6$ ft, $L_c = 8$ ft, $L_d = 10$ ft, $P = 4$ kips, $M = 100$ kip ft, and $w = 20$ kips/ft
$Ra = 66.42$ kips, $Re = 173.57$ kips
$Va = 66.43$ kips, $Vb = Vc = Vd = 26.43$ kips, $Ve = -173.57$ kips
$M_b = 265.71$ kip ft, $M_{ca} = 424.29$ kip ft, $M_{cb} = 524.29$ kip ft, $M_d = 735.71$ kip ft
$M_{max} = 753.18$ kip ft, $X_{bar} = 11.321$ ft.

Procedure:

1) The starting value on the present curve = 
2) As you go from _____ to _____
3) you cross an area under the present curve =
4) The area crossed is POSITIVE/NEGATIVE.
5) thus driving the next diagram UP/DOWN by that amount at the end of the move,
6) thus giving a height of the next diagram =
7) The value on the left end of the present curve =
8) thus the slope on the left end on the next curve =
9) The value on the right end of the present curve =
10) thus the slope on the right end on the next curve =
11) The order of the present curve is
12) thus the order of the next curve is
13) Drawing the next curve thus gives what shape of curve?
14) Concave up or down?
15) Since the starting value of the next curve =
16) and the rate at which it is being pushed down from the curve above =
17) the next curve will cross the x-axis at

\[
\frac{40 \text{ kN}}{8 \text{ kN/m}} = 5 \text{ m}
\]
EXAMPLE 5.01

Draw the shear and bending-moment diagrams for a simply supported beam $AB$ of span $L$ subjected to a single concentrated load $P$ at its midpoint $C$ (Fig. 5.7).

(a) 

(b) 

(c) 

(d)