1) $\Sigma M_A = 0 = -(2)(10)(5) + R_{DV}(12)$
   $R_{DV} = 8.33$ kN

2) $\Sigma F_V = 0 = 8.33 - R_{AV}$
   $R_{AV} = +8.33$ kN

3) $\Sigma F_H = 0 = +2(10) - R_{AH}$
   $R_{AH} = +20$ kN

3) $\Sigma M_A = 0 = -20$ kN (5m) + $R_D(12m)$

Drawn on tension side

$A_{BC} = 0$

$8.33 = V_{BC}$

$V_{CD} = 0$

$8.33$ kN = $A$
\[ \Sigma M_y = -20y + 2 \left( \frac{y}{2} \right) \left( \frac{y}{2} \right) + M_y \]

so

\[ M_y = 20y - y^2 \quad (0 \leq y \leq 10m) \]

\[ \text{Check : } M_{@y=10m} = 20(10) - (10)^2 \]

\[ = 200 - 100 \]

\[ = 100 \text{ kNm} \]

\[ M_{@y=5m} = 20(5) - (5)^2 \]

\[ = 75 \text{ kNm} \]

\[ M_{@y=0m} = 20(0) - (0)^2 \]

\[ = 0 \text{ kNm} \]
2 kN/m

\[ \Delta B_1 = -\frac{5wl^4}{384EI} \]

\[ \Delta B_2 = \frac{R_B l^3}{48EI} \]
## APPENDIX D  Beam Deflections and Slopes

<table>
<thead>
<tr>
<th>Beam and Loading</th>
<th>Elastic Curve</th>
<th>Maximum Deflection</th>
<th>Slope at End</th>
<th>Equation of Elastic Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Image" /></td>
<td>$y = \frac{P L^2}{3 E I}$</td>
<td>$\frac{P L^2}{2 E I}$</td>
<td>$y = \frac{P}{6 E I} \left( x^3 - 3 L x^2 \right)$</td>
</tr>
<tr>
<td>2</td>
<td><img src="image2" alt="Image" /></td>
<td>$y = \frac{w L^4}{6 E I}$</td>
<td>$\frac{w L^3}{6 E I}$</td>
<td>$y = -\frac{w}{24 E I} \left( x^4 - 4 L x^3 + 6 L^2 x^2 \right)$</td>
</tr>
<tr>
<td>3</td>
<td><img src="image3" alt="Image" /></td>
<td>$y = \frac{M L^2}{2 E I}$</td>
<td>$\frac{M L}{E I}$</td>
<td>$y = \frac{M}{2 E I} x^2$</td>
</tr>
<tr>
<td>4</td>
<td><img src="image4" alt="Image" /></td>
<td>$y = \frac{P L^3}{48 E I}$</td>
<td>$\pm \frac{P L^2}{16 E I}$</td>
<td>For $x \leq \frac{L}{2}$: $y = \frac{P}{48 E I} \left( 4 x^3 - 3 L x^2 \right)$</td>
</tr>
<tr>
<td>5</td>
<td><img src="image5" alt="Image" /></td>
<td>For $a &gt; b$: $\frac{P b (L^2 - b^2)^{3/2}}{9 \sqrt{5} E I}$ at $x = \frac{L^2 - b^2}{3}$</td>
<td>$\frac{P b (L^2 - b^2)}{6 E I}$</td>
<td>$\theta_A = \frac{P b (L^2 - b^2)}{6 E I}$ For $x &lt; a$: $y = \frac{P b}{6 E I} \left( x^3 - (L^2 - b^2) x \right)$ For $x = a$: $y = -\frac{P a b^3}{3 E I}$</td>
</tr>
<tr>
<td>6</td>
<td><img src="image6" alt="Image" /></td>
<td>$y = \frac{5 w L^4}{384 E I}$</td>
<td>$\pm \frac{w L^3}{34 E I}$</td>
<td>$y = -\frac{w}{24 E I} \left( x^4 - 2 L x^3 + L^2 x \right)$</td>
</tr>
<tr>
<td>7</td>
<td><img src="image7" alt="Image" /></td>
<td>$y = \frac{M L^2}{9 \sqrt{3} E I}$</td>
<td>$\theta_A = \frac{M L}{6 E I}$</td>
<td>$\theta_B = -\frac{M L}{3 E I}$ $y = -\frac{M}{6 E I} \left( x^3 - L^2 x \right)$</td>
</tr>
</tbody>
</table>
Singularity Functions would be better!

\[ M_x = A \cdot \xi - 2 \frac{\xi^2}{2} + B \cdot \xi - 8 = \frac{d^2 y}{dx^2} \frac{EI}{\xi} \]

\[ \frac{dy}{dx} = +C_1 \]

\[ y = +C_1 \xi + C_2 \]

B.C. @ \( \xi = 0, y = 0 \); \( \xi = 8, y = 0 \); \( \xi = 16, y = 0 \)
A of the column to a given point \( Q \) of its elastic curve, and by \( y \) the deflection of that point (Fig. 10.8a). It follows that the \( x \) axis will be vertical and directed downward, and the \( y \) axis horizontal and directed to the right. Considering the equilibrium of the free body \( AQ \) (Fig. 10.8b), we find that the bending moment at \( Q \) is \( M = -Py \). Substituting this value for \( M \) in Eq. (9.4) of Sec. 9.3, we write

\[
\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI} y
\]  
(10.4)

or, transposing the last term,

\[
\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0
\]  
(10.5)

This equation is a linear, homogeneous differential equation of the second order with constant coefficients. Setting

\[
p^2 = \frac{P}{EI}, \quad p = \sqrt{\frac{P}{EI}}
\]  
(10.6)

we write Eq. (10.5) in the form

\[
\frac{d^2y}{dx^2} + p^2 y = 0
\]  
(10.7)

which is the same as that of the differential equation for simple harmonic motion except, of course, that the independent variable is now the distance \( x \) instead of the time \( t \). The general solution of Eq. (10.7) is

\[
y = A \sin px + B \cos px
\]  
(10.8)

as we easily check by computing \( \frac{d^2y}{dx^2} \) and substituting for \( y \) and \( \frac{d^2y}{dx^2} \) into Eq. (10.7).

Recalling the boundary conditions that must be satisfied at ends \( A \) and \( B \) of the column (Fig. 10.8a), we first make \( x = 0, y = 0 \) in Eq. (10.8) and find that \( B = 0 \). Substituting next \( x = L, y = 0 \), we obtain

\[
A \sin pL = 0
\]  
(10.9)

This equation is satisfied either if \( A = 0 \), or if \( \sin pL = 0 \). If the first of these conditions is satisfied, Eq. (10.8) reduces to \( y = 0 \) and the column is straight (Fig. 10.1). For the second condition to be satisfied, we must have \( pL = n\pi \) or, substituting for \( p \) from (10.6) and solving for \( P \),

\[
P = \frac{\pi^2 EI}{L^2}
\]  
(10.10)

The smallest of the values of \( P \) defined by Eq. (10.10) is that corresponding to \( n = 1 \). We thus have

\[
P_{cr} = \frac{\pi^2 EI}{L^2}
\]  
(10.11)

The expression obtained is known as Euler's formula, after the Swiss mathematician Leonhard Euler (1707–1783). Substituting this
\[ P_a = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA L^2}{L^2} \]

Let: \[ Ar^2 = I \]

\[ r = \sqrt{\frac{I}{A}} \]

\[ \sigma_a = \frac{P_a A}{A} = \frac{\pi^2 EA L^2}{L^2} \]

\[ \sigma_a = \frac{\pi^2 E L^2}{L^2/r^2} = \frac{\pi^2 E (L/r)^2}{r^2} \]

Slenderness ratio

\[ P_a = \frac{m^2 \pi^2 EI}{L^2} \]

\[ I_{yy} = \frac{6a^3}{12} \quad I_{xx} = \frac{a b^3}{12} \]
2" x 4" nominal
1 1/2" x 3 1/2" = True
Douglas fir

E = 1900 k/in²

P = A 14 text

\[ I_{\text{weak}} = \frac{bh^3}{12} = \frac{(3.5)(1.5)}{12} = 0.9844 \text{in}^4 \]

\[ P = \frac{\pi^2 EI_{yy}}{L^2} = \frac{(3.1415)(2.5)}{(8 \text{ft} \times 12 \text{in/ft})^2} \left(1,900 \frac{k}{\text{in}^2}\right)(0.9844 \text{in}^4) \]

= 2.003 k

\[ L_{\text{eff}} = \frac{L_{\text{true}}}{2} \]

8 ft

L_{\text{eff}} = 8 \text{ ft}

L_{\text{true}} = 8 \text{ ft}

8 ft
\[ P_{cr} = \frac{\pi^2 E I}{L_{eff}^2} = \frac{\pi^2 \left( 1900 \text{ksi}\right) \left( 0.9844 \text{in}^4 \right)}{\left( 4 \text{ft} \ast 12 \text{in/ft} \right)^2} \]

\[ = 8.0 \text{ k} \]

**Top View**

\[ \frac{K}{\text{ksi}} \frac{\text{in}^4}{\text{ft}^2} \frac{\text{in}^2}{\text{ft}^2} \]