MOBILITY ALLOWANCE SHUTTLE TRANSIT (MAST) SERVICES: FORMULATION AND SIMULATION COMPARISON WITH CONVENTIONAL FIXED ROUTE BUS SERVICES

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ABSTRACT
The Mobility Allowance Shuttle Transit (MAST) system is an innovative concept that merges the flexibility of Demand Responsive Transit (DRT) systems with the low cost operability of fixed-route bus systems. A MAST service has a fixed base route that covers a specific geographic zone, with one or more mandatory checkpoints; the innovative twist is that, given an appropriate slack time, buses are allowed to deviate from the fixed path to pick up and drop off passengers at their desired locations within a predetermined service area. In this paper we present a mixed integer programming formulation of MAST services and we compare its performance with a conventional fixed-route bus service by a simulation analysis.

KEY WORDS
Transit, Simulation, Heuristic, Pickup and Delivery

1. Introduction

Demands on transit agencies for improved and extended services are increasing. On the other hand, there is little public support for increases in fares or subsidies. Therefore, transit agencies are currently seeking ways to improve service flexibility in a cost-efficient manner.
Most bus transit systems fall into two broad categories: fixed-route and demand responsive transit (DRT) systems. DRT systems tend to be much more costly to deploy. They are largely limited to specialized operations such as Dial-a-Ride service mandated under the Americans with Disabilities Act (paratransit DRT). Fixed-route bus systems are instead much more cost efficient and they require less government subsidy. This is primarily due to the passenger loading capacity of the buses and the consolidation of many passenger trips onto a single vehicle (ridesharing).
However fixed-route bus transit systems, as an alternative to private automobiles, have major deficiencies. The general public considers the service to be inconvenient because of its lack of flexibility since either the locations of pick-up and/or drop-off points or the service's schedule do not match the individual rider's desires. Moreover, the total trip time is perceived as being too long and for longer trips there is often a need of transfers between vehicles. On the other hand, commercial DRT systems, such as taxicabs and shuttle vans, provide much of the desired flexibility, but these improvements in convenience come at a price.
Thus, there is a need for a transit system that provides flexible service at a cost efficient price. The Mobility Allowance Shuttle Transit (MAST) system is an innovative concept that merges the flexibility of DRT systems with the low cost operability of fixed-route bus systems. A MAST service has a fixed base route that covers a specific geographic zone, with one or more mandatory checkpoints conveniently located at major connection points or high density demand zones; the innovative twist is that, given an appropriate slack time, buses are allowed to deviate from the fixed path to pick up and drop off passengers at their desired locations. The MAST system works under a dynamic environment since the majority of the requests occur while the bus is on duty (even though reservations in advance are handled by the system). The only restriction on flexibility is that the deviations must lie within a predetermined distance from the fixed base route.
Such a system somewhat already exists in a reduced and simplified scale. The Metropolitan Transit Authority (MTA) of Los Angeles County has recently introduced MAST as part of its feeder-line 646. During daytime, this line serves as a fixed-route bus system. During nighttime, the line changes to a MAST service and allows specific deviations of half a mile from either side of the fixed route. Since the current demand is low, the bus operator is able to make all the decisions concerning accepting/rejecting customer requests and routing the vehicle. Clearly, in case the service is utilized in a larger scale with heavier (daytime) demand and several requests for deviations, an effective MAST system needs to rely on recent developments in communication and computation technologies that allow real-time information about pick-up/drop-off requests to be used to re-route the bus by

means of a scheduling algorithm. This necessity becomes more evident in case a MAST service relies on a fleet of buses.

In this paper we first provide a mixed integer programming formulation of MAST systems; then we illustrate, through a simulation analysis, a performance comparison between a MAST service and a fixed route bus system, in order to asses the validity of this new type of service versus a conventional one.

Although there has been a significant amount of research on DRT systems, we are unaware of any work performed on studying systems such as the MAST service; however, the systems are related and the MAST system can be considered as a special case of the Pickup and Delivery Problem (PDP) with time windows.

Savelsbergh and Sol (1995) and Desaulniers et al. (2000) provide detailed reviews of the PDP, examining mathematical formulations and solutions approaches presented by different authors. Due to the combinatorial nature of the problem (the PDP is NP-Hard) exact optimization methods are theoretically interesting but practically unsolvable. Therefore most of the research efforts focus on heuristic approaches.

Exact approaches to solve DRT systems provide optimal solutions, but the combinatorial nature of the problem limits the applicability of these methods to very small instances; therefore they provide a good theoretical insight, but practically can not be used to solve real situations. Psaraftis (1980, 1983a) describes an exact backwards and forward dynamic programming solution approach for the single vehicle Dial-a-Ride problem for static and dynamic environments without time windows. Another dynamic programming approach is described in Desrosiers et al. (1986). Sexton and Bodin (1985a, 1985b) and Sexton and Choi (1986) describe a Benders' decomposition approach to solve the single vehicle PDP with time windows and capacity constraints. Dumas et al. (1991) present a Dantzig-Wolfe approach for optimally solving the multiple vehicles PDP with time windows and capacity constraints. Savelsbergh and Sol (1998) propose a branch-and-price based algorithm to solve the dynamic multi vehicle PDP. Fischetti and Toth (1989) develop an additive bounding procedure suitable for a branch-and-bound algorithm for the single vehicle PDP. An exact algorithm approach is described in Lu and Dessouky (2003).

Clustering approaches use the intuitive idea of merging together in a single point requests that are physically close to each other. The problem instances are reduced in size and exact approach can then be applied efficiently. Ioachim et al. (1995) develop a clustering algorithm to solve the multi vehicle PDP with time windows. The work of Daganzo (1984) describes a checkpoint DRT system that combines the characteristics of both fixed route and door to door service. However, the MAST system conceptually differs from the checkpoint only system described, since it allows also for door-to-door requests.

Local search techniques are those heuristics that starts from an initial feasible solution and "moves" locally in the neighborhood of the solution space. The main drawback is that the solution found might be a local optimum, potentially very far from the global optimal. Psaraftis (1983b, 1983c) presents two heuristic approaches for the single vehicle Dial-a-Ride problem with no time windows. Local search procedures are reported in Van Der Bruggen et al. (1993).

Insertion heuristics are probably the most popular techniques. Campbell and Savelsbergh (2003) justify their extensive use in practice, because they are very fast and capable of handling large problems, provide good solutions compared to optimality, can easily handle complicating constraints and can be simply implemented in dynamic environments. Jaw et al. (1986) illustrate a heuristic algorithm for the static multi vehicle PDP with time windows. Madsen et al. (1995) implement an insertion heuristic approach for a partly dynamic multi vehicle PDP. Diana and Dessouky (2004) apply the same concept for the PDP, with an appropriate metric that helps to overcome the myopic behavior that is often the drawback of such a method. A parallel insertion heuristic is proposed by Toth and Vigo (1997) to solve the static multi vehicle PDP with soft time windows. Lu and Dessouky (2003) present a new insertion-based construction heuristic to solve the multi-vehicle pickup and delivery problem with hard time windows.

2. Formulation

A MAST system is represented by a fleet of vehicles serving a set of customers' requests. The vehicles must follow a fixed-route schedule composed by an ordered set of checkpoints, from 1 to TC, and serve each request, defined by pick-up/drop-off stops and a ready time for pick-up. The service stops of each request may be checkpoints or any preferred location within a service area. The distinction from a traditional PDP with time windows is that checkpoints are already scheduled in advance and the departure times from them are preset like in a fixed route service and they can not be violated. Therefore, the time windows associated with them are zero. This is because checkpoints typically represent major transfer centers. Delays at these stops would result in undesirable deviations from a predetermined fixed schedule and passengers missing their connections in case of late arrivals.

To simplify the formulation, we assume a single vehicle MAST service with no capacity constraint operating in a deterministic (no randomness involved) and static (all the requests known in advance) environments, with one customer per request.

The system can be represented by a network. Let:

- np = total non-checkpoint pick-up requests
- nd = total non-checkpoint drop-off requests
- $N_0 = \{1,\ldots,TC\}$, set of TC checkpoints
- $N_{n+} = \{TC+1,\ldots,TC+np\}$, set of np requests
- $N_{n-} = \{TC+np+1,\ldots,TC+np+nd\}$, set of nd requests
- $N_n = N_{n+} \cup N_{n-}$, set of all non-checkpoint requests
- $N = N_0 \cup N_n$, set of all stops in the network

Let also $R = \{1,\ldots,r\}$ be the set of r requests and let *p(k)*, *d(k)* $\in$ N represent the pick-up and drop-off nodes of *k*, $\forall k \in R$.

The network is completed by defining the collection of feasible directed arcs (*i*, *j*) between the pairs of nodes *i* and *j* in N. Let:

- A_0 = arcs in N_0, including only arcs $(i, i+1)$, with $i = 1,\ldots,TC-1$, because the checkpoints are already ordered sequentially in the schedule.
- A_n = arcs in N_n, including all arcs (i, j), $\forall i, j \in N_n$, with $i \neq j$ and with $i=d(k) \neq p(k)=j$, $\forall k \in R$ because for each request the drop-off stop can not be scheduled before the pick-up stop.
- $A_{0,n}$ = arcs between N_0 and N_n, including arcs (i, j), $\forall i \in N_0/\{TC\}$, $\forall j \in N_n$, with $i=d(k) \neq p(k)=j$, $\forall k \in R$ $\forall i \in N_n$, $\forall j \in N_0/\{1\}$ with $i=d(k) \neq p(k)=j$, $\forall k \in R$
- $A = A_0 \cup A_n \cup A_{0,n}$, set of all arcs in the network.

As an example, we consider *k*=4 customers (see Table 1) with their corresponding pick-up and drop-off stops according to the network in Figure 1, describing a simple MAST system with TC=3 checkpoints in N_0 (1^0, 2^0 and 3^0), two pick-up stops in N_{n+} (4^+ and 5^+) and two drop-off requests in N_{n-} (6^- and 7^-).

Table 1 – Sample set of requests

k	*p(k)*	*d(k)*
1	4^+	6^-
2	1^0	7^-
3	5^+	2^0
4	1^0	3^0

The network is almost a complete graph, excluding the arcs violating the conditions described above, namely (2, 1), (3, 2), (3, 1) and (1, 3) that violate the predetermined sequence of checkpoints ($1\rightarrow2\rightarrow3$) and (6, 4), (2, 5), (7, 1) that violate the pick-up before drop-off precedence for each request. In addition, since checkpoints 1 and 3 represent the beginning and the end of the service, there are no arcs to 1 and no arcs from 3.

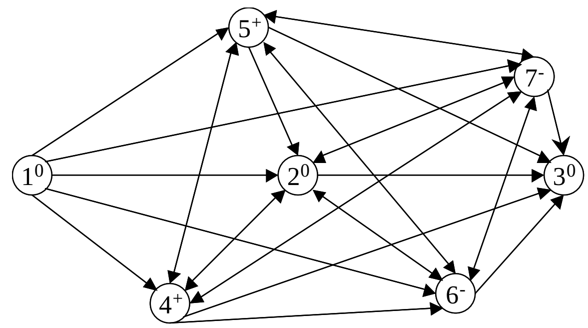

Figure 1 – Sample MAST network

The variables of the system are the following:

- $x_{i,j} = \{0, 1\}$, $\forall (i, j) \in A$, binary variables indicating if an arc (i, j) is used ($x_{i,j} = 1$) or not ($x_{i,j} = 0$).
- ta_i, $\forall i \in N/\{1\}$, arrival time at node *i*.
- td_i, $\forall i \in N$, departure time from node *i*.
- it_i, $\forall i \in N/\{1\}$, idle time spent at node *i*.

We also define the following parameters:

- $d_{i,j}$, $\forall (i, j) \in A$, distance between *i* and *j*.
- *bs*, bus speed
- bt_i, time needed for boardings and disembarkments at stop *i*, $\forall i \in N/\{1\}$.
- τ_i, scheduled departure time $\forall i \in N_0$, and the ready time $\forall i \in N_{n+}$.

Finally, we propose the following mixed integer linear programming formulation for the MAST system:

$$\min \omega_1 \sum_{(i,j)\in A} d_{i,j} x_{i,j} + \omega_2 \sum_{k \in R} \left(ta_{d(k)} - td_{p(k)} \right) + \omega_3 \sum_{i \in N_{n+}} \left(td_i - \tau_i \right) \qquad (1)$$

subject to:

$$\sum_i x_{i,j} = 1 \qquad \forall j \in N/\{1\} \qquad (2)$$

$$\sum_j x_{i,j} = 1 \qquad \forall i \in N/\{TC\} \qquad (3)$$

$$td_i = \tau_i \qquad \forall i \in N_0 \qquad (4)$$

$$td_i \geq \tau_i \qquad \forall i \in N_{n+} \qquad (5)$$

$$ta_{d(k)} \geq td_{p(k)} \qquad \forall k \in R \qquad (6)$$

$$ta_j \geq td_i + x_{i,j} d_{i,j}/bs - (1-x_{i,j})*(\tau_{TC}-\tau_1) \qquad \forall (i, j) \in A \qquad (7)$$

$$td_i = ta_i + bt_i + it_i \qquad \forall i \in N/\{1\} \qquad (8)$$

$$it_i \geq 0 \qquad \forall i \in N/\{1\} \qquad (9)$$

$$x_{i,j} \text{ binary} \qquad \forall (i, j) \in A \qquad (10)$$

The objective function (1) minimizes the weighted sum of three different factors, namely the total miles driven by the vehicle, the total ride time of all customers and the total waiting time at the pick-up stops, defined as the time interval between the ready time and the actual departure time. This definition allows optimizing both the bus variable cost (first term) and the service level (the last two terms); modifying accordingly the weights we can emphasize one factor over the others as needed.

Network constraints (2) and (3) allow each stop (except node 1 and TC) to have exactly one incoming arc and one outgoing arc equal to 1, so that all stops will be visited.

Constraint (4) forces the departure times from the checkpoint to be fixed, while constraint (5) prevents each non-checkpoint pick-up stop from having its departure time earlier than its ready time.

Constraint (6) prevents the drop-off stop to be scheduled earlier than the pick-up stop for each request.

Constraint (7) defines that for each $x_{i,j} = 1$ the arrival time at j should be no less than the departure time from i plus the time needed to travel between i and j. The last term assures that for any $x_{i,j} = 0$ the constraint becomes irrelevant; in fact any value of td_i and ta_j, within the MAST service time interval τ_C–τ_1, would be feasible. This constraint also guarantees that every feasible solution does not contain inner loops, but a single path from node 1 to node TC, visiting all nodes in N only once.

Constraint (8) links together arrival time, departure time and idle time for each stop i in the network.

The following constraint (11) is not necessary to find the optimal solution of the routing problem which will be the same with or without it, but it can be added to provide a more realistic schedule. It allows idling, if necessary, only at stops and not, unrealistically, in between stops.

$$\sum_{(i,j)\in A} x_{i,j}\left(d_{i,j}/bs + bt_i\right) + \sum_{i\in N/\{1\}} it_i = \tau_{TC} - \tau_1 \qquad (11)$$

The following constraint (12) can also be added to the formulation, along with (11), in case we would like to allow idling only at checkpoints:

$$it_i = 0 \qquad \forall i \in N_n \qquad (12)$$

The problem defined by the MAST system is exponential in np+nd (set N_n) and is NP-hard since it is easy to show that our problem can be reduced to a Travel Salesman Problem. Only small instances of the problem with low demand and/or with N_n small enough can be solved optimally by enumeration. More generally we need approximation methods to solve it. The following comparison between a MAST system and a conventional fixed-route service is performed using the insertion heuristic algorithm proposed by Quadrifoglio, Dessouky, and Palmer (2004).

3. MAST/Fixed-route comparison

We now perform a comparison between a MAST service and a fixed-route bus service. For both systems we consider a rectangular service area, with L*W = 10*1 miles (see Figure 2), the same bus speed ($bs = 25$ miles/hour) and the same time for boardings and disembarkements ($bt = 18$ sec) for all stops. A ride r begins either at checkpoint $c = 1$ or $c =$ C and ends at the other one after visiting all the intermediate stops; the vehicle's schedule consists of R rides. We consider a deterministic environment. Hence, we assume the slack time for accommodating random travel times to be zero. We also assume a single vehicle operating in both cases.

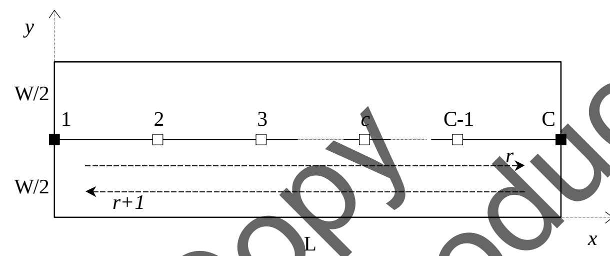

Figure 2 – Service Area

For the fixed-route line service we assume C = 19 fixed stops evenly distributed along the x axis (one stop every 0.5 miles). Since the headway for the fixed route bus is 60 minutes, the scheduled/actual travel time between two consecutive stops is 1.5 minutes.

For the MAST system we consider C = 3 checkpoints evenly distributed along the x axis (5 miles between two consecutive checkpoints) and 25 minutes time interval between the scheduled departure times of two consecutive checkpoints. Note that TC > C for any value of R > 1, because the vehicle stops at each checkpoint c=1,…C more than once at different times. The headway of the MAST system is therefore 100 minutes, which allows the system to have 12.7 minutes of slack time between any pair of consecutive checkpoints to serve the demand in the service area, but off the main route.

Both systems serve the same demand of $\theta = 25$ customers/hour; four different types of customers are possible and their assumed distribution is shown in parenthesis:

- PD: pick-up and drop-off at checkpoints (10%)
- PND: pick-up at checkpoint, drop-off at non-checkpoint (40%)
- NPD: pick-up at non-checkpoint, drop-off at checkpoint (40%)
- NPND: pick-up and drop-off at non-checkpoints (10%)

We also assume that the NP/ND stops are uniformly distributed within the service area, while the P/D stops are uniformly distributed among the 3 checkpoints.

In order to perform the comparison, we define the following performance parameters:

- PT: average ride time per passenger
- PW: average waiting time over NP requests only
- PWK: average walking time per passenger (assumed walking speed = 3 miles/hour)

- M: total miles driven by the bus
- PI: the average time interval between request/show up and the scheduled pick-up time per passenger

While the MAST system serves its customers point to point and no walking occurs, a fixed-route system forces NP and ND requests to walk to/from the nearest fixed stop in order to use the service. Note that the P and D requests could have a certain amount of walking time associated with it, but considering the same demand it would be equivalent for both systems. Consequently, we assume it to be zero. The overall performance Z (in time units) is defined as follows:

$$Z = w_1 * PT * NC_T + w_2 * PW * NC_{NP} + \\ + w_3 * M/bs + w_4 * PWK * NC_T \qquad (13)$$

where NC_T and NC_{NP} stand respectively for the total number of customers and the total number of NP customer requests (NPD and NPND types) served by the system and the last term represents the contribution to Z of the amount of walking time. In setting the weights, we assume that customers perceive the waiting time at stops (w_2) with double discomfort than the ride time on the bus (w_1) and that slack time consumption (w_3) and passengers' ride time (w_1) are equally weighted; we also assume that the weight for walking time (w_4) is conservatively equal to w_2 (even though customers would probably perceive walking time with more discomfort that waiting time at a bus stop especially during nighttime for safety reasons). Hence the weights in Z are set as follows: $w_1 = 0.25$; $w_2 = 0.5$; $w_3 = 0.25$; $w_4 = 0.5$.
NP and ND customer requests are inserted in the schedule dynamically. The heuristic makes use of control parameters to manage the consumption of slack time and improving the overall system performance. We ran the simulations (using Common Random Numbers for the two systems) for 50 hours, so that for the fixed-route service R = 100 and for the MAST system R = 60. The results are shown in Table 2.

Table 2 – MAST/fixed-route comparison

θ (customers/hour)	25	
System	MAST	Fixed
PI (min)	55.98	30.21
PW (min)	1.72	0
PT (min)	23.93	14.00
PWK (min)	0	7.5
M (miles)	983.4	1000
Z	8674.2	9862.1

We observe that the PI values (directly proportional to the headway of the system and not included in Z) clearly are in favor of the fixed-route system. However, it has been shown that for headways larger than 12-13 minutes the majority of the customers are aware of the schedule (Okrent, 1974) and this is true for all P requests showing up at bus stops (for both systems). Furthermore, for NP requests in the MAST system, PI represents the waiting time incurred from the customer's call to the earliest possible pick-up time that people most likely spend at the office, at home or in a comfortable location, not at the bus stop. Therefore, we do not consider PI as a valid parameter for this comparison.
The other figures show that the MAST system compared to the corresponding fixed-route results has a smaller PW (< 2 minutes) and a PT bigger by approximately 10 minutes, but M is lower and there is no walking for the customers as opposed to the fixed-route system where on average customers walk 7.5 minutes. The overall performance Z is clearly in favor of the MAST system, confirming the validity of this innovative service compared to a conventional transportation system for this service region.

4. Conclusion

In this paper we propose a mixed integer programming formulation of Mobility Allowance Transit Shuttle (MAST) services. Since the problem is NP-hard, we need an approximation method to solve it. We also present a performance comparison, through simulation analysis, between MAST and conventional fixed-route bus services. The results show that the innovative hybrid characteristic of MAST services are competitive with conventional ones and perform better under certain demand distributions.

5. Acknowledgements

The research reported in this paper was partially supported by the National Science Foundation under grant NSF/USDOT- 0231665.

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