Arithmetic Gradient Factors ($P/G, A/G$)

Cash flows that increase or decrease by a constant amount are considered arithmetic gradient cash flows. The amount of increase (or decrease) is called the gradient.

$G = \$25$
Base = $\$100$

$G = -\$500$
Base = $\$2000$

Equivalent cash flows:

$G = \$25$
Base = $\$100$

Note: the gradient series by convention starts in year 2.
To find $P$ for a gradient cash flow that starts at the end of year 2 and ends at year $n$:

$$P = \frac{G}{i} \left[ \frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right]$$

or $P = G(P/G,i,n)$

where $(P/G,i,n) =$

$$\frac{1}{i} \left[ \frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right]$$

To find $P$ for the arithmetic gradient cash flow:

$$P = 100(P/A,i,4) + 25(P/G,i,4)$$
To find $P$ for the declining arithmetic gradient cash flow:

\[
\begin{align*}
$P &= $2000(P/A,i,4) - $500(P/G,i,4) \\
\end{align*}
\]

To find the uniform annual series, $A$, for an arithmetic gradient cash flow $G$:

\[
\begin{align*}
$A &= G(P/G,i,n) (A/P,i,4) \\
&= G(A/G,i,n) \\
\text{Where } (A/G,i,n) &= \left[ \frac{1 - \frac{n}{i}}{(1+i)^n - 1} \right]
\end{align*}
\]