Transportation Planning and Technology
Vol. 32, No. 4, August 2009, 377–399

Routledge
Taylor & Francis Group

A methodology for comparing distances traveled by performance-equivalent fixed-route and demand responsive transit services

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(Received 12 February 2008; final version received 12 June 2009)

Public transport systems are confronted by the need to improve their economic effectiveness in order to meet customer requirements at acceptable costs for transit providers, which are often heavily subsidized. Our goal is to understand how the organizational form of the transit system impacts on system productivity. Our methodology consists of comparing performance in terms of distance traveled of two competing transit services, a traditional fixed-route and a demand responsive service, while ensuring a comparable service to the same set of customers. We consider several scenarios, which depend on the road network, service quality level, and demand density. According to our findings, demand responsive transit services perform better for high-quality service levels and low demand density scenarios.

Keywords: transit; bus lines; demand responsive services; service level; distance traveled

Introduction

Public transport systems play a key role in the management of the demand for transport in most urban areas around the world. Such services would rarely be profitable in an open market, at least under the current policy constraints concerning fares and levels of service; but the expected social and environmental benefits push decision makers to heavily subsidize them. However, several different economic factors are demanding greater efficiency of public expenditures, including transit agency budgets. The challenge is thus to provide public transport services that minimize operating costs while maintaining a service quality that is comparable to that of the dominant mode, i.e. the private car. The strategy to achieve this usually relies on investing in new infrastructure. Where this is neither possible nor convenient,

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ISSN 0308-1060 print: ISSN 1029-0354 online

DOI: 10.1080/03081060903119618
http://www.informaworld.com

other ‘physical’ characteristics of the systems are improved, such as the creation of reserved bus lanes.

In this paper we explore an alternative way of lowering the operating costs of a transit service, by focusing on the different ways it can be organized. Most services are in fact fixed-route (FIX) services operated by a fleet of vehicles that run with pre-defined paths and schedules. However, since the 1970s, a different form of public transport system has been theorized and experimented with, that is usually termed ‘demand responsive transit’ (DRT). Our paper studies whether a DRT service would compare favorably against a traditional FIX service with respect to distance traveled, and thus operating costs, within different urban contexts, for different service qualities and demand levels, using a simulation methodology approach. In a companion paper we draw further comparisons between the two kinds of service from the point of view of emissions of pollutants, by using the same methodology presented here (Diana *et al.* 2007).

A DRT service is a public transportation service in which users have to book their trip in advance. The operator then collects the reservations and schedules the vehicle paths to serve the requests. Large systems were initially envisioned as a less fuel-consuming alternative to traditional large buses, considering the then oil crisis. Nowadays in the USA, beyond the flourishing market niche of airport feeders, these systems are mainly used to provide a mobility service to those that cannot use traditional public transport lines, such as elderly and disabled people.

Only a few papers draw a comparison between FIX and DRT services from this perspective. An introductory framework comparing the performance of different forms of transit systems is provided by Diana and Pronello (2004), who developed an experimental plan to determine the best system in terms of vehicle running and emissions with a large set of scenarios. This paper partially builds on their work, by proposing a new and more rigorous methodology for tackling the difficult problem of comparing service qualities between FIX and DRT services. Diana (2003) has developed a detailed application to understand how public transport running would be affected if the actual evening bus service in the city of Turin (Italy) were to be partially substituted by a DRT service. However, the results of this study cannot be easily generalized, since they refer to the specific situation of that city concerning both the service organization and the demand patterns. Quadrifoglio and Dessouky (2004) proposed a comparison between a FIX service and a fixed and flexible hybrid transit solution, applying a methodology based on a weighted multi-term objective function, partially related to the one used in this paper. Over the years there have been many other papers that, broadly speaking, have drawn comparisons between fixed and flexible route services, although their focus is on more general economic aspects; see for example, Daganzo (1984) and Aldaihani *et al.* (2004) or Chang and Yu (1996) for a review.

In the next Section we describe our simulation framework, defining the idealized service areas and the characteristics of travel demand. The systems studied are then presented in Section 'Competing transit systems.' Section 'Performance comparability of the systems and virtual travel time' describes how we can establish equivalences between different public transport systems with respect to level of service, which of course must be kept constant in order to draw meaningful comparisons. In Section 'Experimental design,' we present our experimental design and present our results. We then present our conclusions in the final Section.

The simulation framework

In this section we present the framework in which we will perform our simulation analyses, defining the assumptions about the service areas and demand. We will consider different urban structures, each one with its own road network and demand distribution, in order to cover different operational contexts and appreciate their influence on the results.

Service areas and road networks

The three cases under investigation are the following:

- Case G. A square area of 25 km^2 (with an edge $L = 5$ km), evenly divided into 100 square sub-areas with edges of 0.5 km and a bus stop in the middle of each. The road network is represented by a grid connecting all stops (Figure 1).

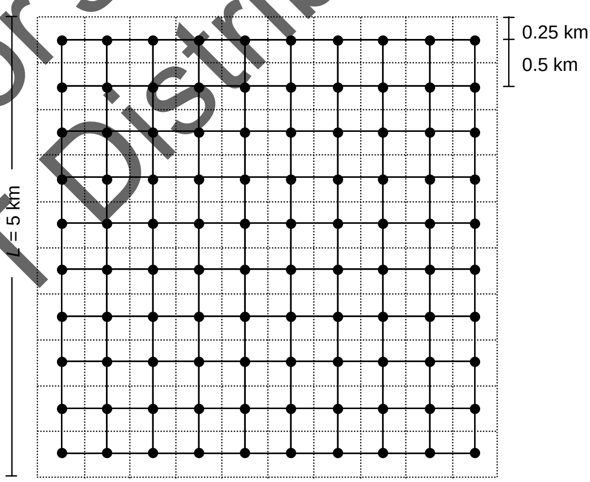

Figure 1. Case G: service area and road network.

- Case R. A circular area of $25\,km^2$ (with a radius $r=2.82$ km), composed of a central sub-area with a diameter of 0.51 km and by five concentric and adjacent rings with the same width of 0.51 km (ring 1 being adjacent to the central sub-area and ring 5 being the outer ring). Each ring is evenly divided in 32 slices. As a result, there are 161 sub-areas and in the middle of each there is a bus stop. The road network is composed of 16 straight roads connecting the 10 stops along the same diameter and the middle stop, which is common to all roads. On the outer edge, the spacing between the terminal stops of two adjacent lines is about 0.5 km (Figure 2a).
- Case RR. The same configuration as case R, but with five additional circular roads connecting the 32 stops in the middle of each ring. The lengths of these roads are, respectively, 3.22, 6.45, 9.67, 12.89, and 16.11 km; while the spacing between stops in these roads is, respectively, about 0.1, 0.2, 0.3, 0.4, and 0.5 km (Figure 2b).

Case G (grid) is a good approximation of most US cities, whose road networks are roughly designed as a grid, and it is probably the most studied in research dealing with the optimal supply of public transport services, because it is mathematically easier (cf. Aldaihani *et al.* 2004). In addition, it can be seen as a good starting point for more realistic analyses, for example, by dividing an urban area into several regions in which demand is nearly constant and then applying this analysis to each region. The ring-radial network RR can represent the classic monocentric 'European style' city and

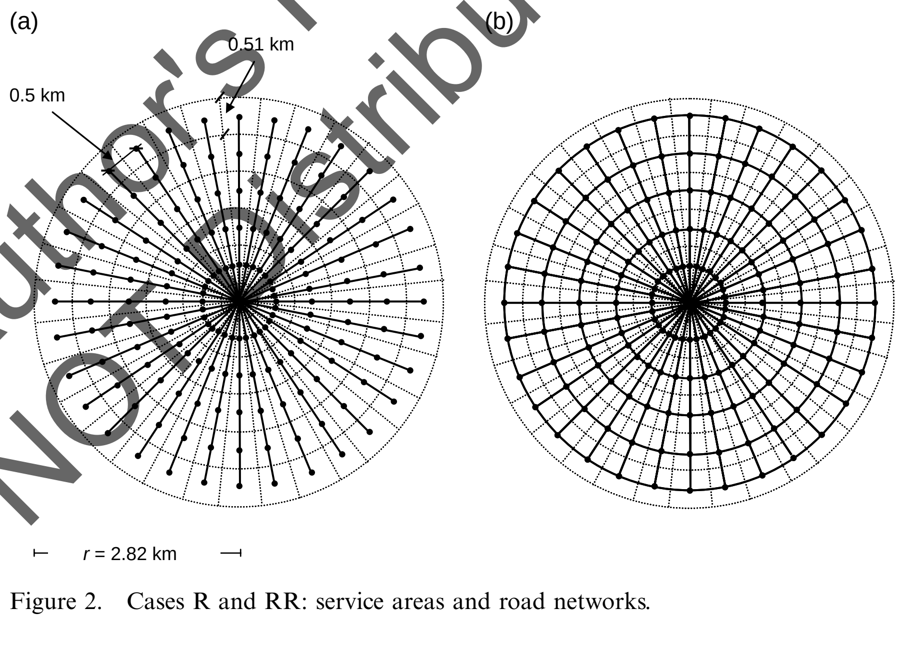

Figure 2. Cases R and RR: service areas and road networks.

is still of interest when urban sprawl processes have not completely reversed territorial dynamics. The intermediate R (purely radial) case is more an abstraction, but it has been considered because in many cities the public transport configuration is strongly monocentric, whereas services between peripheral sectors are weaker and more problematic. We have chosen to take this phenomenon to extreme consequences by eliminating any service that does not pass through the center. Of course, it would not be wise to organize a DRT service that can move only along radial lines, so it could be interesting to study a hybrid situation between R and RR, in which rings are used only by DRT services.

Demand

For Case G, demand is uniformly distributed across the whole service area; therefore, each sub-area has 1/100 chance to be selected as a pick-up or drop-off point.

For Cases R and RR, the demand distribution is shaped as a cone and linearly decreases from a maximum in the center to zero on the outer edge. Thus, the distance from the center of each demand point is drawn from a symmetric triangular distribution $[-2.82, +2.82]$ and the orientation of each is drawn from a uniform distribution $[0, 2\pi]$. Because of the geometry of the service area and the assumed partitions, the probability that a demand point (either a pick-up or a drop-off) would be in the central sub-area is 2.33%; while the probability that it will be in each of the 32 sub-areas located in ring 1–5 is, respectively, 0.5%, 0.78%, 0.84%, 0.67%, and 0.27%.

In all cases, each demand point is assigned to the nearest bus stop in the grid and we assume that trip origins and destinations are statistically independent. However, we will exclude from consideration requests having pick-up and drop-off assigned to the same bus stop.

The temporal distribution of the demand is modeled as a Poisson process with rate λ and we assume a static environment, with all demand known in advance.

This demand modeling, both temporal and spatial, is clearly a simplification and can be modified and refined accordingly in future research, including applied case studies. In this paper, we focus on simplified but reasonable modeling assumptions, making sure that the same set of customers, drawn from the above distributions, are used to test and compare the competing transit system, to guarantee a meaningful comparison.

Competing transit systems

All vehicles in both systems are assumed to move at the same operating speed $v = 20$ km/hour, which also includes the time spent for customer boarding and disembarkment.

Traditional fixed-route (FIX) bus services: configuration, distance traveled, transfers, and waiting time

In this section we derive the expected values and variances for distance traveled, and waiting time experienced by customers for each of the three cases described above. The values calculated analytically have been verified by simulation.

Case G

There are 20 lines (10 horizontal and 10 vertical) covering the whole grid of stops. Each line is 4.5 km long and passes by 10 stops. All the lines are assumed to have the same headway h_G, which depends on fleet size. Vehicles in each line move back and forth between terminals located at the edges.

It is possible to derive the distribution of distance d_G traveled by the service customers by enumerating all the possible pick-up/drop-off pairs of the demand. The resulting average distance traveled and its variance are

$$\mathrm{E}[d_G] = 0.667L, \tag{1}$$

$$\mathrm{Var}[d_G] = 0.109L^2. \tag{2}$$

We can also derive the portion of customers which will need a line transfer to reach their destination. A pick-up point can randomly be in any of the 100 stops in the grid. The corresponding drop-off can be any of the other 99 stops (since we are excluding the stop assigned to the pick-up). Each line passes by 10 stops and each stop in the grid is served by two lines: a horizontal line and a vertical one. Therefore, given a pick-up stop, there are $9+9=18$ stops which, if chosen as drop-off, would allow the customer to avoid a line transfer, since both service stops would be served by the same line (either horizontally or vertically). Thus, the portion of 'no-transfer' (nt_G) and 'transfer' (t_G) customers are:

$$nt_G = 18/99 = 18.2\%, \tag{3}$$

$$t_G = 1 - nt_G = 81/99 = 81.8\%. \tag{4}$$

Since h_G is equal for all vehicles in the network and assuming no synchronization among the lines, no-transfer customers will have to wait a uniform U[0, h_G] time for their pick-up, with an expected value of $h_G/2$. Transfer customers can board indifferently either the horizontal or the vertical line at their pick-up (whichever comes first) and then switch to the other one at their transfer stop. Hence, they will have to wait $h_G/3$ at their pick-up stop (that is the expected value of the minimum waiting time between the horizontal and vertical line, both having a U[0, h_G] distribution); however, they will have to wait $h_G/2$ time at their transfer stop (at this point, they cannot choose the line anymore). Therefore, their total expected waiting time is $h_G/3+h_G/2$. The average waiting time WT_G and its variance (analytically derivable by conditioning) are given by

$$\mathrm{E}[WT_G] = nt_G\frac{h_G}{2} + t_G\left(\frac{h_G}{3} + \frac{h_G}{2}\right) = \frac{3 + 2t_G}{6}h_G = 0.773h_G, \tag{5}$$

$$\mathrm{Var}[WT_G] = 0.145h_G^2. \tag{6}$$

Case R

There are 16 lines, each one of them following a diametric road, about 5.13 km long and passes by 11 stops, one every 0.51 km. All the lines are assumed to have the same headway h_R. Vehicles in each line move back and forth between terminals located at the edges.

Given the demand distributed as described above (Section 'Demand'), we can derive the distribution of the distance traveled by customers by enumerating all possible pick-up/drop-off pairs. Note that only radial trips are possible. The resulting average distance traveled by customers and its variance are

$$\mathrm{E}[d_R] = 0.984r, \tag{7}$$

$$\mathrm{Var}[d_R] = 0.114r^2. \tag{8}$$

No-transfer customers need to have both pick-up and drop-off in the same diameter, thus served by the same line. Customers having their pick-up in the middle stop (2.33%) clearly are no-transfer (regardless of where their drop-off stop is, since all lines pass the middle). Customers having their pick-up anywhere else (100–2.33% = 97.67%) also may not need a transfer if their drop-off stop falls in the same diameter, which will happen 97.67%/16 + 2.33% = 8.43% of the time. Therefore, the portion of no-transfer customers is 2.33% + 97.67% × 8.43% = 10.56% and transfer customers are 100–10.56% = 89.44%. However, the count of the no-transfer customers actually includes the ones having pick-up and drop-off at the same stop, which are not considered by assumption and are 0.72% of the total. After subtracting them and normalizing the values we have

$$nt_R = 9.9\%, \tag{9}$$

$$t_R = 1 - nt_R = 90.1\%. \tag{10}$$

No-transfer customers will wait an average time of $h_R/2$ at their pick-up stop; the remainder will need a transfer in the center point and they will wait a total time of $2 \times h_R/2 = h_R$ (note that in this case, customers do not have a choice for their boarding at pick-up, like in Case G). The average waiting time WT_R and its variance are therefore given by

$$\mathrm{E}[WT_R] = nt_R\frac{h_R}{2} + t_R h_R = 0.95h_R, \tag{11}$$

$$\mathrm{Var}[WT_R] = 0.181h_R^2. \tag{12}$$

Case RR

In addition to the lines of Case R, there are five more lines; each one follows one of the five ring roads. All the diametric lines are assumed to have the same headway h_d. The ring lines instead have different headways, namely h_1, h_2, h_3, h_4, and h_5, from the inner to the outer line, respectively. The values clearly depend on fleet size.

Demand is distributed as Case R, but the road network is more complex. However, we can still derive the distribution of the distance traveled with the additional reasonable assumption that customers, when facing a choice among different possible paths from pick-up to drop-off, would first minimize the number of transfers needed and then the total distance traveled ('*least-transfer*' *policy*). This means that customers prefer a longer path if this makes them avoid a line transfer, such as using only a circular line from pick-up to drop-off in the same ring, instead of using two diametric lines with a transfer in the middle stop, even though the latter option would slightly shorten the total trip (with respect to distance, not necessarily time). Under this assumption, the resulting average distance traveled by customers and its variance are

$$\mathrm{E}[d_{RR}] = 0.787r, \tag{13}$$

$$\mathrm{Var}[d_{RR}] = 0.148r^2. \tag{14}$$

The portion of no-transfer customer would be in this case much higher than in Case R, because of the additional circular lines. After enumerating all the possibilities, we have the portion of no-transfer customers' nt_{RR}:

$$nt_{RR} = 30.0\%, \tag{15}$$

which includes 9.9% of them using only a diametric line; 2.4%, 5.9%, 6.8%, 4.3%, and 0.7% of them using only a circular line, respectively, in ring 1, 2, 3, 4, and 5. The portion of transfer customer t_{RR} is the complement

$$t_{RR} = 1 - nt_{RR} = 70.0\%, \tag{16}$$

which includes customers using all possible combinations among two diametric lines (23.4%) or using a diametric line and one of the circular lines (specifically 16.4%, 17.8%, 10.1%, and 2.3% for line, respectively, in ring 1, 2, 3, and 4).

From the above information, we can also calculate the usage of the lines, defined as the probability that a request drawn from the assumed demand distribution will make use of each of them for its trip. After normalization, the usages are as follows: 60.8% for the diametric lines considered altogether; 11%, 14%, 9.9%, 3.9%, and 0.4% for ring line 1, 2, 3, 4, and 5, respectively. This would allow defining the average headway h_{RR}, which we will later use as a measure of the service quality and is calculated in proportion to the usage of the lines as follows:

$$h_{RR} = 60.8\% \times h_d + 11\% \times h_1 + 14\% \times h_2 + 9.9\% \times h_3 + 3.9\% \times h_4 + 0.4\% \times h_5. \tag{17}$$

Finally, we can calculate the average waiting time WT_{RR}. Each line will have an expected waiting time given by half of its headway. By considering all the possible requests and rearranging all the terms, the resulting value is given by

$$\mathrm{E}[WT_{RR}] = 0.52h_d + 0.094h_l + 0.12h_2 + 0.084h_3 + 0.034h_4 + 0.0036h_5. \tag{18}$$

The variance of WT_{RR} is analytically derivable by conditioning; however, since it is a cumbersome function of the headways, we will not show it here, but we will provide its values along with the others in Section 'Performance of the fixed-route (FIX) system' (Table 2).

The waiting times WT derived above in all three cases might appear overestimated especially for high headway scenarios, since transit customers usually adjust their arrival times at their pick-up stops according to the service schedule. However, in our model, WT represents the entire time from the moment a customer would be ready to be picked up to the moment the pick-up actually occurs. Most of this time may be spent at home/office or any other comfortable location and not only at the bus stop. For a meaningful comparison we will use the same waiting time definition for the DRT service in the following section.

At the end of this section, we want to emphasize that we are not attempting to define the optimal FIX transit service for each of the above cases. For example, it is well known that the optimal configuration of a bus system in case G is not the one we assumed, being L-shaped paths across the grid more efficient (Newell 1979). In addition, schedule synchronization between lines often occurs (such as timed transfer systems), especially for low frequency services, reducing transfer times. Our goal is rather to provide simplified models of likely public transport assets that can be observed when the urban structure is similar to the one we introduced. Those assets are the product of successive historical evolutions and of social groups' interaction, rather than that of a completely rational optimization approach, and it is on such systems that we are focusing our attention. Readers interested in the optimization approach for transit systems are referred to the abundant literature in the sector, see for example, Ceder and Wilson (1986) for an introductory overview, Kim and Barnhart (1999) and Ceder (2002) for a more updated state of the art review or Baaj and Mahmassani (1995) and Ceder *et al*. (2002) for applied studies.

Demand responsive transit (DRT) services

As an alternative to the FIX services described above, we consider a DRT system operating in the same area and serving the same demand for each case. DRT systems are 'many-to-many' types of service without pre-defined paths or schedules that can serve requests between any origin/destination pair without vehicle changes, while allowing ridesharing. While DRT systems (such as Paratransit) are generally door-to-door services, in our study we assume that customers walk to the closest stop in the network in order to simplify our comparison between the services, since total walking time will be the same for both systems.

Scheduling DRT systems is a known NP-Hard problem in the Operations Research literature: problem instances of realistic size cannot be solved to optimality within a reasonable computational time. Many heuristics have thus been developed to give approximate solutions to be able to operate these systems. For the purpose of our simulations, we will use the insertion heuristic presented in Diana and Dessouky (2004), where it has been referred to as 'Algorithm 1.'

We assume a static environment, that is, all requests come to a dispatch center before starting the scheduling phase. Customers have to book their ride specifying the origin, destination, number of passengers, and pick-up time. Trip origin and destination can be any stop on the road network of the case considered. The operator fixes (or negotiates) the maximum ride time MT and the maximum wait time MW at the pick-up point. MT and MW control the quality of the system. Tightening them ensures a higher quality to the customers, but decreases the probabilities of sharing a ride, thus increasing both the number of needed vehicles and the kilometers driven, and ultimately the operating costs. MT is defined as follows:

$$MT = a \times DT + b, \tag{19}$$

where DT represents the direct ride time from pick-up to the delivery point, and a and b are two parameters specified by the scheduler. All requests are then scheduled by the algorithm, which starts from an initial feasible solution and attempts to minimize fleet size by progressively lowering the number of used vehicles, until some request cannot be served without violating some of the constraints. The final schedule defines the paths that the needed vehicles will have to follow through the given street network in order to serve all requests in due time. Vehicles can stop and be idle while waiting for customers at any node of the street network, provided that no customers are already onboard.

The calculation of distance traveled and waiting time for DRT services is dependent upon the setting of the parameters a, b, and MW, and cannot be simply analytically derived as done for the FIX case, but has to be computed by simulation. This will be done in Section 'Simulations and performance of the demand responsive transit (DRT) system.'

Performance comparability of the systems and virtual travel time

To compare the systems in terms of total distance traveled, we need to make sure that the two systems provide an analogous service to customers, so that a potential efficiency gain would not come at a cost in terms of overall service quality level, which we will refer to as the performance of the system.

Customers using either one of the services would face a waiting time at their pick-up stop and a ride time spent onboard. FIX transfer customers will have additional waiting time at their transfer stop. Note that DRT customers would generally have a longer ride time, because of ridesharing; however, they would never need a transfer. We disregard other possible sources of noise, such as the comfort onboard the vehicle or any other element that could influence customers' perceptions and opinions about the service performance.

Hence, we can measure the performance of either one of the systems by the following function Z_i, for each customer $i = 1$ to N, representing the *virtual travel time* and defined as

$$Z_i = RT_i + \omega_1 \times WT_i + \omega_2 \times T_i, \tag{20}$$

where $RT_i = d_i/v$ is ride time (d_i is distance traveled), WT_i is total waiting time, and $T_i = \{0, 1\}$ indicates whether i needs a transfer or not. $T_i = 1$ with probability t_G, t_R, or t_{RR} ($T_i = 0$ with probability nt_G, nt_R, or nt_{RR}), depending on the road network, for a customer drawn by the corresponding assumed demand distribution and using the FIX service; while $T_i = 0$ for all customers using the DRT service, since there are no transfers.

The weights ω_1 and ω_2 are needed to obtain a uniform and meaningful value for Z_i, which is measured in ride time units. A lot of research has been performed with the aim of giving correct estimations of them and different field studies recommend different values. Beyond the different research perspectives, these discrepancies are probably due to interaction effects between the perceived inconvenience of waiting for the bus or having to transfer and other situation-specific factors such as the quality of the scheduling, meteorological conditions, security concerns at bus stops, or even trip purpose and the activity patterns of travelers (Evans 2004, Evans and Pratt 2004, Vande Walle and Steenberghen 2006). On the basis of two recent studies (Guo and Wilson 2004, Wardman 2004) and taking into account that in our idealized framework, we do not consider the above disturbance effects, we assume $\omega_1 = 1.8$ and $\omega_2 = 10$ min, meaning that a unit of waiting time is perceived by customers (on average) to be about 1.8 units of ride time and that the discomfort due to a line transfer can be compared to 10 min of ride time.

The following mean value over all customers served provides an indication of the overall performance of the system considered:

$$Z = \frac{1}{N} \sum_{i=1}^{N} Z_i. \tag{21}$$

Thus, the two systems can be considered comparable in terms of performance if their Z values are similar when they serve the same set of customers.

Experimental design

For each road network case (G, R, and RR), we build a 2^2 factorial design, where the two factors under control are service quality and demand density; hence, we create 12 operating scenarios. In each scenario, we first calculate the distribution of distances traveled, waiting time, ride time, and ultimately the resulting Z_i and its mean Z for the FIX service; then, we perform simulations of the DRT service, properly adjusting the parameters a, b, and MW, defined in Section 'Demand responsive transit (DRT) services,' in order to have an overall performance (measured by the Z value) that is analogous to the FIX service, ensuring in this way a proper comparison of services.

The first factor considered is service quality, which is identifiable by the mean headway of each FIX case (h_G, h_R, and h_{RR}); in fact, a smaller headway provides a better service quality to customers by lowering the overall waiting time and vice versa. We recall that when setting the levels of the factors for a 2^k factorial design, it is wise to consider wide ranges for each variable. Thus, the two levels of headway considered are about 5–7 min for the high-quality service level and about 27–30 min for the low-quality service level. These represent extreme values typically encountered in most urban bus transit services. We can determine the FIX fleet sizes needed in each case: specifically, in Case G, we consider a service with one vehicle/line and another one with four vehicles/line; in Case R, we consider a service with one vehicle/line and another one with five vehicles/line; in Case RR, we consider a service with one vehicle/line in the diametral lines and two vehicles/line (one clockwise and one counterclockwise) in each circular line and a another case with five vehicles/line in the diametric lines and 2, 4, 6, 8, and 10 vehicles/line (half clockwise and half counterclockwise) in each circular line, respectively, from the outer line to the inner line, to better respond to the triangular demand distribution. The resulting headways for each service configuration are shown in Table 1.

The other factor is demand density, which is expressed by the Poisson arrival rate λ of customers requesting the service. We consider $\lambda = 2$ requests/min for the low demand level and $\lambda = 50$ requests/min for the high demand level. The low demand level corresponds to approximately 4.8 requests/hour/km^2

Table 1. FIX service quality settings: headways (min).

Case	Headway	Low quality (q)	High quality (Q)
G	h_G	27.00	6.75
R	h_R	30.77	6.15
RR	h_d	30.77	6.15
	h_1	9.67	1.93
	h_2	19.34	4.83
	h_3	29.00	9.67
	h_4	38.67	19.34
	h_5	48.34	48.34
	h_{RR}	27.05	6.54

(within our 25 km^2 service areas). This value is typically encountered in many DRT services operating either in sparsely populated rural areas or in urban areas to serve specific social groups. The high demand level is instead roughly the maximum allowable for a FIX system working with the maximum headway of 30 min, before reaching the vehicle capacity limit, which we assume to be about 85 people. In fact, let us consider the most heavily loaded system, which is Case R, since it has the minimum number of lines (16) and only one transfer point. We have shown in Section 'Case R' that about 90% of customers use two lines to accomplish their trip. Thereby there are 5400 trips per hour to serve when we have $\lambda = 50$ requests/min, which is about 338 people/hour traveling on each line. The vehicle cycle time, given our operating speed $v = 20$ km/hour, is 30 min. With a 30 min headway, each bus will thus have to carry about 85 people at maximum, assuming that all passengers are onboard when the center is crossed.

For the remainder of the paper, we refer to the above described 12 scenarios through the following notation: (a) the initial letters indicate the considered case (G, R, or RR); (b) then, a 'q' is added for low service quality scenarios (headways are set as shown in the third column of Table 1) and a 'Q' is added for high service quality scenarios (headways are set as shown in the last column of Table 1); and finally, (c) the demand density of each scenario will be indicated with either a 'L' for low ($\lambda = 2$ requests/min) or a 'H' for high ($\lambda = 50$ requests/min).

Performance of the fixed-route (FIX) system

With the above headway values (Table 1) and recalling that $L = 5$ km for Case G and $r = 2.82$ km for Cases R and RR, we can analytically calculate distance traveled d, waiting times WT, and the quality function Z for each scenario, by using Eqs. (1)–(21). Their expected values are summarized in Table 2 along with their variances. The variances of Z (last column) were

Table 2. FIX service: distance traveled, waiting time, percentage of transfers, and virtual travel time.

Scenario	E[d] (km)	Var[d] (km^2)	E[WT] (min)	Var[WT] (min^2)	t (%)	E[Z] (min)	Var[Z] (min^2)
G_q_L, G_q_H	3.33	2.72	20.9	105.9	81.8	55.7	474
G_Q_L, G_Q_H	3.33	2.72	5.2	6.6	81.8	27.6	96
R_q_L, R_q_H	2.78	0.91	29.2	171.2	90.1	70.0	644
R_Q_L, R_Q_H	2.78	0.91	5.8	6.8	90.1	27.9	59
RR_q_L, RR_q_H	2.22	1.18	23.0	169.4	70.0	55.1	699
RR_Q_L, RR_Q_H	2.22	1.18	5.6	16.7	70.0	23.7	108

obtained through simulation. We also include the percentage t of transfer customers in the sixth column. Note that the demand level does not influence the figures.

Simulations and performance of the demand responsive transit (DRT) system

At this point, we run the simulations for the DRT system. The goal is to adjust the DRT parameters a, b, and MW to provide customers with a service comparable to the corresponding FIX system in terms of performance, measured by Z. Theoretically speaking, the setting of these parameters can be arbitrary and several combinations of the values of the parameters may provide desired Z values. However, we tried to follow logic and use reasonable assumptions to set them. In each scenario, we fixed the maximum wait time for the DRT service MW equal to double the FIX headway, which corresponds to the maximum possible waiting time for a customer using FIX and needing a transfer in Cases G and R. This is not the case for Case RR, since there is not a unique headway, but for uniformity and simplicity we follow the same rule and we set MW equal to double the h_{RR} values. Thus, we derive the MW values for the DRT service by doubling the headway values reported in the second, third, and last row of Table 1 and rounding them to the nearest integer.

Two more parameters must then be set, namely a and b from Eq. (19), which define maximum ride time MT. We chose to take b equal to the above derived maximum wait time MW, and we run several trial and error simulations adjusting the last parameter a, until the values of Z for FIX and DRT do not differ more than 10%. In doing so, we also maintain the same a value for DRT systems operating in the same road network and at the same service quality level, even though at different demand levels; that is, for example, the DRT parameters will be the same for scenarios G_q_L and G_q_H; similarly for the other pairs. Each DRT simulation was of two hours duration and the number of scheduled requests was 240 for smaller problems and 6000 for larger ones.

The results of this simulation process are shown in Table 3. The second, third, and fourth column show the values of the three above parameters; whereas the following six columns report for the DRT service the same information that is contained in Table 2 for the FIX service. The second to last column provide the Z values for the FIX system serving the same demand considered for the corresponding DRT system; we note that these figures are slightly different than the analytical values shown in Table 2, since they are calculated by simulation. The last column shows the differences in virtual travel time between the two services for each scenario.

It can be seen that the absolute values of $\Delta Z\%$ are all below 5%, except in one case, ensuring a solid comparability between the two systems in terms of performance. It is interesting to note that the a values vary substantially across the different scenarios. Moreover, the sensitivity of Z to a is higher for scenarios with low service quality, particularly when the RR network is considered, where we had to set a much more precisely in order to obtain comparable virtual travel times. A possible explanation is that in low service quality scenarios, the parameters b and MW take on much larger values, due to our assumptions. Each time a DRT simulation is run, an approximate solution to a combinatorial optimization problem has to be found, as described in Section 'Demand responsive transit (DRT) services,' respecting several different constraints (for their analytical definition see Dumas *et al.* 1991). Since the scheduling constraints are essentially dependent on a, b, and MW, if b and MW are 'loose,' then the a value has a greater chance to bound the problem solution and become the key parameter, to which the outcome Z is more sensitive.

Distributions of ride time, waiting time, and virtual travel time

To further evaluate the comparability of the two systems, an inspection of some of the distributions can be helpful. We show in the following charts the histograms of the ride time for scenarios G_q_H (Figure 3) and RR_Q_L (Figure 4), the waiting time for scenarios G_Q_H (Figure 5) and RR_q_L (Figure 6), and the virtual travel time for scenarios R_q_L (Figure 7) and RR_Q_H (Figure 8). In each chart we compare FIX and DRT systems serving the same set of customers.

As shown in Figures 3 and 4, ride time is shorter for the FIX system than for DRT. As noted earlier, this is expected, since DRT vehicles deviate from the shortest path from pick-up to drop-off of each customer, because of ridesharing. However, DRT customers do not need to transfer line like the majority of FIX customers.

Figures 5 and 6 show different shapes of the waiting time distributions for FIX and DRT, even though the means are comparable (see Tables 2 and 3). In

Table 3. DRT service: parameters, distance traveled, waiting time, and virtual travel time.

Scenario	a (−)	*b* (min)	*MW* (min)	E[*d*] (km)	Var[*d*] (km^2)	E[*WT*] (min)	Var[*WT*] (min^2)	E[*Z*] (min)	Var[*Z*] (min^2)	*Z*(FIX) (min)	Δ*Z*%
G_q_L	1.0	54	54	7.90	27.6	18.9	246	57.6	683	55.7	+3.2%
G_q_H	1.0	54	54	7.02	19.5	16.3	186	50.4	569	55.7	−9.7%
G_Q_L	1.7	13	13	5.52	8.8	5.6	17	26.7	107	27.6	−3.3%
G_Q_H	1.7	13	13	5.55	9.9	5.9	15	27.4	104	27.6	−0.7%
R_q_L	1.1	62	62	7.34	34.9	27.7	434	71.9	1028	69.9	+2.7%
R_q_H	1.1	62	62	8.00	32.5	24.8	306	68.8	796	69.9	−1.7%
R_Q_L	4.5	12	12	5.65	16.8	5.9	17	27.6	197	27.8	−0.7%
R_Q_H	4.5	12	12	6.29	19.9	5.3	14	28.3	217	27.8	+1.8%
RR_q_L	1.005	54	54	6.61	22.7	19.8	253	55.5	733	55.0	+0.5%
RR_q_H	1.005	54	54	5.26	17.9	22.2	249	55.8	839	55.0	+1.1%
RR_Q_L	2.5	13	13	3.59	5.8	6.5	19	22.4	91	23.6	−4.7%
RR_Q_H	2.5	13	13	4.00	7.0	6.3	15	23.4	80	23.6	−0.4%

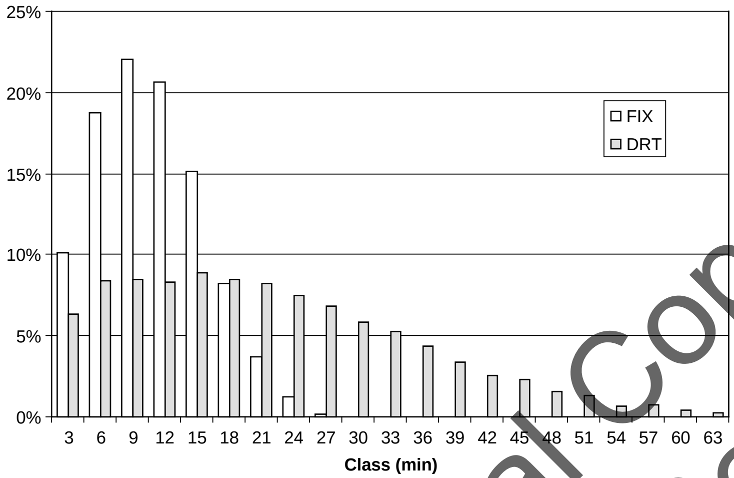

Figure 3. Scenario G_q_H: ride time distribution.

particular, the distributions for DRT present a much higher frequency in the first class, due to the fact that our scheduling algorithm tentatively inserts all requests with zero waiting time, and then shifts them forward only if this is needed to accommodate further trips. This is more likely to happen when demand density is higher, as it can be seen by comparing the two charts. Note that while DRT waiting time is referred to the pick-up stop only, the waiting

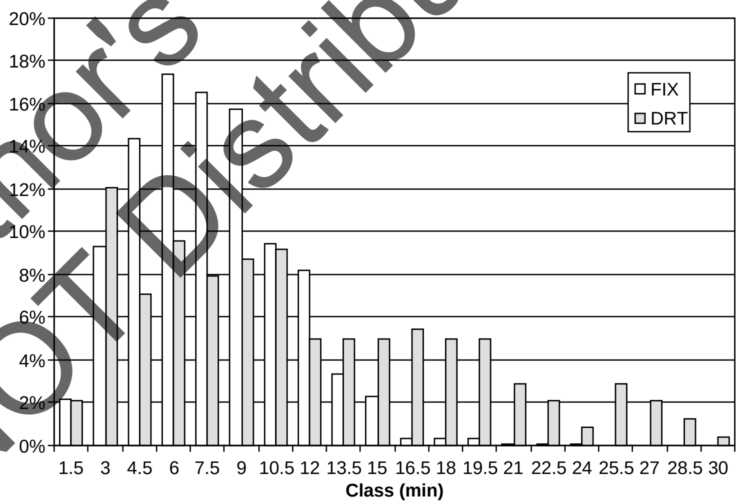

Figure 4. Scenario RR_Q_L: ride time distribution.

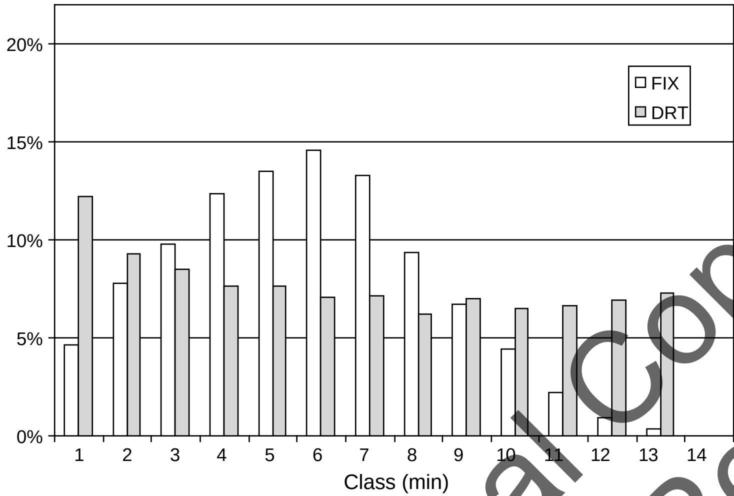

Figure 5. Scenario G_Q_H: waiting time distribution.

time for FIX customers includes the time at pick-up and transfer stop (if any) combined.

The distributions of the virtual travel time Z_i (Figures 7 and 8) show a close similarity between the FIX and DRT histograms: therefore, not only the mean values Z are close (Table 3), but also their distributions. This is true

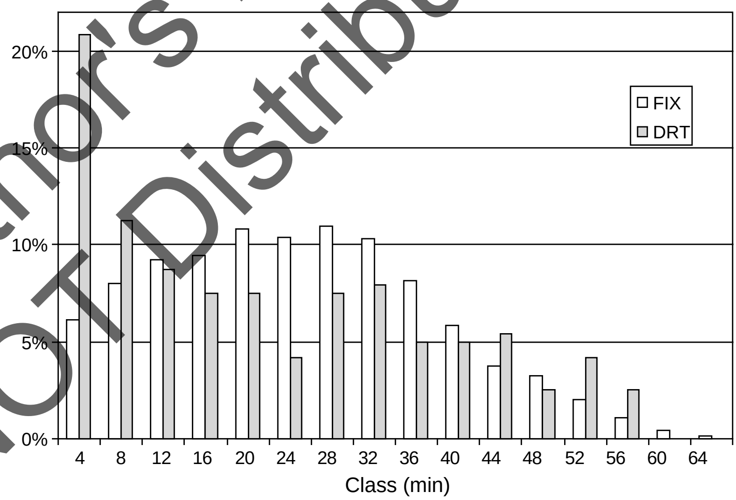

Figure 6. Scenario RR_q_L: waiting time distribution.

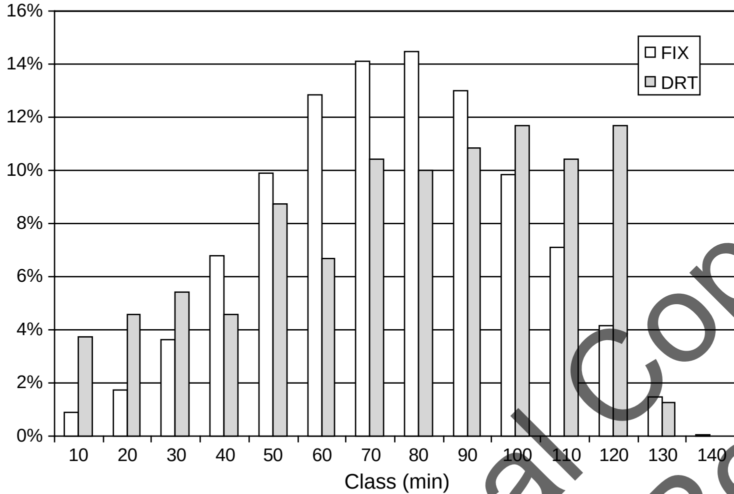

Figure 7. Scenario R_q_L: virtual travel time distribution.

for all the scenarios of our experimental plan, beyond the ones that are considered here.

To summarize, we can conclude that the above settings of the DRT parameters *a*, *b*, and *MT*, as shown in Table 3, ensure that FIX and DRT systems would have very similar performance in each considered scenario. The unavoidable slight differences (a few minutes of virtual travel time on

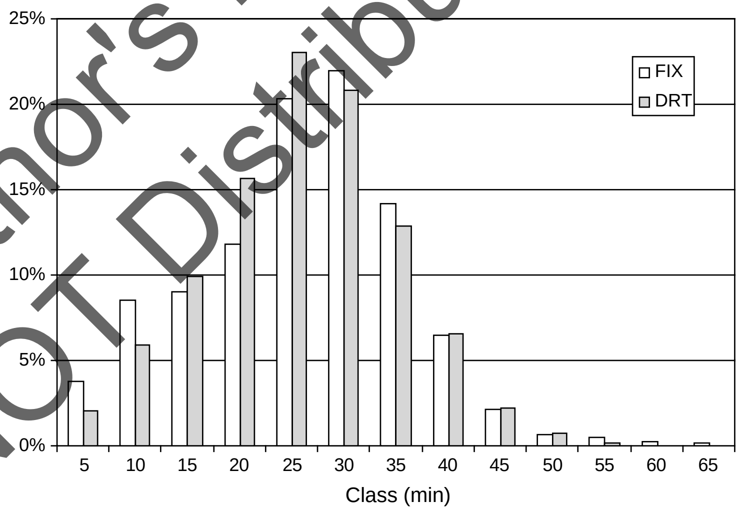

Figure 8. Scenario RR_Q_H: virtual travel time distribution.

average) would hardly be noticed by travelers, also considering the degree of approximation in determining the weights ω_1 and ω_2 in Eq. (20). We are finally able to evaluate and compare the two systems in terms of total distance traveled.

Total distance traveled

The computations of total distance traveled by FIX services are straightforward and depend only on the geometry of the service area, described in Section 'Service areas and road networks,' and on the headways of Table 1. On the other hand, distances traveled for the DRT fleet are clearly dependent upon the particular problem instance. In Table 4 we show the total distance traveled by vehicles of the two competing systems based on the simulations performed in the preceding sections. The last column shows the percentage variation of the DRT service compared to the FIX one.

For each scenario, regardless of demand level (i.e. G_Q scenarios), the kilometers traveled by the FIX fleets are in between those traveled by the DRT services in the same scenario for low (i.e. G_Q_L) and high (i.e. G_Q_H) demand density levels, except for the RR_Q scenarios, where the DRT service always performs better in terms of distance traveled. This is consistent with the results reported in Diana and Pronello (2004), where DRT services were found to be relatively more effective in comparison with FIX services in monocentric road networks.

It should also be mentioned that high service quality (Q) scenarios allow for a better performance of the DRT service. For example, the improvement of the DRT service compared to the corresponding FIX in the R_Q_L

Table 4. Total distance traveled (km) for each scenario.

Scenario	FIX	DRT	Δ%
G_q_L	800	459	−43%
G_q_H	800	3290	+311%
G_Q_L	3200	666	−79%
G_Q_H	3200	5185	+62%
R_q_L	640	528	−18%
R_q_H	640	4800	+650%
R_Q_L	3200	667	−79%
R_Q_H	3200	7505	+135%
RR_q_L	1040	279	−73%
RR_q_H	1040	2501	+140%
RR_Q_L	4400	400	−91%
RR_Q_H	4400	3589	−18%

scenario (−79%) is more significant than the one noted in R_q_L (−18%). The same pattern can be noted for the other scenarios.

In more general terms, our findings are in line with the well-known fact that DRT services perform best when the demand density is low and a good service quality is sought.

Conclusions

In this paper we studied how the organizational form of a public transport service can affect distances traveled, and thus operating costs, in different urban contexts and for different levels of service quality and demand. Particular attention has been devoted to address the difficult problem of adequately characterizing the service performance of public transport systems that have radically different organizational forms. Our methodology builds on past research to define an overall performance of transit services from a customer's point of view and allows comparing different systems. Results indicate that DRT services are more effective than the FIX services in minimizing traveled distances when the demand density is not too high and a good level of service is sought. In particular, demand responsive services show better behavior in a ring-radial network.

However, caution should be taken when defining policies or taking operational decisions. The economic feasibility of such a change should be assessed, since the distances traveled, and the corresponding number of drivers, would sharply increase when the demand density is moderate to high. Moreover, the demand for public transport would not be insensitive to such a radical change in the organizational form of the service, even if the service performance does not change significantly according to our methodology. A deeper evaluation and comparison of the two systems in an operational context would require, among other factors, a demand-offer equilibrium model. The research presented here can be seen as a comparative analysis of FIX and DRT systems that only considers the supply side. This analysis could be embedded in a more comprehensive decision support system in order to fully explore the possibility of minimizing distances traveled through a better use of the two systems.

Finally, we would like to reiterate that in performing our comparisons we assumed a static environment with all demand known in advance for both systems. Since in practice the demand for public transit services generally arises in real time during operations, an interesting extension of the present research would be comparing the two systems in a dynamic environment. In this respect, the work of Diana (2006), which quantifies the loss of efficiency in the scheduling process of a dynamic DRTs compared to the static case, would help in developing such research.

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