2.9 **STATICALLY INDETERMINATE PROBLEMS**

In use int giv we of
car prc car me eq de! Be the inc tyF
\( S_B = S_{AL} \)

(2) \( S_B = \frac{P_B L_B}{A_B E_B} \)

\( S_{AL} = \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} \)

(2) \( \frac{P_B L_B}{A_B E_B} = \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} \)

(3) \( \Sigma F_H = 0 = + P_A + P_B - 100 \text{ kN} \)

\( P_A = 100 - P_B \)

(2) \( \frac{P_B L_B}{A_B E_B} = \frac{(100- P_B) L_{AL}}{A_{AL} E_{AL}} \)

\( C_B = \frac{P_B}{A_B} \)
EXAMPLE 2.03

Given supports \( \text{P}_1 \) and \( \text{P}_2 \) in \( AC \) and \( BC \), we obtain

\[
\begin{align*}
(2.14) \\
\text{from the notation} \ BC, \ we \\
\end{align*}
\]

\[
\begin{align*}
\text{forces} \ \text{P}_1 \text{ and} \ \text{R}_B; \\
(2.15) \\
\text{and} \ \text{R}_B; \\
(2.16) \\
\text{in} \ AC \text{ and} \ \text{P}_2 = \\
\end{align*}
\]

Fig. 2.23

Obtained

Fig. 2.22
\[ s = \frac{P \cdot L_1}{A \cdot E} = \frac{PBL_B}{AEB} + \frac{PBL_T}{AET} \]

\[ s = \frac{P \cdot L_1}{A \cdot E} = P_B \left( \frac{L_B + L_T}{A} \right) \]
1) \( R_A - P + R_B = 0 \)

2) \(-S_{AB} + S_{AC} = 0 \)

\[
\frac{PL_1}{AE} + \frac{R_B(L_1 + L_2)}{AE} = 0
\]

\[
R_B = \frac{PL_1}{(L_1 + L_2)} \quad \text{so} \quad R_A = P - R_B \quad (I)
\]

\[
R_A = P \left( \frac{L_1 + L_2}{L_1 + L_2} - \frac{L_1}{L_1 + L_2} \right) = P \left( \frac{L_2}{L_1 + L_2} \right)
\]
2.10 PROBLEMS INVOLVING TEMPERATURE CHANGES

\[ \epsilon_T = \frac{\delta_T}{L} = \frac{\alpha \Delta T L}{L} \]

\[ \epsilon_T = \alpha \Delta T \]

\[ \delta_T = \alpha (\Delta T)L \] (2.21)

The strain \( \epsilon \) is the change in length here, there is:

Let us consider between two points 2.31a. Again, the rod is thin because of the cross section zero. However, and \( P' \) on...
2.10 Problems Involving Temperature Changes

Fig. 2.31 Rod with ends restrained against thermal expansion.

Fig. 2.32 Superposition method applied to rod restrained against thermal expansion.

\[
P = \frac{AE}{L} \Delta \mu T L - P L = 0
\]

\[
\Delta S = \Delta S_L - \Delta S_B = 0
\]
EXAMPLE 2.06

Determine the values of the stress in portions AC and CB of the steel bar shown that a clo is +75°F. steel.

We is statical let it und

The corre

\[ S = \alpha \Delta T \frac{L}{L} \]

\[ S = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} \]

Applying to express into Eq. 1

Expressin of the im
Forces in Members

\[ \sum F_x = 0 = -F_{BC} - 20kN + 40 + 60 \]

\[ F_{BC} = +80 \text{ kN} \]

\[ \sum F_y = 0 \times \]

\[ \sum M = 0 \times \]

\[ 2M = 0 \times \]
\[ S_{AB} = \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} \quad S_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} \]

\[ \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} \]

1. \[ F_{AB} = F_{BC} \frac{A_{AB}}{A_{BC}} \frac{E_{AB}}{E_{BC}} \frac{L_{BC}}{L_{AB}} \]

2. \[ \sum F_H = 0 = -F_{AB} + 20k - F_{BC} \]
$P = 100 \, \text{kN}$

Steel core:

- $E = 200 \, \text{GPa}$
- $A = 0.1 \, \text{m}^2$

Al:

- $E = 70 \, \text{GPa}$
- $A = 0.06 \, \text{m}^2$

$P = 100 \, \text{kN}$

2m

$ZF_V = 0 = -P_0 + P_{al} + P_a$

$S_{al} = S_a$

1. $\frac{P_{al} \, L_{al}}{A_{al} \, E_{al}} = \frac{P_a \, L}{A_a \, E_a}$

$P_{al} = 100 - P_a$

2. $P_{al} = \frac{P_a \, D_a \, L_{al} \, E_{al}}{L_{al} \, A_a \, E_a} = P_a \frac{0.06 \, \text{m}^2}{0.10 \, \text{m}^2} \frac{70 \, \text{GPa}}{200 \, \text{GPa}}$

$S_a = \frac{P_a}{A_a}$
A = 2 \text{in}^2
E = 29,000 \text{K/in}^2

\begin{align*}
\delta_B &= \frac{F_{AC} L_{AC}}{A_{AC} E_{AC}} + \frac{F_{CB} L_{CB}}{A_{CB} E_{CB}} \\
F_{AC} &= 200 \text{kN} \\
F_{CB} &= 0 \\
&S_B = \frac{200 \text{kN} \times (8 \text{ft} \times 12 \text{in/ft})}{(2 \text{in}^2) \times (29,000 \text{K/in}^2)} = 0.331 \text{in}
\end{align*}

S_B = \frac{P_{AB} L_{AC}}{A_{AC} E_{AC}} = \frac{B_H (16' \times 12')}{(2 \text{in}^2) \times (29,000 \text{K/in}^2)}

0.331 = \frac{B_H (16 \times 12)}{Z (29,000)}

B_H = 100 \text{kN}
\[ A = 0.01 \text{ m}^2 \]

Steel

Green \( \Delta T = 300 \text{ C} \)

\[ L = 2 \text{ m} \]

\[ T_1 = T_0 \]

\[ T_2 = T_0 + \Delta T \]

\[ E = 9\% \]

\[ \varepsilon = \frac{\Delta L}{L} \]

\[ \sigma_T = \alpha \Delta T L \]

\[ \alpha = \frac{\sigma_T}{\Delta T L} \]

\[ \sigma_T = \alpha \Delta T L = \left(11.7 \times 10^{-6} \text{ m/m C}\right) \left(300 \text{ C}\right) \left(2 \text{ m}\right) = 0.00702 \text{ m} \]

\[ 2 \text{ m} \]

\[ F_B \]

\[ S_p = \frac{F_B L}{\frac{A E}{m^2 \text{ N}}} = \frac{F_B 2 \text{ m}}{0.01 \text{ m}^2 \left(200 \times 10^9 \text{ N/m}^2\right)} = 0.00702 \text{ m} \]

\[ S_p = \sigma_T \]

\[ F_B = 7.02 \text{ MN} \]
The diagram illustrates a force analysis problem involving a bolt. The forces are denoted as follows:

- **T** = \( \frac{P}{A_{\text{bolt}}} \)

There are two types of bearing capacity calculations:

1. **BEARING**
   - \( D_{\text{bolt}} \) (t_{\text{plate}})

2. **CONCRETE**
   - \( 36,000 \text{ psi} \)
   - \( 200 \text{ kips} \)
   - \( 3000 \text{ psi} \)
   - \( \frac{P}{A} = \frac{200}{a\times a} = 3000 \text{ psi} \)

The soil is indicated to have a capacity of \( 2 \text{ kips/ft}^2 \).
Solve for the maximum normal stress in the 3/4" diameter bar above.

\[ \sigma = \frac{P}{A} = \frac{50 \text{K}}{\pi (0.75 \text{ in})^2} = \frac{50 \text{K}}{0.442 \text{ in}^2} = 113 \text{ ksi} \]

Tension

Pop Quiz #1 - Max tensile stress:

\[ \sigma = \frac{P}{A} = \frac{400 \text{K}}{\pi (6 \text{ in})^2 - \frac{\pi}{4} (5 \text{ in})^2} = 46.3 \text{ ksi} \]

Tension
Pop Quiz #2

Determine the maximum normal stress in the W12x50 shown.

\[ \sigma = \frac{P}{A} = \frac{500 \text{k}}{14.6 \text{in}^2} = 34.2 \text{ksi} \]

Compression

(14.7 in. in our text)

\[ \sigma = \frac{P}{A} \]
Determine the max stress in the steel 4" x 3" x 1/2" angle:

\[ \sigma = \frac{P}{A} = \frac{60k}{3.25\text{in}^2} = 18.46\text{ k/in}^2 \]

Tension
Pop Quiz #4

Maximum tensile stress in copper L 4" x 3" x ½" with 3/4" bolt holes drilled as shown:

\[ A = 3.25\text{in}^2 - (2\text{holes})(0.75\text{"} \times \frac{1}{2}\text{") = 2.5in}^2 \]

\[ \sigma = \frac{P}{A} = \frac{60\text{kN}}{2.5\text{in}^2} = 24\text{ksi} \]

\[ \text{Tension} \]

Wrong units

30.4 psi

-2 no units

Pop Quiz #5

Determine the maximum shearing stress in the 5/8" diameter bolts used in the connection above. \[ A = (6\text{bolts})\left(\frac{\pi}{4}\right)\left(\frac{5}{8}\text{in}\right)^2 \]

\[ \tau = \frac{60\text{kN}}{1.84\text{in}^2} = 32.6\text{ksi} \]

Shear

Pop Quiz #6

Determine the maximum bearing stress in the 4" x 3" x 1/2" angle bolted as shown.

\[ A_{\text{bearing}} = (6\text{bolts})\left(D_{\text{bolt}} = 5/8\text{"ight)}\left(t_{\text{plate}} = 1/2\text{"ight)} \]

\[ \tau_{\text{plate}} = \frac{60\text{kN}}{1.875\text{in}^2} = 32\text{ksi} \]

Bearing compressive
$F_{AB} = 40 \text{kN Compressive}$

$F_{DE} = 10 \text{kN Tension}$

$F_{CD} = 30 \text{kN Tension}$

$F_{BC} = 60 + 20 + 10$

$F_{BC} = +60 + 20 + 10$

$= +30 \text{kN Comp}$

$+60 + 80 = 20 - ? - 40 + 100 = 0$

$T = \frac{T_c}{J}$
\[ \tau = \frac{P}{A} \]

\[ \sigma = \frac{60k}{\pi (1.6\text{in})^2} = \frac{127.324}{k/\text{in}^2} \]

NORMAL \{ \text{COMPRESSIVE} \}\{ \text{TENSION} \}

\[ \sigma = \frac{6k}{(1.6\text{in})(2\text{in})} = 1.875 \text{k/in}^2 \]

\[ \tau = \frac{200\#}{\pi(0.15\text{in})(0.3\text{in})} \]
**BOLT**

NORMAL STRESSES

\[ \sigma = \frac{P}{A} \text{Tensile} \]

**BAR**

\[ A = \frac{\pi D_o^2}{4} - \frac{\pi D_i^2}{4} \]

\[ \sigma = \frac{P}{A} \text{Compressive} \]

SHEAR STRESS

BEARING

\[ \tau = \frac{P}{A} \]

**BOLT**

\[ \tau = \frac{P}{A} \text{(Bolt Head)} \]

\[ (\pi D_b^2 \text{(Thread)} + P) \]

**ANGLE**

\[ \sigma = \frac{P}{A} \text{Tension} \]

L \( 3'' \times 2'' \times 1/2'' \)

\[ L \text{ W12x50} \]

**INDEX**