# TABLE C-A.7.1

**APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, K**

<table>
<thead>
<tr>
<th>BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEORETICAL K VALUE</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
<td>2.10</td>
<td>2.0</td>
</tr>
<tr>
<td>END CONDITION CODE</td>
<td><img src="image1" alt="Rotation Fixed and Translation Fixed" /></td>
<td><img src="image2" alt="Rotation Free and Translation Fixed" /></td>
<td><img src="image3" alt="Rotation Fixed and Translation Free" /></td>
<td><img src="image4" alt="Rotation Free and Translation Free" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sigma_a &= \frac{KL}{r} \\
\sigma_f &= \frac{KL}{r}
\end{align*}
\]

CASE (a) or (c)
Specific formulas for the design of steel, aluminum and wood columns under centric loading will now be considered. Figure 10.27 shows examples of columns that would be designed using these formulas. The design for the three different materials using Allowable Stress Design is first presented. This is followed with the formulas needed for the design of steel columns based on Load and Resistance Factor Design.†

\[
\gamma = \sqrt{\frac{I}{A}}
\]

Fig. 10.27  The water tank in (a) is supported by steel columns and the building in construction in (b) is framed with wood columns.

Structural Steel—Allowable Stress Design. The formulas most widely used for the allowable stress design of steel columns under a centric load are found in the Specification for Structural Steel Buildings of the American Institute of Steel Construction.‡ As we shall see, an exponential expression is used to predict \( \sigma_{cr} \) for columns of short and intermediate lengths, and an Euler-based relation is used for long columns. The design relations are developed in two steps:

1. First a curve representing the variation of \( \sigma_{cr} \) with \( L/r \) is obtained (Fig. 10.28). It is important to note that this curve does not incorporate any factor of safety.§ The portion \( AB \) of this curve is defined by the equation

\[
\frac{K L}{\gamma_{xy}} = 10
\]

\[
\frac{K L}{\gamma_{yy}} = 8
\]

\[
\sigma_{cr} = \left[ 0.658 (\gamma_{xy} \gamma_{yy}) \right] \sigma_y
\]

\[
\sigma_y = [0.092 \gamma_{xy}] \sigma_y
\]

\[
\sigma_y = 36
\]

†In specific design formulas, the letter \( L \) will always refer to the effective length of the column.


§In the Specification for Structural Steel for Buildings, the symbol \( F \) is used for stresses.
where

\[
\sigma_\text{cr} = \frac{\pi^2 E}{(L/r)^2}
\]  
\[\text{(10.39)}\]

The portion BC is defined by the equation

\[
\frac{L}{r} = 4.71 \frac{E}{G_y} \sigma_y + 0.877 \sigma_r
\]  
\[\text{(10.40)}\]

We note that when \(L/r \neq 0\), \(\sigma_\text{cr} = \sigma_r\) in Eq. (10.38). At point B, Eq. (10.38) joins Eq. (10.40). The value of slenderness \(L/r\) at the junction between the two equations is

\[
\frac{L}{r} = 4.71 \sqrt{\frac{E}{G_y}} = 8 \text{ (10.41)}
\]

If \(L/r\) is smaller than the value in Eq. (10.41), \(\sigma_\text{cr}\) is determined from Eq. (10.38), and if \(L/r\) is greater, \(\sigma_\text{cr}\) is determined from Eq. (10.40). At the value of the slenderness \(L/r\) specified in Eq. (10.41), the stress \(\sigma_\text{cr} = 0.44 \sigma_y\). Using Eq. (10.40), \(\sigma_\text{cr} = 0.877 (0.44 \sigma_y) = 0.39 \sigma_y\).

2. A factor of safety must be introduced to obtain the final AISC design formulas. The factor of safety specified by the specification is 1.67. Thus

\[
\sigma_{\text{all}} = \frac{\sigma_\text{cr}}{1.67}
\]  
\[\text{(10.42)}\]

The formulas obtained can be used with SI or U.S. customary units.

We observe that, by using Eqs. (10.38), (10.40), (10.41), and (10.42), we can determine the allowable axial stress for a given grade of steel and any given value of \(L/r\). The procedure is to first compute the value of \(L/r\) at the intersection between the two equations from Eq. (10.41). For given values of \(L/r\) smaller than that in Eq. (10.41), we use Eqs. (10.38) and (10.42) to calculate \(\sigma_{\text{all}}\), and for values greater than that in Eq. (10.41), we use Eqs. (10.40) and (10.42) to calculate \(\sigma_{\text{all}}\). Figure 10.29 provides a general illustration of how \(\sigma_\text{cr}\) varies as a function of \(L/r\) for different grades of structural steel.
\( \tau_c = 0.877 \tau_{Euler} \) (To the right of break point)

\( \tau_c = [0.658 \sigma_y / \sigma_e] \tau_y \) (To the left of break point)

\( \tau_{all} = \frac{\tau_{cr}}{1.67} \)

\( \tau_{all} = \tau_{all} \cdot A \)
How much load can a W10 x 39 carry according to AISC specs if supported?

\[ C_{cr} = \begin{cases} 0.877 & \text{cy} \\ \frac{[0.658 \sqrt{\frac{E}{G_y}}]}{C_y} & \text{cy} \end{cases} \]

\[ \frac{L_{\text{break}}}{Y_{\text{point}}} = 4.71 \left( \frac{29,000 \text{ psi}}{80 \text{ ksi}} \right) \]

\[ S_{cr} = \frac{1}{L_{\text{break}}} \left( \frac{2E}{(L/r)^2} \right) \]

\[ P_{\text{all}} = \left( \frac{46.97 \text{ kN}}{11.5 \text{ in}^2} \right) \left( 11.5 \text{ in}^2 \right) = \frac{F_{Q,S} = 1.67}{\text{See next page}} \]
\[
\frac{L_{\text{break}}}{r} = 4.71 \sqrt{\frac{E}{K_y}}
\]
\[
= 4.71 \sqrt{\frac{29,000 \text{ ksi}}{80 \text{ ksi}}} = 89.67 \text{ break point}
\]

Strong axis slenderness ratio:
\[
\frac{K_L}{K_w} = \frac{24 \text{ ft}(12 \text{ in}/\text{ft})}{4.271 \text{ in}} = 67.44
\]

Weak axis slenderness ratio:
\[
\frac{K_L}{K_Y} = \frac{8 \text{ ft}(12 \text{ in}/\text{ft})}{1.98 \text{ in}} = 48.48
\]

So buckles about strong axis (x-x)

\[
\sigma_c = \frac{29,000 \text{ ksi}}{26} = 1,077 \text{ ksi}
\]

\[
\sigma_c = \frac{K_W}{K_w} \left( \frac{24 \text{ ft} \times 12 \text{ in/ft}}{4.271 \text{ in}} \right)^2 \times 80 \text{ ksi} = 46.97 \text{ k/ln}^2
\]

\[
P_c = 2\pi A = 46.97 \text{ k/ln}^2 \times (11.5 \text{ in}^2) =
\]

\[
\text{Pallowed} = \frac{P_c}{1.67} = \frac{(46.97 \text{ k/ln}^2)(11.5 \text{ in}^2)}{1.67} = 323 \text{ k}
\]
What is the theoretical critical buckling load for a W18x71 supported as shown.
\[ P_y = 50 \text{ kips} \]
\[ A = 20.8 \text{ in}^2 \]
\[ F_{xx} = 7.5 \text{ in} \]
\[ F_{yy} = 1.7 \text{ in} \]
\[ E = 29,000 \text{ ksi} \]
Determine max theoretical AISC load for a W18×71 supported as shown with $f_y = 50$ ksi.
\[ \text{KL} = 26 \text{ ft} \times 12'' \text{ ft} = 312 \text{ in}^2 \]
\[
\frac{\text{KL}}{r} = \frac{312 \text{ in}}{7.50 \text{ in}} = 41.6 \\
A = 20.8 \text{ in}^2
\]

\[ \text{KL} = 26 \text{ ft} \times 12'' \text{ ft} \times 0.5 = 156 \text{ in}^2 \]
\[
\frac{\text{KL}}{r} = \frac{156 \text{ in}}{1.7 \text{ in}} = 91.8 \text{ Critical}
\]

So buckles about weak axis

\[
P_{cr} = \frac{\pi^2 EI_{yy}}{(L_{eff})^2} = \frac{\pi^2 (29,000 \text{ ksi})^2 (60.3 \text{ in}^4)}{(156 \text{ in})^2} = 709 \text{ k}
\]

\[
P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 (29,000 \text{ ksi}) (20.8 \text{ in}^2)}{(91.8)^2} = (33.96 \text{ ksi})(20.8 \text{ in}^2) = 706 \text{ k}
\]

\[ P_{all} = \frac{P_{cr}}{1.6} \]
\[ T_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(91.8)^2} \]

Critical column

\[ = 33.96 \text{ ksi} \]

\[ \frac{KL}{r} \text{ break point} = 4.71 \sqrt{\frac{E}{Gy}} \]

\[ = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 113.4 \]

Our \( KL/r = 91.8 < 113.4 \) so

\[ T_{cr} = \left[ 0.658 \left( \frac{Gy}{T_e} \right) \right] Gy \]

\[ = \left[ 0.658 \left( \frac{50}{23.96} \right) \right] 50 = 0.537 \left( 50 \text{ ksi} \right) \]

\[ = 26.86 \text{ ksi} \]

So \( P_{cr} = T_{cr} A = 26.86 \text{ ksi} \left( 20.8 \text{ in}^2 \right) \)

\[ P_{\text{death}} = 559 \text{ kips} \]

\[ P_{\text{all}} = P_{\text{cr}}/1.6 \]
PROBLEM 10.12

Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (a) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (b) Using $E = 105 \text{ GPa}$, determine the critical load of each rod.

**SOLUTION**

(a) Same area:

\[
\frac{\pi}{4} (d_2^2 - d_1^2) = b_1^2 - b_0^2
\]

\[
b_1^2 = b_0^2 - \frac{\pi}{4} (d_2^2 - d_1^2)
\]

\[
= 60^2 - \frac{\pi}{4} (60^2 - 40^2) = 2.0292 \text{ mm}^2
\]

\[
b_1 = 45.047 \text{ mm} \quad t = \frac{1}{2} (b_0 - b_1) \quad t = 7.48 \text{ mm}
\]

(b) Circular:

\[
I = \frac{\pi}{64} (d_0^4 - d_1^4) = 510.51 \times 10^3 \text{ mm}^4 = 510.51 \times 10^{-9} \text{ m}^4
\]

\[
P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9)(510.51 \times 10^{-9})}{(3.0)^2} = 58.8 \times 10^3 \text{ N}
\]

\[
P_{cr} = 58.8 \text{ kN}
\]

Square:

\[
I = \frac{1}{12} (b_0^4 - b_1^4) = 736.85 \times 10^3 \text{ mm}^4 = 736.85 \times 10^{-9} \text{ m}^4
\]

\[
P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9)(736.85 \times 10^{-9})}{(3.0)^2} = 84.8 \times 10^3 \text{ N}
\]

\[
P_{cr} = 84.8 \text{ kN}
\]
Case I:

\[ P_{ca, X-X} = \frac{\pi^2 EI}{(L_{eff})^2} = \frac{\pi^2 (2.2 \times 10^3 \text{kips})(36 \text{ in}^4)}{(0.5 \times 18 \text{ ft} \times 12 \text{ in/ft})^2} = 67 \text{k} \]

Case II:

\[ P_{ca} = \frac{\pi^2 (2.2 \times 10^3 \text{kips})(6 \times 2) \text{ in}^3}{(0.7 \times 8 \text{ ft} \times 12 \text{ in/ft})^2} = 19.2 \text{k} \]

Case III:

\[ P_{ca} = 16.7 \text{k} \text{ using } L_{eff} = L_{TRUE} = 6 \text{ ft} \text{ and } EI = 6.2 \times 10^3 \text{ in}^3/\text{ft} \]

Case IV:

\[ P_{ca} = 76.9 \text{k} = (P_{ca, II} \times 4 \text{ times higher}) \]

(Case II is 4 times higher than Case IV)
\[ W_{36 \times 302} \]
\[ T_y = 100 \text{ kpsi} \]
\[ 10 \text{ ft} \times K = 1.0 \]
\[ I_{yy} = 13,000 \text{ in}^4 \]
\[ V_{yy} = 3.82 \text{ in} \]
\[ 12 \text{ ft} \times K = 0.8 \]

**For X - X**

\[ K = 2.1 \]
\[ I_{xx} = 21,000 \text{ in}^4 \]
\[ V_{xx} = 15.4 \text{ in} \]

\[ \frac{K L}{V_{xx}} = \frac{(2.1)(22 \text{ ft})(12''/\text{ft})}{15.4 \text{ in}} = 36. \]

**For Y - Y**

Top:
\[ KL = \frac{(1.0)(10 \text{ ft})(12''/\text{ft})}{V_{yy}} \]
\[ 3.82 \text{ in} \]
\[ = 31.41 \]

Bottom:
\[ KL = \frac{(0.8)(12 \text{ ft})(12''/\text{ft})}{V_{yy}} \]
\[ 3.82 \text{ in} \]
\[ = 30.12 \]
\[ \tau_e = \frac{\pi^2 E}{(KL)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(36)^2} = 220 \text{ ksi} \]

\[ \frac{KL}{r_{\text{breakpoint}}} = 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{100 \text{ ksi}}} = 80.2 \]

Our \( \frac{KL}{r} = 36 < 80.2 \) so

\[ \tau_n = \left[ \frac{0.658 \sigma_y}{E} \right] \tau_y \\
= \left[ \frac{0.658 \times 100 \text{ ksi}}{220 \text{ ksi}} \right] 100 \text{ ksi} \]

\[ = 82.7 \text{ ksi} \]

\[ P_n = \tau_n A = 82.7 \text{ ksi} (88.8 \text{ in}^2) = 7343 \text{ K} \]

\[ P_{\text{safe}} = 7343 \text{ K} / 1.67 = \]