UNIAXIAL STRESS-STRAIN

Stress-Strain Curve for Mild Steel

The slope of the linear portion of the curve equals the modulus of elasticity.

**DEFINITIONS**

**Engineering Strain**
\[ \varepsilon = \frac{\Delta L}{L_o}, \]
where
- \( \varepsilon \) = engineering strain (units per unit),
- \( \Delta L \) = change in length (units) of member,
- \( L_o \) = original length (units) of member.

**Percent Elongation**
\[ \text{% Elongation} = \left( \frac{\Delta L}{L_o} \right) \times 100 \]

**Percent Reduction in Area (RA)**
The % reduction in area from initial area, \( A_i \), to final area, \( A_f \), is:
\[ \text{%RA} = \left( \frac{A_i - A_f}{A_i} \right) \times 100 \]

**Shear Stress-Strain**
\[ \gamma = \frac{\tau}{G}, \]
where
- \( \gamma \) = shear strain,
- \( \tau \) = shear stress,
- \( G \) = shear modulus (constant in linear torsion-rotation relationship).

**True Stress and True Strain**
\[ \sigma = \frac{P}{A}, \]
\[ \varepsilon = \frac{\delta}{L}, \]
where
- \( \sigma \) = stress on the cross section,
- \( P \) = loading, and
- \( A \) = cross-sectional area.

**THERMAL DEFORMATIONS**
\[ \delta_t = \alpha L (T - T_o), \]
where
- \( \delta_t \) = deformation caused by a change in temperature,
- \( \alpha \) = temperature coefficient of expansion,
- \( L \) = length of member,
- \( T \) = final temperature, and
- \( T_o \) = initial temperature.

**CYLINDRICAL PRESSURE VESSEL**

**Cylindrical Pressure Vessel**
For internal pressure only, the stresses at the inside wall are:
\[ \sigma_t = \frac{P}{r_i^2 - r_o^2} \quad \text{and} \quad \sigma_r = -P; \]

For external pressure only, the stresses at the outside wall are:
\[ \sigma_t = -\frac{P_o}{r_o^2 - r_i^2} \quad \text{and} \quad \sigma_r = -P_o, \]
where
- \( \sigma_t \) = tangential (hoop) stress,
- \( \sigma_r \) = radial stress,
- \( P_i \) = internal pressure,
- \( P_o \) = external pressure,
- \( r_i \) = inside radius, and
- \( r_o \) = outside radius.

For vessels with end caps, the axial stress is:
\[ \sigma_a = \frac{P_o}{r_o^2 - r_i^2} \]

\( \sigma_t, \sigma_r, \) and \( \sigma_a \) are principal stresses.

When the thickness of the cylinder wall is about one-tenth or less of its inside radius, the cylinder can be considered as thin-walled. In which case, the internal pressure is resisted by the hoop stress and the axial stress.

\[ \sigma_t = \frac{Pr}{t} \quad \text{and} \quad \sigma_a = \frac{Pr}{2t} \]

where \( t \) = wall thickness.

**STRESS AND STRAIN**

**Principal Stresses**

For the special case of a *two-dimensional* stress state, the equations for principal stress reduce to

\[ \sigma_u, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \]

\[ \sigma_c = 0 \]

The two nonzero values calculated from this equation are temporarily labeled \( \sigma_u \) and \( \sigma_b \) and the third value \( \sigma_c \) is always zero in this case. Depending on their values, the three roots are then labeled according to the convention:

- algebraically largest = \( \sigma_1 \)
- algebraically smallest = \( \sigma_3 \)
- other = \( \sigma_2 \)

A typical 2D stress element is shown below with all indicated components shown in their positive sense.

**Mohr’s Circle – Stress, 2D**

To construct a Mohr’s circle, the following sign conventions are used:

1. Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are negative.
2. For constructing Mohr’s circle only, shearing stresses are plotted above the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, forms a clockwise couple. Shearing stresses are plotted below the normal axis when the shear stresses form a counterclockwise couple.

The circle drawn with the center on the normal stress (horizontal) axis with center, \( C \), and radius, \( R \), where

\[ C = \frac{\sigma_x + \sigma_y}{2} \quad \text{and} \quad R = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \]

The two non-zero principal stresses are then:

\[ \sigma_a = \sigma_x + \sigma_y \quad \text{and} \quad \sigma_b = \sigma_x - \sigma_y \]

The maximum inplane shear stress is \( \tau_{xy} = R \). However, the maximum shear stress considering three dimensions is always

\[ \tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} \]

**Hooke’s Law**

Three-dimensional case:

\[ \varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \gamma_{xy} = \tau_{xy}/G \]

\[ \varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \quad \gamma_{yz} = \tau_{yz}/G \]

\[ \varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \quad \gamma_{xz} = \tau_{xz}/G \]

Plane stress case (\( \sigma_z = 0 \)):

\[ \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad \gamma_{xy} = \frac{1}{1-\nu} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 - \nu \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau_{xy} \end{bmatrix} \]

Uniaxial case (\( \sigma_y = \sigma_z = 0 \)):

\[ \sigma_x = E\varepsilon_x \text{ or } \sigma = E\varepsilon_x \]

where \( \varepsilon_x, \varepsilon_y, \varepsilon_z \) = normal strain,

\( \sigma_x, \sigma_y, \sigma_z \) = normal stress,

\( \gamma_{xy}, \gamma_{yz}, \gamma_{xz} \) = shear strain,

\( \tau_{xy}, \tau_{yz}, \tau_{xz} \) = shear stress,

\( E \) = modulus of elasticity,

\( G \) = shear modulus, and

\( \nu \) = Poisson’s ratio.

---


34  **MECHANICS OF MATERIALS**


\[ V(0, 50.96 \text{ ksi}) \]

\[ \Sigma_t = \begin{pmatrix} \sigma_{yy} \cr \tau_{xy} \end{pmatrix} \]

\[ H(+25.48 \text{ ksi}, -50.96 \text{ ksi}) \]

\[ 40k/0.785 \text{ in}^2 = 50.96 \text{ ksi} \]

\[ \Sigma_{yy} = 20\% \times 0.785 \text{ in}^2 = 25.48 \text{ ksi} \]

\[ \sin 2\theta \]

\[ \frac{12.74, 0}{52.53} = \tan \alpha \]

\[ \alpha = 13.6^\circ \]
\[ T_{\text{max}} = 15 \text{ ksi} \]

\[ T_{\text{max}} = R_{\text{Mohr's Circle}} \]

\[ T_{\text{max}} = 20 \text{ ksi} \]

\[ T_{\text{max}} = 10 \text{ ksi} \]

\[ T_{\text{max}} = 15 \text{ ksi} \]
STATIC LOADING FAILURE THEORIES

See MATERIALS SCIENCE/STRUCTURE OF MATTER for Stress Concentration in Brittle Materials.

Brittle Materials

Maximum-Normal-Stress Theory
The maximum-normal-stress theory states that failure occurs when one of the three principal stresses equals the strength of the material. If \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \), then the theory predicts that failure occurs whenever \( \sigma_1 \geq S_{ut} \) or \( \sigma_2 \leq -S_{uc} \). Where \( S_{ut} \) and \( S_{uc} \) are the tensile and compressive strengths, respectively.

Coulomb-Mohr Theory
The Coulomb-Mohr theory is based upon the results of tensile and compression tests. On the \( \sigma, \tau \) coordinate system, one circle is plotted for \( S_{ut} \) and one for \( S_{uc} \). As shown in the figure, lines are then drawn tangent to these circles. The Coulomb-Mohr theory then states that fracture will occur for any stress situation that produces a circle that is either tangent to or crosses the envelope defined by the lines tangent to the \( S_{ut} \) and \( S_{uc} \) circles.

If \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \) and \( \sigma_3 < 0 \), then the theory predicts that yielding will occur whenever
\[
\frac{\sigma_1 - \sigma_3}{S_{ut} - S_{uc}} \geq 1
\]

Ductile Materials

Maximum-Shear-Stress Theory
The maximum-shear-stress theory states that yielding begins when the maximum shear stress equals the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield. If \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \), then the theory predicts that yielding will occur whenever \( \tau_{max} \geq S_y/2 \) where \( S_y \) is the yield strength.
\[
\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}
\]

Distortion-Energy Theory
The distortion-energy theory states that yielding begins whenever the distortion energy in a unit volume equals the distortion energy in the same volume when uniaxially stressed to the yield strength. The theory predicts that yielding will occur whenever
\[
\left( \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2} \right)^{1/2} \geq S_y
\]
The term on the left side of the inequality is known as the effective or Von Mises stress. For a biaxial stress state the effective stress becomes
\[
\sigma' = \left( \sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 \right)^{1/2}
\]
or
\[
\sigma' = \left( \sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \right)^{1/2}
\]
where \( \sigma_x \) and \( \sigma_y \) are the two nonzero principal stresses and \( \sigma_y \), \( \sigma_x \), and \( \tau_{xy} \) are the stresses in orthogonal directions.

VARIABLE LOADING FAILURE THEORIES

Modified Goodman Theory: The modified Goodman criterion states that a fatigue failure will occur whenever
\[
\frac{\sigma_f}{S_e} + \frac{\sigma_m}{S_{ut}} \geq 1 \quad \text{or} \quad \frac{\sigma_{max}}{S_y} \geq 1, \quad \sigma_m \geq 0,
\]
where
\( S_e \) = fatigue strength,
\( S_{ut} \) = ultimate strength,
\( S_y \) = yield strength,
\( \sigma_f \) = alternating stress, and
\( \sigma_m \) = mean stress.
\( \sigma_{max} \) = \( \sigma_m + \sigma_f \)

Soderberg Theory: The Soderberg theory states that a fatigue failure will occur whenever
\[
\frac{\sigma_f}{S_y} + \frac{\sigma_m}{S_y} \geq 1 \quad \sigma_m \geq 0
\]

Endurance Limit for Steels: When test data is unavailable, the endurance limit for steels may be estimated as
\[
S_e = \begin{cases} 0.5 S_{ut}, & S_{ut} \leq 1,400 \text{ MPa} \\ 700 \text{ MPa}, & S_{ut} > 1,400 \text{ MPa} \end{cases}
\]
Fatigue Strength

Effect of *mean stress*

Fluctuating stress = static stress + completely reversed stress

mean + alternating

\[ \sigma_m = \text{mean stress}; \sigma_a = \text{alternating stress (or stress amplitude)} \]

\[ \sigma_{\text{max}} = \text{maximum stress}; \sigma_{\text{min}} = \text{minimum stress} \]

\[ \sigma_m = (\sigma_{\text{max}} + \sigma_{\text{min}})/2 \]

\[ \sigma_a = (\sigma_{\text{max}} - \sigma_{\text{min}})/2 \]
Endurance Limit Modifying Factors: Endurance limit modifying factors are used to account for the differences between the endurance limit as determined from a rotating beam test, $S'_e$, and that which would result in the real part, $S_e$.

$$S_e = k_a k_b k_c k_d k_e S'_e$$

where

Surface Factor, $k_a = aS^b$

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>Factor $a$</th>
<th>Exponent $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kpsi</td>
<td>MPa</td>
</tr>
<tr>
<td>Ground</td>
<td>1.34</td>
<td>1.58</td>
</tr>
<tr>
<td>Machined or CD</td>
<td>2.70</td>
<td>4.51</td>
</tr>
<tr>
<td>Hot rolled</td>
<td>14.4</td>
<td>57.7</td>
</tr>
<tr>
<td>As forged</td>
<td>39.9</td>
<td>272.0</td>
</tr>
</tbody>
</table>

Size Factor, $k_s$:

- For bending and torsion:
  - $d \leq 8$ mm; $k_b = 1$
  - $8 \leq d \leq 250$ mm; $k_b = 1.189d^{-0.097}$
  - $d > 250$ mm; $0.6 \leq k_b \leq 0.75$
- For axial loading: $k_b = 1$

Load Factor, $k_c$:

- $k_c = 0.923$ axial loading, $S_{tu} \leq 1,520$ MPa
- $k_c = 1$ axial loading, $S_{tu} > 1,520$ MPa
- $k_c = 0.577$ bending
- $k_c = 1$ torsion

Temperature Factor, $k_d$:

For $T \leq 450^\circ$C, $k_d = 1$

Miscellaneous Effects Factor, $k_e$:

Used to account for strength reduction effects such as corrosion, plating, and residual stresses. In the absence of known effects, use $k_e = 1$.

TORSION STRAIN

$$\gamma_{kz} = \lim_{kz \to 0} r(\Delta \phi/\Delta z) = r(d\phi/dz)$$

The shear strain varies in direct proportion to the radius, from zero strain at the center to the greatest strain at the outside of the shaft. $d\phi/dz$ is the twist per unit length or the rate of twist.

$$\tau_{kz} = G \gamma_{kz} = G r(d\phi/dz)$$

$$T = G(d\phi/dz) \int r^2 dA = GJ(d\phi/dz)$$

$$\phi = \int_0^L \frac{T}{GJ} dz = \frac{Tl}{GJ}$$, where

$\phi$ = total angle (radians) of twist,

$T$ = torque, and

$L$ = length of shaft.

$T/l\phi$ gives the twisting moment per radian of twist. This is called the torsional stiffness and is often denoted by the symbol $k$ or $c$.

For Hollow, Thin-Walled Shafts

$$\tau = \frac{T}{2A_m t}$$ where

$t$ = thickness of shaft wall and

$A_m$ = the total mean area enclosed by the shaft measured to the midpoint of the wall.

BEAMS

Shearing Force and Bending Moment Sign Conventions

1. The bending moment is positive if it produces bending of the beam concave upward (compression in top fibers and tension in bottom fibers).

2. The shearing force is positive if the right portion of the beam tends to shear downward with respect to the left.

TORSION

Torsion stress in circular solid or thick-walled ($t > 0.1 r$) shafts:

$$\tau = \frac{T_r}{J}$$

where $J$ = polar moment of inertia (see table at end of STATICS section).

---

The relationship between the load \( q \), shear \( V \), and moment \( M \) equations are:

\[
q(x) = -\frac{dV(x)}{dx} \\
V = \frac{dM(x)}{dx} \\
V_2 - V_1 = \int_a^x [-q(x)] dx \\
M_2 - M_1 = \int_a^x V(x) dx
\]

**Stresses in Beams**

\[ \varepsilon_x = -\frac{y}{\rho}, \text{ where} \]
\[ \rho = \text{the radius of curvature of the deflected axis of the beam, and} \]
\[ y = \text{the distance from the neutral axis to the longitudinal fiber in question.} \]

Using the stress-strain relationship \( \sigma = E\varepsilon \),

**Axial Stress:** \( \sigma_x = -\frac{Ey}{\rho} \), where

\[ \sigma_x = \text{the normal stress of the fiber located y-distance from the neutral axis.} \]
\[ 1/\rho = M/(EI), \text{ where} \]
\[ M = \text{the moment at the section and} \]
\[ I = \text{the moment of inertia of the cross section.} \]

\[ \sigma_x = -\frac{My}{l}, \text{ where} \]
\[ y = \text{the distance from the neutral axis to the fiber location above or below the axis. Let } y = c, \text{ where } c = \text{distance from the neutral axis to the outermost fiber of a symmetrical beam section.} \]

\[ \sigma_x = \pm \frac{Mc}{l} \]

Let \( S = I/c: \text{ then, } \sigma_x = \pm M/S \), where

\[ S = \text{the elastic section modulus of the beam member.} \]

**Transverse shear flow:** \( q = VQ/I \) and

**Transverse shear stress:** \( \tau_{xy} = VQ/(lb) \), where

\[ q = \text{shear flow,} \]
\[ \tau_{xy} = \text{shear stress on the surface,} \]
\[ V = \text{shear force at the section,} \]
\[ b = \text{width or thickness of the cross-section, and} \]
\[ Q = A'y', \text{ where} \]
\[ A' = \text{area above the layer (or plane) upon which the desired transverse shear stress acts and} \]
\[ y' = \text{distance from neutral axis to area centroid.} \]

---

**Deflection of Beams**

Using \( 1/p = M/(EI) \),

\[
EI \frac{d^2y}{dx^2} = M, \text{ differential equation of deflection curve} \\
EI \frac{d^3y}{dx^3} = \frac{dM(x)}{dx} = V \\
EI \frac{d^4y}{dx^4} = \frac{dV(x)}{dx} = -q
\]

Determine the deflection curve equation by double integration (apply boundary conditions applicable to the deflection and/or slope).

\[
EI \left( \frac{dy}{dx} \right) = \int M(x) \ dx \\
Ely = \int \left[ \frac{dM(x)}{dx} \right] \ dx
\]

The constants of integration can be determined from the physical geometry of the beam.

**COLUMNS**

For long columns with pinned ends:

**Euler's Formula**

\[
P_{cr} = \frac{\pi^2 EI}{l^2}, \text{ where} \]

\[ P_{cr} = \text{critical axial loading,} \]
\[ l = \text{unbraced column length.} \]

substitute \( I = r^2 A \):

\[
\frac{P_{cr}}{A} = \frac{\pi^2 E}{(\psi r)^2}, \text{ where} \]

\[ r = \text{radius of gyration and} \]
\[ \psi r = \text{slenderness ratio for the column.} \]

For further column design theory, see the CIVIL ENGINEERING and MECHANICAL ENGINEERING sections.
### Table C-C2.2
**Approximate Values of Effective Length Factor, K**

<table>
<thead>
<tr>
<th>Buckled Shape of Column Is Shown by Dashed Line.</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical K Value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended Design Value When Ideal Conditions Are Approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**End Condition Code**
- Rotation fixed and translation fixed
- Rotation free and translation fixed
- Rotation fixed and translation free
- Rotation free and translation free

For column ends supported by, but not rigidly connected to, a footing or foundation, G is theoretically infinity but unless designed as a true friction-free pin, may be taken as 10 for practical designs. If the column end is rigidly attached to a properly designed footing, G may be taken as 1.0. Smaller values may be used if justified by analysis.

**AISC Figure C-C2.3**
Alignment chart, sidesway prevented

**AISC Figure C-C2.4**
Alignment chart, sidesway not prevented

ELASTIC STRAIN ENERGY
If the strain remains within the elastic limit, the work done during deflection (extension) of a member will be transformed into potential energy and can be recovered.

If the final load is $P$ and the corresponding elongation of a tension member is $\delta$, then the total energy $U$ stored is equal to the work $W$ done during loading.

$$U = W = P\delta/2$$

The strain energy per unit volume is

$$u = U/AL = \sigma^2/2E$$

(for tension)

MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>Units</th>
<th>Steel</th>
<th>Aluminum</th>
<th>Cast Iron</th>
<th>Wood (Fir)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity, $E$</td>
<td>Mpsi</td>
<td>29.0</td>
<td>10.0</td>
<td>14.5</td>
<td>1.6</td>
</tr>
<tr>
<td>GPa</td>
<td>200.0</td>
<td>69.0</td>
<td>100.0</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>Modulus of Rigidity, $G$</td>
<td>Mpsi</td>
<td>11.5</td>
<td>3.8</td>
<td>6.0</td>
<td>0.6</td>
</tr>
<tr>
<td>GPa</td>
<td>80.0</td>
<td>26.0</td>
<td>41.4</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>Poisson's Ratio, $v$</td>
<td></td>
<td>0.30</td>
<td>0.33</td>
<td>0.21</td>
<td>0.33</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion, $\alpha$</td>
<td>10^{-6}/F</td>
<td>6.5</td>
<td>13.1</td>
<td>6.7</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>10^{-6}/C</td>
<td>11.7</td>
<td>23.6</td>
<td>12.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Beam Deflection Formulas – Special Cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(δ is positive downward)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Formula</th>
<th>ϕ&lt;sub&gt;max&lt;/sub&gt;</th>
<th>δ&lt;sub&gt;max&lt;/sub&gt;</th>
<th>δ(x) = \frac{wx^2}{24EI} (L^2 - 2Lx + x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \delta = \frac{Pa^2}{6EI} (3x - a) ), for ( x &gt; a )</td>
<td>( \frac{P \delta}{2EI} )</td>
<td>( \frac{P \delta}{2EI} )</td>
<td>( \frac{P \delta}{2EI} )</td>
</tr>
<tr>
<td>2</td>
<td>( \delta = \frac{Px}{6EI} (-x + 3a) ), for ( x \leq a )</td>
<td>( \frac{P \delta}{2EI} )</td>
<td>( \frac{P \delta}{2EI} )</td>
<td>( \frac{P \delta}{2EI} )</td>
</tr>
<tr>
<td>3</td>
<td>( \delta = \frac{wL^4}{24EI} \left( x^2 + 6L^2 - 4Lx \right) )</td>
<td>( \frac{wL^4}{24EI} )</td>
<td>( \frac{wL^4}{24EI} )</td>
<td>( \frac{wL^4}{24EI} )</td>
</tr>
<tr>
<td>4</td>
<td>( \delta = \frac{ML^2}{2EI} )</td>
<td>( \frac{ML^2}{2EI} )</td>
<td>( \frac{ML^2}{2EI} )</td>
<td>( \frac{ML^2}{2EI} )</td>
</tr>
<tr>
<td>5</td>
<td>( \delta = \frac{Pb(L^2 - b^2)}{6EI} ) at ( x = \sqrt{\frac{L^2 - b^2}{3}} )</td>
<td>( \frac{Pb(L^2 - b^2)}{6EI} )</td>
<td>( \frac{Pb(L^2 - b^2)}{6EI} )</td>
<td>( \frac{Pb(L^2 - b^2)}{6EI} )</td>
</tr>
<tr>
<td>6</td>
<td>( \delta = \frac{wL^4}{24EI} \left( L^2 - 2Lx + x^2 \right) )</td>
<td>( \frac{wL^4}{384EI} )</td>
<td>( \frac{wL^4}{384EI} )</td>
<td>( \frac{wL^4}{384EI} )</td>
</tr>
<tr>
<td>7</td>
<td>( \delta = \frac{wL^4}{384EI} ) at ( x = \frac{L}{2} ), ( \frac{wL^4}{384EI} ) at ( x = \frac{1}{2} \pm \frac{L}{\sqrt{12}} )</td>
<td>( \frac{wL^4}{384EI} )</td>
<td>( \frac{wL^4}{384EI} )</td>
<td>( \frac{wL^4}{384EI} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
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<td><img src="image1" alt="Diagram 1" /></td>
<td>$A = bh/2$  $x_c = 2b/3$  $y_c = h/3$</td>
<td>$I_x = bh^3/36$  $I_y = b'h/36$  $I_n = bh^1/12$  $I_v = b^3h/4$</td>
<td>$r_x^2 = h^2/18$  $r_y^2 = b^2/18$  $r_n^2 = h^2/6$  $r_v^2 = b^2/2$</td>
<td>$I_{x,y} = Abh/36 = b^2h^2/72$  $I_{x,y} = Abh/4 = b^2h^2/8$</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram 2" /></td>
<td>$A = bh/2$  $x_c = b/3$  $y_c = h/3$</td>
<td>$I_x = bh^3/36$  $I_y = b'h/36$  $I_n = bh^1/12$  $I_v = b^3h/12$</td>
<td>$r_x^2 = h^2/18$  $r_y^2 = b^2/18$  $r_n^2 = h^2/6$  $r_v^2 = b^2/6$</td>
<td>$I_{x,y} = -Abh/36 = -b^2h^2/72$  $I_{xy} = Abh/12 = b^2h^2/24$</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram 3" /></td>
<td>$A = bh/2$  $x_c = (a+b)/3$  $y_c = h/3$</td>
<td>$I_x = bh^3/36$  $I_y = [bh(b^2 - ab + a^2)]/36$  $I_n = bh^1/12$  $I_v = [bh(b^2 + ab + a^2)]/12$</td>
<td>$r_x^2 = h^2/18$  $r_y^2 = (b^2 - ab + a^2)/18$  $r_n^2 = h^2/6$  $r_v^2 = (b^2 + ab + a^2)/6$</td>
<td>$I_{x,y} = [Ah(2a-b)]/36$  $I_{x,y} = -[bh^2(2a-b)]/18$  $I_{xy} = [Ah(2a+b)]/12$  $I_{xy} = -[bh^2(2a+b)]/24$</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram 4" /></td>
<td>$A = bh$  $x_c = b/2$  $y_c = h/2$</td>
<td>$I_x = bh^3/12$  $I_y = b'h/12$  $I_n = bh^1/3$  $I_v = b^3h/3$  $J = bh(b^2 + h^2)/12$</td>
<td>$r_x^2 = h^2/12$  $r_y^2 = b^2/12$  $r_n^2 = h^2/3$  $r_v^2 = b^2/3$  $r_p^2 = (b^2 + h^2)/12$</td>
<td>$I_{x,y} = 0$  $I_{xy} = Abh/4 = b^2h^2/4$</td>
</tr>
<tr>
<td><img src="image5" alt="Diagram 5" /></td>
<td>$A = h(a+b)/2$  $y_c = h(2a+b)/(3(a+b))$</td>
<td>$I_x = h^3(a^2 + 4ab + b^2)/(36(a+b))$  $I_y = h^3(3a + b)/(12(a+b))$</td>
<td>$r_x^2 = h^2(a^2 + 4ab + b^2)/(18(a+b))$  $r_y^2 = h^2(3a + b)/(6(a+b))$</td>
<td>$I_{x,y} = 0$  $I_{xy} = Abh/4 = b^2h^2/4$</td>
</tr>
<tr>
<td><img src="image6" alt="Diagram 6" /></td>
<td>$A = ab\sin\theta$  $x_c = (b + a\cos\theta)/2$  $y_c = (a\sin\theta)/2$</td>
<td>$I_x = (ab\sin^3\theta)/12$  $I_y = [ab\sin\theta(b^2 + a^2\cos^2\theta)]/12$  $I_n = (a^2b\sin^3\theta)/3$  $I_v = [ab\sin\theta(b + a\cos\theta)]/3$  $I_x = (a\sin\theta)^2/12$  $I_y = (b^2 + a^2\cos^2\theta)/12$  $I_n = (a^2\sin^3\theta)/3$  $I_v = (b + a\cos\theta)^2/3$  $I_x = -ab\cos^2\theta)/6$  $I_y = (a\sin\theta^2\cos\theta)/6$</td>
<td>$r_x^2 = (a^2\sin^2\theta)/12$  $r_y^2 = (b^2 + a^2\cos^2\theta)/12$  $r_n^2 = (a^2\sin^3\theta)/3$  $r_v^2 = (b + a\cos\theta)^2/3$  $r_p^2 = -ab\cos^2\theta)/6$  $r_m^2 = (a\sin\theta^2\cos\theta)/6$</td>
<td>$I_{x,y} = (a^2\sin^2\theta\cos\theta)/12$  $I_{xy} = (a\sin\theta^2\cos\theta)/6$</td>
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| ![Circle](image) | $A = \pi a^2$  
$x_c = a$  
$y_c = a$  
$J = \pi a^4/2$ | $I_{x_c} = I_{y_c} = \pi a^4/4$  
$I_x = I_y = 5\pi a^4/4$  
$J = \pi a^4/2$ | $r_x^2 = r_y^2 = a^2/4$  
$r_x^2 = r_y^2 = 5a^2/4$  
$r_p^2 = a^2/2$ | $I_{x_c y_c} = 0$  
$I_{x y} = A a^2$ |
| ![Annulus](image) | $A = \pi(a^2 - b^2)$  
$x_c = a$  
$y_c = a$  
$J = \pi(a^4 - b^4)/2$ | $I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$  
$I_x = I_y = (5\pi a^4/4 - \pi a^2 b^2 - \pi b^4/4)$  
$J = \pi(a^4 - b^4)/2$ | $r_x^2 = r_y^2 = (a^2 + b^2)/4$  
$r_x^2 = r_y^2 = (5a^2 + b^2)/4$  
$r_p^2 = (a^2 + b^2)/2$ | $I_{x_c y_c} = 0$  
$I_{x y} = A a^2$  
$= \pi a^2(a^2 - b^2)$ |
| ![Quarter Circle](image) | $A = \pi a^2/2$  
$x_c = a$  
$y_c = 4a(3\pi)$  
$J = 2\pi a^4/8$ | $I_{x_c} = a^4(\theta - \sin \theta \cos \theta)/4$  
$I_{y_c} = a^4/8$  
$I_x = \pi a^4/8$  
$I_y = 5\pi a^4/8$ | $r_x^2 = a^2(\theta - \sin \theta \cos \theta)/4$  
$r_y^2 = a^2/4$  
$r_x^2 = a^2/4$  
$r_y^2 = 5a^2/4$ | $I_{x_c y_c} = 0$  
$I_{x y} = 2a^4/3$ |
| ![Circular Sector](image) | $A = a^2\theta$  
$x_c = 2a \sin \theta/3$  
$y_c = 0$  
$J = \pi a^4/8$ | $I_x = a^4(\theta - \sin \theta \cos \theta)/4$  
$I_y = a^4(\theta + \sin \theta \cos \theta)/4$ | $r_x^2 = a^2(\theta - \sin \theta \cos \theta)/4$  
$r_y^2 = a^2/4$  
$r_x^2 = a^2/4$  
$r_y^2 = 5a^2/4$ | $I_{x_c y_c} = 0$  
$I_{x y} = 0$ |
| ![Circular Segment](image) | $A = a^2\left[\theta - \sin 2\theta/2\right]$  
$x_c = 2a \sin \theta/3$  
$y_c = 0$  
$J = \pi a^4/8$ | $I_x = A a^2/4 \left[1 - \frac{2\sin^3 \theta \cos \theta}{3\theta - 3\sin \theta \cos \theta}\right]$  
$I_y = A a^2/4 \left[1 + \frac{2\sin^3 \theta \cos \theta}{\theta - \sin \theta \cos \theta}\right]$  
$J = \pi a^4/8$ | $r_x^2 = a^2/4 \left[1 - \frac{2\sin^3 \theta \cos \theta}{3\theta - 3\sin \theta \cos \theta}\right]$  
$r_y^2 = a^2/4 \left[1 + \frac{2\sin^3 \theta \cos \theta}{\theta - \sin \theta \cos \theta}\right]$ | $I_{x_c y_c} = 0$  
$I_{x y} = 0$ |
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| ![Parabola](image) | $A = 4ab/3$  
$x_c = 3a/5$  
$y_c = 0$ | $I_x = I_y = 4ab^3/15$  
$I_{x_c} = 16a^3b/175$  
$I_{y_c} = 4a^3b/7$ | $r_x^2 = r_y^2 = b^2/5$  
$r_{x_c}^2 = 12a^2/175$  
$r_{y_c}^2 = 3a^2/7$ | $I_{x,y_c} = 0$  
$I_{x,y} = 0$ |
| ![Half a Parabola](image) | $A = 2ab/3$  
$x_c = 3a/5$  
$y_c = 3b/8$ | $I_x = 2ab^3/15$  
$I_y = 2ba^3/7$ | $r_x^2 = b^2/5$  
$r_y^2 = 3a^2/7$ | $I_{x,y} = Aab/4 = a^2b^2$ |
| ![n° Degree Parabola](image) | $A = bh/(n+1)$  
$x_c = n+1/n+b$  
$y_c = h/n+1/2n+1$ | $I_x = bh^3/3(n+1)$  
$I_y = bh^3/n+3$ | $r_x^2 = h^2(n+1)/3(n+1)$  
$r_y^2 = n+1/b^2$ | |
| ![n° Degree Parabola](image) | $A = n+1/bh$  
$x_c = n+1/2n+1$  
$y_c = n+1/2(n+2)b$ | $I_x = n+1/3(n+3)b^3h$  
$I_y = n+1/3n+1b^3h$ | $r_x^2 = n+1/3(n+1)b^2$  
$r_y^2 = n+1/3n+1b^2$ | |