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Reliability analysis of centralized versus decentralized zoning strategies for paratransit services

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Abstract

ADA paratransit services are a very large and ever-growing industry providing door-to-door transportation services for people with disability and elderly customers. Paratransit system, however, just like all other public transportation systems, suffers from travel time variability due to various factors and as a result gives its customers unreliable services. Although service reliability is a very important aspect in transportation study, it has not received much attention in the paratransit research community. A quantitative study evaluating the paratransit service reliability under different zoning strategies is yet to be found. This research filled this gap.

Statistical models were proposed to represent travel time variability. Simulation experiments based on real demand data from Houston, Los Angeles and Boston were performed to quantitatively compare the reliability performance of centralized and decentralized operating strategies under different travel time variability levels. Results showed that the decentralized strategy, compared to the centralized no-zoning strategy, substantially improves the reliability of paratransit in terms of on-time performance. This research provides a framework for paratransit agencies to evaluate the service reliability of different organizational strategies through the simulation method.

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Keywords: paratransit; reliability; zoning strategy

1. Introduction

The passage of the American with Disabilities Act (ADA) in 1990 essentially prohibited discrimination based on

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disability, revolutionizing the requirements and expectations for transit agencies. Section 223 of the Act requires that public entities which operate non-commuter fixed-route transportation services also provide complementary paratransit service for individuals unable to use the fixed route system, as their mental and/or physical disability prevents them “to get to or from the system or to board, ride, and disembark from the vehicles.”

As a consequence, the demand for this type of service has experienced a tremendous growth in the last years (8%/yr), more than tripling their ridership in a 15-year period [1]. There are over 5,500 providers of paratransit services for the elderly and persons with disabilities as of today nationwide. Meanwhile however, the transit agencies are facing two common challenges. First, the operating costs have raised even more (12%/yr) than the ridership growth, probably due to the enlarged service areas as a result of urban sprawl. Second, customers of

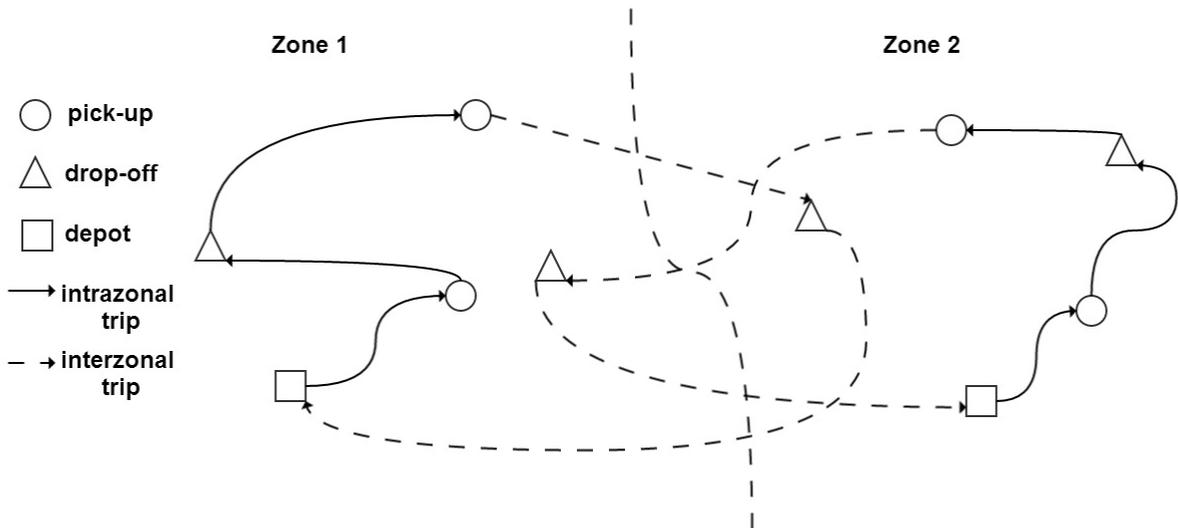


Fig. 1. decentralized (zoning) strategy for paratransit services

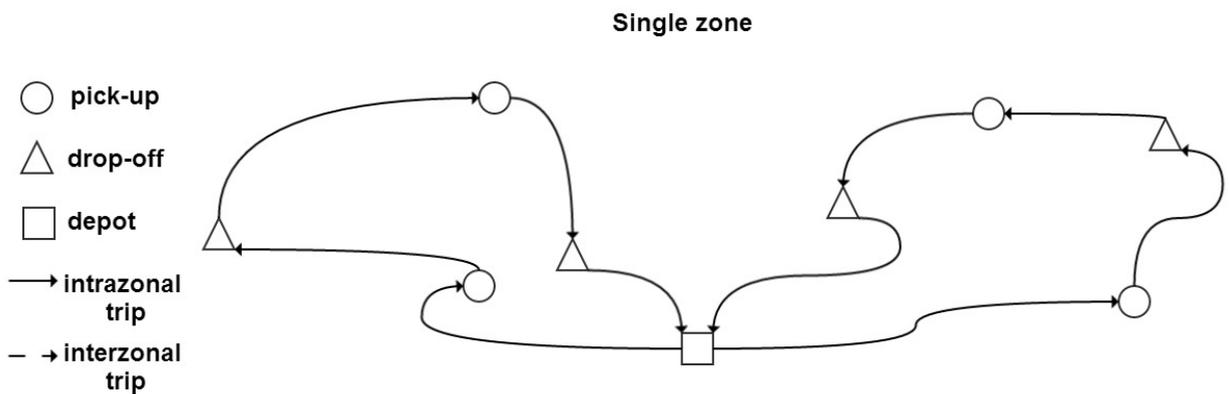


Fig. 2. centralized strategy for paratransit services

paratransit services often experience unpunctuality due to the lateness of vehicles caused by congestion and other random events.

Paratransit services are operated according to different policies. Maximum service time-windows and maximum ride times (related to direct “taxi” ride time) are usually enforced to ensure service quality to customers. To address

the aforementioned challenges, transit agencies strive to improve the efficiency of paratransit services and at the same time maintain an adequate level of service. Different operating practices and innovative ways to organize these services have been adopted to ease the management of these large daily operations. Among these operating strategies, centralized and decentralized (zoning) strategies are the most commonly seen. In the former strategy, the whole service area is operated as one single zone; in the latter strategy the whole service area is divided into adjacent zones and operated by independent providers, and vehicles are allowed to traverse boundaries only to pick up or drop off customers, so maintaining at least starting point or final destination of customers in relative zone (see Fig. 1, Fig. 2).

Zoning strategy is reported to have many advantages: lower management cost, lower operation complexity, and better job satisfaction among drivers due to a more contained and familiar driving range. In addition, a zoning strategy is more likely to provide on-time services for customers as smaller zones tend to have shorter trips and shorter trips tend to have smaller travel time variability. As a result, compared to a centralized strategy, a zoning strategy is adopted by more and more transit agencies, especially in large metropolitan areas. Unfortunately, these advantages of zoning strategy come at a price: it usually results in larger operating costs compared to its centralized strategy counterpart. Since centralized and decentralized operating strategies each has some advantages and disadvantages, a quantification regarding the tradeoffs of performance under zoning/centralized strategies is non-trivial to determine and would help transit planners and operators make more informed decisions on organizational strategies. However, although comparative studies on the operations cost and productivity of paratransit services under different zoning strategies have received some attention from the transportation research community (see for example [2] and [3]), a quantitative study on the reliability performance under centralized/zoning strategies is still missing in the literature. Note that reliability study is a very important research topic in transportation systems as in the real world the link travel times are more likely to be random variables than being deterministic. As a result, the reliability of paratransit services is an important issue the transit agencies are facing.

Motivated by the above fact, this research aims to provide a quantitative study to compare the reliability performance of paratransit services under centralized and decentralized operating strategies. Because an analytic model of the reliability problem is very difficult to develop without making strong assumptions or significantly simplifying the model, we resort to simulation as the investigation methodology. We use real-world paratransit demand data from Houston, Los Angeles and Boston as input to our simulation model. Under reasonable travel time randomness assumptions, the reliability performance of centralized and zoning strategies are compared and analyzed.

This paper is organized as follows. In the next section related work on paratransit is reviewed. Section three introduces our simulation model including the travel time variability models we adopt, the zoning/centralized operating strategies we investigate, and the scheduling algorithm we develop. Section four is devoted to the simulation experiments with a description of the demand data and the analysis of the experimental results. The paper ends with concluding remarks.

2. Literature review

2.1. DARP and paratransit

The paratransit scheduling problem, formally known as the dial-a-ride-problem (DARP), is a special case of vehicle routing problem (VRP). In a general sense, VRP studies the problem that dispatches a fleet of vehicles to serve a set of customers at different locations. More specifically, in DARP the customers are riders with pick-up and drop-off locations, ride time constraints, and time-window constraints. The objective of DARP is usually to minimize the operations cost (related to vehicles' travel distance) while maintaining a level of service to the customers. The DARP is a difficult optimization problem due to its intrinsic combinatorial nature. In fact, it's a well-known NP-hard problem which means there is no known (and highly unlikely will be) efficient algorithms that can solve an instance of meaningful size to optimality in a reasonable time. A lot of studies on optimization algorithms for DARP can be found in the literature. Readers are referred to [4, 5] for comprehensive surveys on the existing efforts of mathematical modelling techniques and solution methods for DARP.

In the paratransit research community, two commonly-adopted methodologies for paratransit performance analysis are analytical modeling method and simulation method. An empirical model relating vehicle fleet size,

travel demand and level of service was first developed by Wilson et al. [6] for dial-a-ride system. This model was statistically calibrated by simulation results. Later, by assuming a demand distribution and routing algorithm, Daganzo [7] derived a theoretical model characterizing the required fleet size given the demand rate, vehicle speed and service area. Combining the above two models, Fu developed a more elaborative analytical model that is able to predicate the fleet size with more quality of service parameters [8]. More recently, Li and Quadrifoglio [9] developed a closed-form formula to determine the optimal zoning strategy for feeder transit services considering both customer service quality and operating costs. Although it is known that analytical modeling is more expressive and easier to conduct parametric analysis, it is also considerably difficult to derive due to its intrinsic complexity of scheduling problem and its exogenous complexity associated with parameters. Due to this very reason, all the aforementioned analytical models have adopted strong assumptions on some parameters (for instance assumptions on demand distribution). On the other hand, simulation models are very powerful tool for analyzing complex systems such as paratransit services and can produce plausible analysis as long as sufficient input data is available. As a result, compared to analytical models, simulation methods have become the more favorable for paratransit performance analysis [10-15].

2.2. Zoning strategies

There aren't many studies on the performance comparison of operation strategies such as decentralized vs. centralized strategy on DARP. In the earlier time, a "Zonal" service pattern was proposed for the purpose of easing dispatching and fare determination [16]. It is widely accepted that the size of the service area is one of the keys that affect the productivity and quality of service of Demand Responsive Transit (DRT) systems, to which paratransit services belong to. Deka [17] analysed how performance measures are associated with local environmental characteristics such as density and the characteristics of the trip makers, using a dataset containing a very large volumes of trips. The productivity aspect was studied in [14, 18, 19]. McKnight and Pagano [20] surveyed 42 service providers in the US and found out that quality increases with size of ridership. Paquette et al. [21] reviewed the up-to-date definitions of quality in the paratransit services and concluded that further study is needed to understand the trade-offs between costs, quality, and operational policies in DARP services. More recently, Shen and Quadrifoglio [2] explicitly compared decentralized strategies and centralized strategy and found that decentralized strategies result in higher total vehicle usage and higher empty backhaul miles driven. Lu et al. [3] proposed new decentralized strategies and showed that these strategies improve productivity and lower operating costs compared to the traditional ones.

Intuitively, a larger service area tends to have more trips with longer length, and will compromise the consistency of the service provided by a paratransit system. This is because longer trips are more vulnerable to random events such as congestion. Although this was supported empirically in [22]. A quantitative model relating the service area size and the service consistency is yet to be found.

3. Methodology

3.1. Travel time variability modeling

The reliability of transportation networks is of increasing interests in both theoretical and application studies. Traditionally, travel time reliability is defined as the probability that travel times experienced by travelers are within an acceptable range. Thus, travel time reliability is an important aspect of customer level of service (LOS). The reliability of paratransit system, as all other transportation systems, is defined as its ability to continuously perform at a pre-specified level of service. In transportation system reliability studies, three different types of travel time variability are often considered based how the sample of the data is formed: (i) day-to-day, (ii) period-to-period and (iii) vehicle-to-vehicle [23]. Note that while the first two are associated with temporal distribution of travel times over different days or over the course of a day, vehicle-to-vehicle variability concerns how different travelers experience travel time differently. Thus this kind of variability is the one to be concerned in this research.

Note that previous research on paratransit zoning strategies assumed the travel times to be deterministic. However, the real world is full of uncertainty and traffic delay happens all the time. Major factors that are accounted for include: traffic incidents, work zones, weather, events, traffic control facilities, etc. As a result a system that

assumes the travel time to be deterministic is overly idealistic and simplified. Thus it's necessary to introduce uncertainty to the system to examine its reliability.

Numerous probability distributions have been proposed in order to describe the distribution of travel times. Normal, Log-normal, Gamma and Weibull distributions are among the most widely used models and discussions about their accuracy and applicability can be found in [24-30]. Although arguably lognormal distribution is observed to be a better fit in field tests, normal distribution is indeed a better choice in modeling travel time variability a priori from a practical perspective because of its computational simplicity. Moreover, several studies did support that normal distribution is a sufficiently accurate representation of travel time [26, 27, 29, 31, 32].

Herman and Lam [31] found the coefficient of variation of travel times to be less than 20 percent in most cases. They also observed the coefficient of variation to decrease with the mean trip time for trips up to 30 minutes and to be constant for longer trips. Several other studies [26, 27, 32] also found the standard deviation of travel times to increase linearly with the average travel times, meaning the coefficient of variation is constant. We also found that some other studies observed the variance-to-mean ratio of travel time (defined as the variance of travel time divided by the mean) is more likely to be constant than coefficient of variance [33, 34]. As a result, we propose two types of stochastic travel time models. Let T denote the random variable of travel times and let t denote the mean travel time. In the first type of model, T is defined by (1).

$$T = t + X \cdot \sqrt{c \cdot t} \tag{1}$$

where $X \sim Normal(0,1)$ and c is the variance-to-mean constant.

For the second type of variability model, T is given by (2).

$$T = t \cdot Y \tag{2}$$

where $Y \sim Normal(1, c_v)$ and c_v is the coefficient of variance.

3.2. Reliability evaluation measures

In the general context of transportation engineering, a wide range of reliability measures have been identified. These measures were broadly categorized into three types: (i) statistical range, (ii) buffer time measures and (iii) tardy trip indicators [35]. Lomax et al. recommended the use of percent variation, misery index, and buffer time index for practical performance measures [35]. For more simplicity and better understanding for laymen, the U.S. Department of Transportation (DOT) recommended for reliability measures the use of 90th and 95th percentile travel time, buffer index, planning time index, and frequency of congestion in [36]. Similarly, buffer index, planning time index, percent variation, percent on-time arrival and misery index were identified as five reliability measures in a recent NCHRP report [37].

In the context of paratransit services, not many studies have been conducted on the reliability issue. Readers are referred to Paquette et al. [21] for a recent comprehensive survey on the quality of service measures in DARP operations. One of the first studies on the quality of service in paratransit was due to Pagano and McKnight [38]. They used dimensions and attributes of quality in the general public transportation to develop those specific to paratransit. From the users' perspective, they identified 41 attributes that belong to eight service quality dimensions. Those related to the reliability dimension are listed in Table 1.

Table 1 Reliability attributes used by Pagano and McKnight [38]

Dimension	Attributes
Reliability	Notification of delays or cancellation of service Wait time for pickup at home Wait time for pickup away from home Arriving at destination on time Few delays while on the vehicle

Based on the above attributes, we determine our paratransit reliability measures to be:

- Total tardiness: defined as the sum of all the time differences between the actual pickup/drop-off times and the scheduled latest pickup/drop-off times.
- Number of delayed trips: a delayed trip is defined as a trip that is not picked up or dropped off before its scheduled latest time.

3.3. Customer generation

Each customer trip includes the following information: pick-up and drop-off locations, requested pick-up time, passenger No., and the need of a wheelchair accessible vehicle. The pick-up and drop-off locations, and pick-up time, are presumably random but chosen from a distribution of locations and trip start time.

3.4. Algorithm description

The algorithm for trip distribution, insertion scheduling and after-processing is summarized as follows.

Step 0.

- (a) Generate customers according to the pre-specified distribution.
- (b) Distribute the trips of each customer to different zones.

Step 1. For each of the zones, set $i=0$. (i represents the number of vehicles that are used)

While unassigned trips not equal to 0 do:

- (a) For each depot, generate one empty route from it.
- (b) Choose first trip in the unassigned trip list.
- (c) Check all the possible insertions for feasibility
- (d) If more than one feasible insertions are found, select the one that minimizes the additional travel distance for the existing route
- (e) Update the schedule of the inserted route and delete the trip that is just inserted from the unassigned trip list.

Step 2. If feasible insertion cannot be found, set $i=i+1$ then go to Step 1(a).

Step 3. Record the basic schedule after inserted all the static requests.

Step 4. Update the basic schedule with stochastic travel times.

4. Simulation experiments

4.1. Demand data

The customer demand information is generated according to actual demand data from Houston, Los Angeles and Boston. The data was provided by METROLift in Houston, Access Services Inc. (ASI) in the Los Angeles County and MBTA in Boston. On an average weekday, there are about 5,000 trips for METROLift, 8000 trips for ASI and 6000 trips for MBTA. The zoning strategies adopted by Houston and Boston are shown in Fig. 3 and Fig. 4.

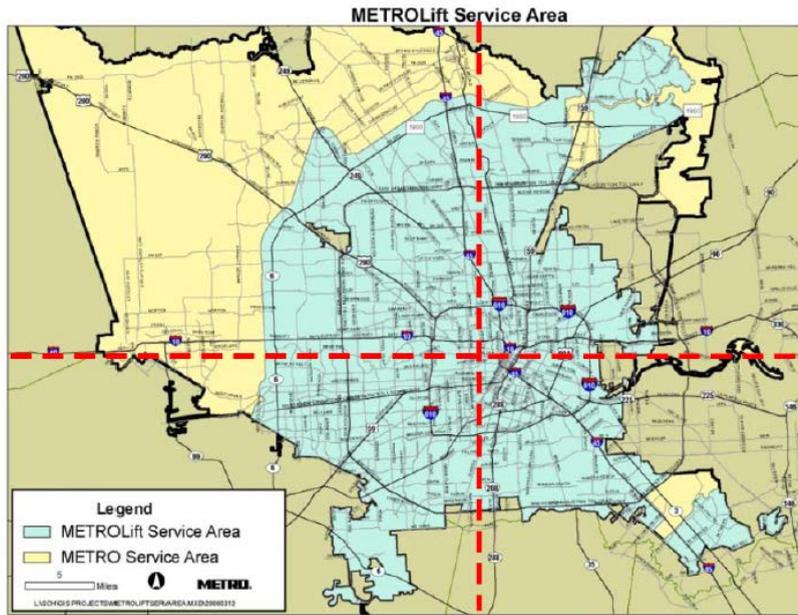


Fig. 3. Houston paratransit service area

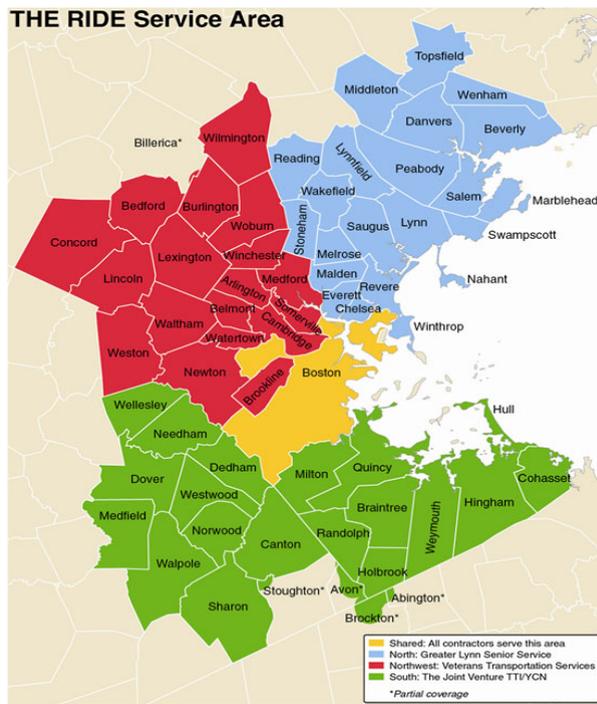


Fig. 4. Boston paratransit service area [39]

Note that currently METROLift adopts a centralized operating strategy, so Fig. 3 shows the artificial zones generated according to the rules developed in [2]. For MBTA, it has three providers covering four zones in the whole service area, with the central Boston area shared by all providers (see Fig. 4). For ASI, it has six zones over the service area. We consider only the Northern (N), Southern (S), Eastern (E) and West/Central (W) zones because

the demand of the other two zones is less than 5% of the total daily average demand. The daily average number of trips in each zone for each city is shown in Table 2.

Table 2 Geographic demand distribution

Houston	
Zone	Daily average trips
Northwest	1208
Southwest	1510
Southeast	1272
Northeast	1010
Los Angeles	
Zone	Daily average trips
Northern	1813
Southern	2780
Eastern	2253
West/Central	1402
Santa Clarita	144
Antelope Valley	273
Boston	
Zone	Daily average trips
North	1886
Northwest	2400
South	1918

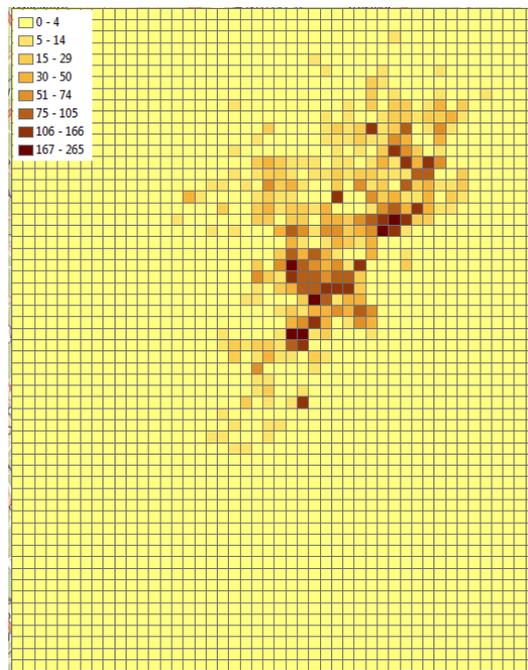


Fig. 5. distribution of pickup location from northern zone - Boston

To illustrate the demand distribution of Boston, we show the pickup distribution for northern zone as an example in Fig. 5. This distribution will be used to generate the input data for the simulation model. Each square in the figures represents a one-by-one mile area. The counted number in each square area represents the number of trip ending in each area.

We use Manhattan (rectilinear) distance to calculate the travel distance between different locations, meaning the travel distance between two stops $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated as $|x_1 - x_2| + |y_1 - y_2|$. Although the

Manhattan distance is an approximation of the real distance, several studies showed it to be a reasonably accurate representation of the actual travel distance (see for example [40, 41]).

As mentioned before, we assume the travel times between all O-D pairs independent normally distributed random variables. The mean travel time is defined as the travel distance divided by the vehicle speed. To analyse the impact of stochastic variations of travel times, two types of variation models are proposed.

Table 3 Travel time variability models

Model Type	Assumption	Parameter
M1	Variance-to-Mean Ratio is constant	10s
		20s
		30s
M2	CV is constant	0.1
		0.15
		0.2

The first type of model (M1) assumes that the variance to mean ratios (defined as the variance of travel time divided by the mean) for all O-D pairs are constant, as was observed in [33, 34]. The second type of model (M2) assumes that the coefficient of variation (defined as the ratio of standard deviation to mean) of travel times is constant, as is supported by [26, 27, 32]. Fig. 6 illustrates how standard deviation of travel time changes with mean travel time for each of the two models. Also note that while the CV (the slope in this figure) of M1 decreases as the mean increasing, the CV of M2 stays unchanged. For each of the models, different parameter values are used to represent three different levels of travel time variability, as listed in Table 3. Fig. 7 gives an illustration of how dispersed the travel time is around an average trip length of 30 minutes at various levels of variability.

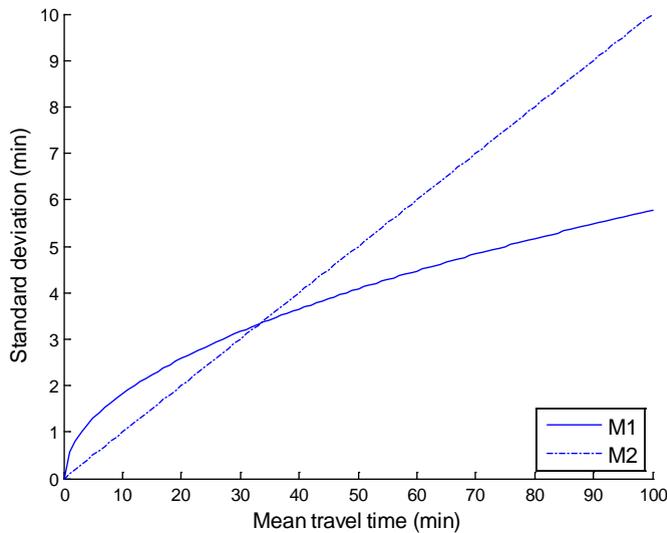


Fig. 6. Standard deviation – mean for M1 (20s) and M2 (CV=0.1).

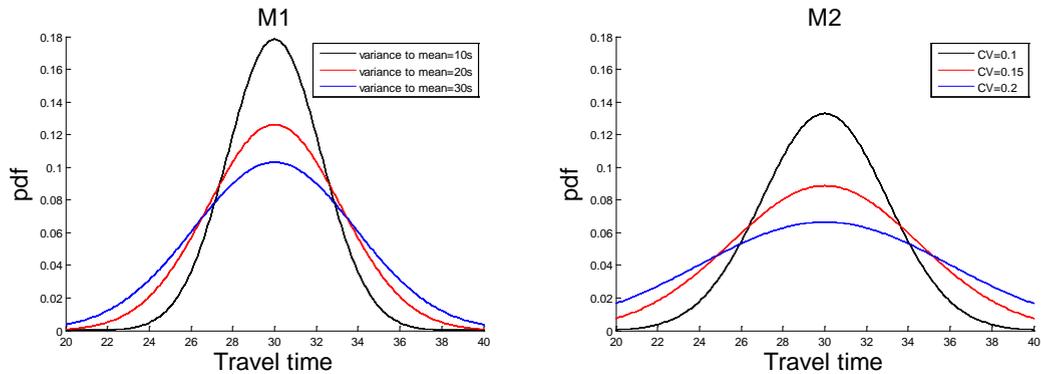


Fig. 7. M1 versus M2 around travel time=30 min.

4.2. Parameters

The following system parameters are used in the simulation:

- Vehicle travel speed: 25 miles/hour
- Service time of each customer: 1 minute
- Maximum ride time factor: 2.5 (the ratio of actual ride time divided by direct ride time, as mandated by law)
- Unlimited number of vehicles are available
- Vans capacity: 4 wheelchairs or 10 ambulatory persons
- Cabs capacity: 1 wheelchair or 4 ambulatory persons
- Service time period: 24 hours. The paratransit service responds to customers' demand 24 hours a day.

4.3. Result analysis

As mentioned in previous section, the performance measures of reliability we select are total tardiness (min) and number of delayed trips. Recall that total tardiness is defined as the summation of the differences between the actual pickup or drop-off times and the latest acceptable pickup and drop-off times (LT) specified by the users' time windows. And a delayed trip is defined as a trip that is not picked up or dropped off before its LT.

The performance of alternative zoning strategies is compared based on the demand data from Boston, Los Angeles and Houston. For each case 10 simulation replications are conducted. For each city, two types of stochastic travel time models are considered. M1 assumes the variance-to-mean ratio of travel times is constant and M2 assumes the coefficient of variance (CV) of travel times is constant. For each type of model, three different parameters are used to represent different levels of variability. The results are summarized in Table 4, Table 5 and Table 6.

It is observed that zoning strategy has lower total tardiness and lower number of delayed trips than no-zoning strategy in all cases. Taking Boston as an example, zoning strategy reduces the total tardiness and number of delayed trips by up to 32 percent and 23 percent, respectively under the M1 assumption. Under the M2 assumption, the reduction is 14 percent and 10 percent, for total tardiness and number of delayed trips respectively. Also note that this reduction in delay is consistent under various levels of travel time variability. The results imply that adopting zoning strategy can significantly improve the reliability of paratransit services.

Table 4 Reliability performance - Boston

M1									
Reliability measure	Variance-to-Mean Ratio=								
	10s			20s			30s		
	No_Z ^a	Z ^b	% ^c	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	525.2	359.8	-31.5%	1298	886	-31.7%	2086.8	1414.9	-32.2%
Number of delayed trips	297.1	230	-22.6%	509.7	408.3	-19.9%	667.3	534.1	-20.0%

M2									
Reliability measure	CV=								
	0.1			0.15			0.2		
	No_Z	Z	%	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	980.8	857.7	-12.6%	4018.8	3451.6	-14.1%	9505.9	8174	-14.0%
Number of delayed trips	292.6	277.5	-5.2%	753.6	679.2	-9.9%	1232.6	1114.2	-9.6%

a. No-zoning (centralized strategy).

b. Zoning strategy.

c. Zoning strategy as compared to no-zoning strategy.

Table 5 Reliability performance – Los Angeles

M1									
Reliability measure	Variance-to-Mean Ratio=								
	10s			20s			30s		
	No_Z ^a	Z ^b	% ^c	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	874.9	611.6	-30.1%	2172.4	1449.3	-33.3%	10031	8055.3	-19.7%
Number of delayed trips	430.1	331.8	-22.9%	730.8	561.2	-23.2%	1399.4	1164.2	-16.8%

M2									
Reliability measure	CV=								
	0.1			0.15			0.2		
	No_Z	Z	%	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	2137	2118	-0.9%	8522.6	7909.9	-7.2%	19537	18016	-7.8%
Number of delayed trips	565.4	533.1	-5.7%	1323.7	1207.2	-8.8%	2101.6	1922.7	-8.5%

Table 6 Reliability performance - Houston

M1									
Reliability measure	Variance-to-Mean Ratio=								
	10s			20s			30s		
	No_Z ^a	Z ^b	% ^c	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	336.8	315	-6.5%	891.8	769.3	-13.7%	1487.6	1279.9	-14.0%
Number of delayed trips	194.1	180.1	-7.2%	350.1	308.4	-11.9%	464.6	413.4	-11.0%

M2									
Reliability measure	CV=								
	0.1			0.15			0.2		
	No_Z	Z	%	No_Z	Z	%	No_Z	Z	%
Total tardiness (min)	1090.1	909.7	-16.5%	4109.5	3749.1	-8.8%	9690.2	9019.3	-6.9%
Number of delayed trips	302	273.4	-9.5%	680	667.8	-1.8%	1101	1090.7	-0.9%

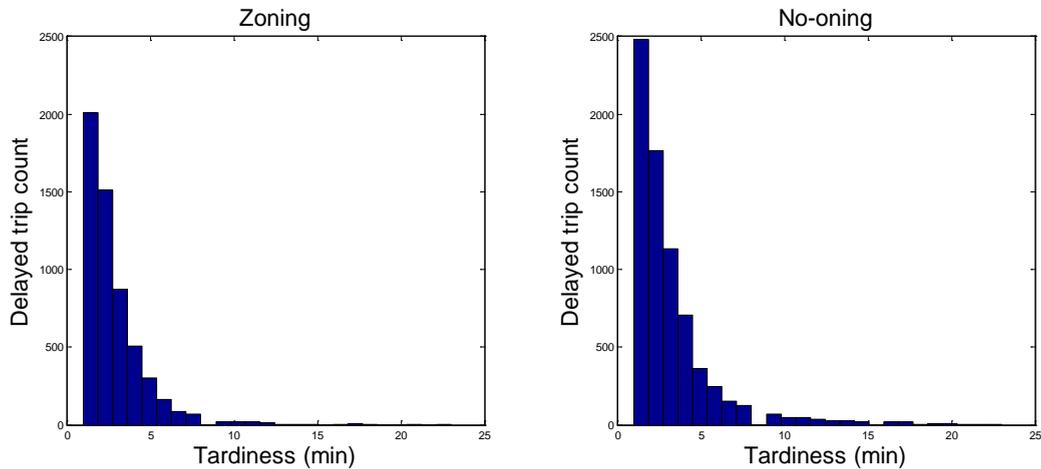


Fig. 8. Tardiness distribution - LA

Fig. 8 gives a close look on how the delayed trips of different level of tardiness are distributed under decentralized and centralized strategy in the Los Angeles case.

5. Conclusions

The intent of this research was to evaluate the effects of zoning control strategies on the reliability of paratransit systems under the stochastic travel time assumption. The particular strategies of our interest are centralized strategy in which the whole large service region is operated by one central agency, and decentralized strategy in which the entire service area is divided into independently managed zones. Two types of travel time variability models were proposed. Type 1 model (M1) assumes the variance-to-mean ratio of travel time to be constant and Type 2 model (M2) assumes the coefficient of variance to be constant. A simulation model was introduced and a series of numerical experiments were conducted using three real-world demand datasets collected from Houston, Los Angeles and Boston. By analyzing the experimental results we had the following findings.

1. Adopting a zoning can significantly reduce the total tardiness and the number of delayed trips experienced by users. This means that compared to the centralized strategy, decentralized strategy can improve the service reliability of paratransit systems from users' perspective.
2. Comparing the two types of stochastic travel time models, it was observed that under the assumption of M1 the improvement in reliability by adopting zoning strategy was greater and more consistent than under the assumption of M2. This was perhaps due to the fact that M2 introduces a more dispersed travel time distribution than M1. Further study is needed to before more determinate conclusion can be reached.

Finally, future research directions include developing paratransit scheduling algorithms that take stochastic travel times into account and investigating paratransit reliability based on more travel time variation patterns. Analytical models that take demand, a scheduling algorithm and an operating strategy as input and reliability measurements as output are also of both theoretical and practical interests.

Acknowledgements

Acknowledgements and Reference heading should be left justified, bold, with the first letter capitalized but have no numbers. Text below continues as normal.

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