Optimal Zone Design for Feeder Transit Services

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Feeder transit services generally operate within residential service areas and move customers to and from a transfer point that connects to a major fixed-route transit network. Feeders can operate in a traditional fixed-route or in an emerging demand-responsive fashion. In designing such systems, planners may divide the entire service area into zones independently served by a single feeder line to provide better customer service, lower operating cost, and make management of the operations easier. An analytical model is developed to help decision makers determine the number of zones in a residential service area while balancing customer service quality and vehicle operating costs. For fixed-route and demand-responsive feeder transit, closed-form expressions and numerical procedures are used to derive the optimal number of zones as a function of the main parameters. Analytical expressions are validated by simulation runs.

General public demand-responsive transit (DRT) services, often known as dial-a-ride, have experienced tremendous growth in recent years. The National Transit Summaries and Trends (NTST) report for 2005 indicates that DRT vehicle revenue miles had increased 91% (the second-highest-increasing rate among all modes of transit) from 307.9 million miles to 589.2 million miles in the previous 10 years (*1*). DRT systems provide much of the desired flexibility with a door-to-door type of service compared to the inconvenience of traditional fixed-route transit (FRT) systems. The general public considers FRT systems to be inconvenient because of their lack of flexibility, since either the locations of pickup and drop-off points or the services' schedules do not match the individual rider's desires.

FRT systems typically are more cost-efficient because of the predetermined schedule, the large loading capacity of the vehicles, and the consolidation of many customer trips onto a single vehicle (ridesharing). However, DRT systems are much more costly to deploy and, therefore, are largely limited to specialized operations, such as taxicabs, shuttle vans, and dial-a-ride services, other than paratransit services. According to the NTST, in 2005, the national total fare revenue earned was about 9.7% of the operating expense for DRT systems, which is much less than the percentage of 27.9% for FRT systems (*1*). Urban sprawl and the associated increasing dispersion of the growing urban population causes conventional fixed-route transit systems to become progressively more inefficient and relegated to a marginal role.

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Thus, there is an increased interest in improving the efficiency of the public transportation systems by means of a better integration between DRT and FRT services. The broad category of flexible transit services includes all types of hybrid services that combine pure demand responsive services and fixed-route services. These services have established stop locations or established schedules or both, combined with some degree of demand-responsive operations. However, unlike regular fixed-route systems, their use has been limited in practice so far.

The demand-responsive connector (DRC), also known as the feeder transit line, is considered a flexible transit service because it operates in a demand-responsive fashion within a service area and moves customers to and from a transfer point that connects to a FRT network. A survey conducted by Koffman for a TCRP project found that the DRC transit service has been operating in quite a few cities and is one of the most frequently used types of flexible transit services (*2*), especially within low-density residential areas, which is the result of urban sprawl, one of the most evident phenomena occurring in the last few decades. For instance, in Denver the call-n-Ride system provides demand-responsive services in zones connected to stations of a light rail system. Depending on the circumstances (i.e., demand, shape of service area, time of day), feeders can also operate in a traditional fixed-route fashion.

In designing such systems for large communities, planners may divide the whole service area into zones for easier management of the operation, to reduce operating cost, and to provide a better level of service to customers. In each zone, an independent feeder line would provide the service to its customers (see Figure 1). For example, the best number of zones is difficult to determine because the balance between operating costs and service quality is frequently difficult to evaluate, especially within areas with low and sparse demands. However, a nonoptimal structure is often adopted, and sometimes there is a lack of zone design, simply because these services are considered a niche market. However, trends suggest that these services will progressively increase their market share and importance within transit agencies, demanding a more rigorous and methodological design approach to the problem.

This research study builds on previous work performed by the authors (*3*), in which an analytical model was developed to determine the best operating policy for adoption in a residential zone to maximize the level of service. That work defined and derived the critical demand density representing the switching point between the fixed-route and demand-responsive competing policies. The present paper develops an analytical model to help planners determine the optimal number of zones while balancing customer level of service and operating costs. Simulations are developed to validate the results of the analytical model. The main purpose is to develop simple analytical equations to guide planners in their decisions with as little information as possible. These are powerful tools that can help solve the complex feeder transit design problem.

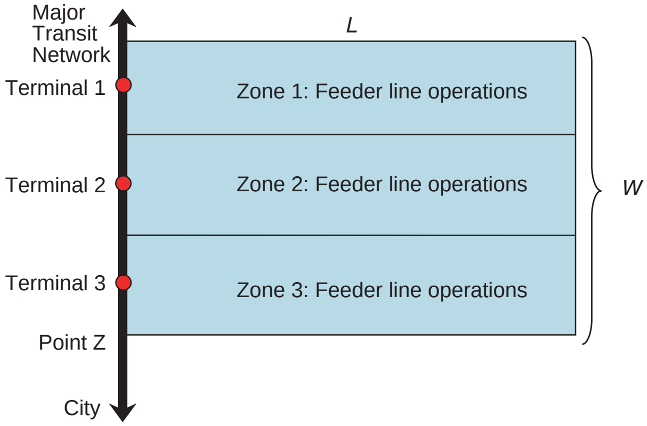

FIGURE 1 Feeder line service area with three zones.

LITERATURE REVIEW

Research on the DRC system is limited and to the authors' knowledge, no research has been performed on methodologies to determine the best number of service zones for the DRC. Work that specifically addresses DRC was conducted by Cayford and Yim (*4*), who surveyed customer demand for DRC for the city of Millbrae, California. They also designed and implemented an automated system used for DRC services. The service uses an automated phone-in system for reservations, computerized dispatching over a wireless communication channel to the bus driver, and an automated callback system for customer notifications.

Although there has been little research on DRC, the purely DRT systems have been extensively studied. Savelsbergh and Sol (*5*), Desaulniers et al. (*6*), and Cordeau and Laporte (*7*) provided comprehensive reviews of the proposed methodologies and solutions to deal with these difficult problems. Dessouky et al. used computer-simulation methods to investigate the effect of using a strategy of zoning versus no-zoning and time-window settings on performance measures such as total traveled miles, deadhead miles, and fleet size (*8*). Dessouky et al. demonstrated through simulation that it is possible to reduce environmental impact substantially while increasing operating costs and service delays only slightly for a DRT system (*9*). Sandlin and Anderson presented a procedure for calculating a serviceability index for DRT operators based on regional socioeconomic conditions and internal operation data (*10*). Palmer et al. studied the DRT system consisting of dial-a-ride programs that transit agencies use for point-to-point pickup and delivery of the elderly and handicapped (*11*). Diana et al. studied the problem of determining the number of vehicles needed to provide a DRT service with a predetermined quality for the user for waiting time at stops and maximum allowed detour (*12*).

Flexible transit service merges the flexibility of pure DRT with the low-cost operability of FRT. Flexible transit services may involve checkpoints. Daganzo describes a flexible system in which the pickup and drop-off points are concentrated at centralized locations called checkpoints (*13*). Quadrifoglio et al. developed bounds on the maximum longitudinal velocity to evaluate the performance and help the design of mobility allowance shuttle transit (MAST) services (*14*). This type of service is defined as route deviation and is another often-used flexible transit service summarized in the survey conducted by Koffman (*2*). Quadrifoglio and others developed an insertion heuristic for scheduling MAST services by using control parameters, which properly regulate the consumption of the slack time (*15*), and formulated the scheduling of the MAST services as a mixed integer program with added logic constraints (*16*).

Aldaihani et al. developed an analytical model that helps decision makers design a hybrid grid network that integrates a flexible demand-responsive service with a fixed-route service (*17*). Their model is to determine the optimal number of zones in an area, where each zone is served by on-demand vehicles. But ridesharing in an on-demand vehicle is not considered in the model, and there are no waiting costs associated with on-demand vehicles. Cortés and Jayakrishnan proposed and simulated one type of flexible transit called high-coverage point-to-point transit, which requires the availability of a large number of transit vehicles (*18*). Pagès et al. identified the real-time mass transport vehicle routing problem and developed a global solution algorithm (*19*). The mass transport network design problem was formulated and solved by the developed algorithm. Khattak and Yim explored the demand for a consumer-oriented personalized DRT (PDRT) service in the San Francisco Bay Area (*20*). About 60% of those surveyed were willing to consider PDRT as a viable option, and about 12% reported that they were "very likely" to use PDRT. Many were willing to pay for the service and highly valued the flexibility in scheduling the service.

SYSTEM DESCRIPTION

Service Area and Demand

The service area is a representation of residential communities and is modeled as a rectangle of width W and length L (see Figure 1). The service area is divided into n zones with length L and width W/n. Within each zone the terminal connecting with the outside fixed-route major transit network is located at the half width on the far left of the zone. The temporal distribution of the demand is assumed to be a Poisson process with constant average arrival rate λ for the whole service area. It is assumed that a fraction α of the customers need to be transferred from the service area to a major attraction destination (such as a city's downtown) through the terminals (pick up customers) and a fraction $1 - \alpha$ of them vice versa (drop off customers). The customers' location, for either a pickup or a drop-off, has a uniform distribution within the service area.

Transit Operation Policies

As shown in Figure 1, the major fixed-route transit service connects terminals and transfers customers from the service area to the city or vice versa. Although the average headway of the major transit can be slightly dependent on the number of zones, it is reasonably assumed to be a constant.

Within each service zone, an FRT policy or a DRC policy would be adopted to operate the feeder service. For each operating policy only one vehicle moving at average speed v_b mph and stopping at each station for a period of s (dwelling time) is considered.

FRT Policy

As shown in Figure 2, within each zone the FRT policy offers continuous service with the vehicle moving back and forth along the

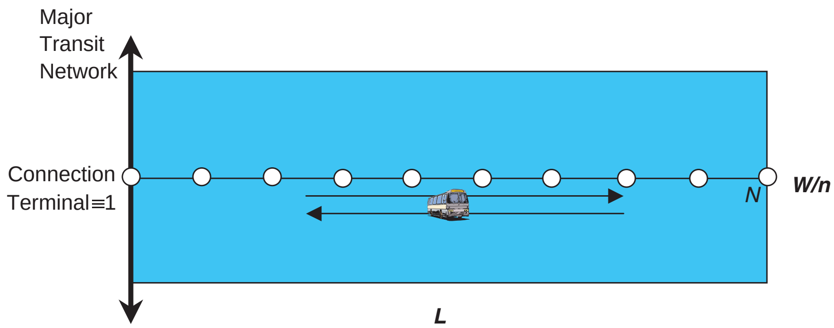

FIGURE 2 FRT service in one zone.

route between Bus Station 1 (connection terminal) and station N (located at the middle of the right boundary of the service area). There are $N - 2$ stations between 1 and N, and the distance between adjacent stations is a constant d miles. The pickup customers show up at random within the service zone, walk to the nearest station, and wait for the bus. The drop-off customers show up and wait at the terminal, take a ride, and then walk to their final destinations at random within the service zone.

DRC Policy

Within each zone, the DRC policy provides a demand-responsive terminal-to-door (and vice versa) service to customers, by picking them up and dropping them off at their desired locations. The vehicle begins and ends each of its trips at the terminal. It is assumed that pickup customers are able to notify their presence by means of a phone or Internet booking service. Immediately before the beginning of each trip, waiting customers (both pickup and drop-off) are scheduled and the route for the trip in the service zone is constructed. An insertion heuristic algorithm (described in the section on simulation development) is used to schedule the requests. There is no real-time scheduling; customers who show up while the DRC vehicle is performing a trip are scheduled and served in the following trip.

In this paper, the rectilinear distance is used because it is more similar to the road network than the Euclidean distance. In fact, as shown by Dessouky et al. (*8*), a rectilinear movement of the vehicle is a good approximation of the reality.

There is no planned idle time between trips. The DRC vehicle keeps running except for the following condition: if the vehicle returns to the terminal and no customer is requesting a pickup or a drop-off at any location, then it waits until a customer shows up.

ANALYTICAL MODEL

Development of the analytical model needed to determine the optimal number of zones, n, is described next. For the FRT policy, a customer can be in the following states: walking between the destination and the nearest bus station, waiting for the FRT, riding the FRT, waiting for the major transit, and riding the major transit. For the DRC policy, a customer can be in states of waiting for an on-demand vehicle, riding an on-demand vehicle, waiting for the major transit, and riding the major transit. It is assumed that the different customer states may have different costs to the customer.

Parameters and Notation

The parameters of the model are as follows:

λ = average demand in the whole residential area (customer/hour);
α = fraction of customers traveling from the residential area to the city; $1 - \alpha$ is the fraction of customers traveling from the city to the residential area;
L = length of the residential service area (mi);
W = width of the residential service area (mi);
d = distance between FRT bus stations within a zone (mi);
a_k = customer cost of walking between a FRT bus station and a house within a zone (\$/customer/hour);
a_w = customer cost of waiting at terminals (\$/customer/hour);
a_w^h = customer cost of waiting at houses (\$/customer/hour);
a_v = customer cost of traveling in an on-demand vehicle (\$/customer/hour);
a_b = customer cost of traveling in a fixed-route bus in the zones (\$/customer/hour);
a_B = customer cost of traveling in a major transit vehicle between the city and terminals (\$/customer/hour);
F_v = total cost of an on-demand vehicle (\$/vehicle/hour);
F_b = total cost of a fixed-route bus (\$/bus/hour);
v_{wk} = average speed of customer walking (mph);
v_b = average speed of an on-demand vehicle or a fixed-route bus (mph);
v_B = average speed of a major transit vehicle (mph);
s = dwelling time of a fixed-route bus or an on-demand vehicle (h); and
S = dwelling time of a major transit vehicle at terminals (h).

The computed variables in the model, which are a function of n, are

$E(T_{wk})$ = expected walking time in a zone for pickup or drop-off customers,
$E(T_{wt}^p)$ = expected waiting time for pickup customers in a zone,
$E(T_{rd}^p)$ = expected ride time for pickup customers in a zone,
$E(T_{rd-B}^p)$ = expected ride time for pickup customers in a major transit vehicle,
$E(T_{wt}^d)$ = expected waiting time for drop-off customers at a terminal,
$E(T_{rd}^d)$ = expected ride time for drop-off customers in a zone, and
$E(T_{rd-B}^d)$ = expected ride time for drop-off customers in a major transit vehicle.

Total Cost Function

The total cost of the designed system includes customer and vehicle cost. The vehicle cost of the major transit is dependent on the number of vehicles determined by the headway and customer demand. They are not a function of n and are independent of the FRT or DRC policy in a zone. Therefore, this vehicle cost of the major transit is not counted to determine the optimal number of zones.

Since the major transit has a constant headway, the customer waiting time for the major transit is independent of the number of zones. For drop-off customers, this waiting time is also independent of the FRT or DRC policy in a zone. No coordination is assumed between the major transit headway and the FRT headway in a zone; the expected waiting time of pickup customers at a terminal for the FRT policy is approximately the same as that for the DRC policy. Therefore, the customer waiting time for the major transit is not included in the total cost definition.

Then the total costs of the system for FRT policy and DRC policy are as follows:

FRT total cost = customer cost + FRT bus cost

$$= n\frac{\lambda}{n}\alpha\left\{a_k E\left(T_{wk}\right) + a_w E\left(T_{wt}^p\right) + a_b E\left(T_{rd}^p\right) + a_B E\left(T_{rd-B}^p\right)\right\} + n\frac{\lambda}{n}(1-\alpha)\left\{a_B E\left(T_{rd-B}^d\right) + a_w E\left(T_{wt}^d\right) + a_b E\left(T_{rd}^d\right) + a_k E\left(T_{wk}\right)\right\} + nF_b \tag{1}$$

Direct total cost = customer cost + DRC vehicle cost

$$= n\frac{\lambda}{n}\alpha\left\{a_w^h E\left(T_{wt}^p\right) + a_v E\left(T_{rd}^p\right) + a_B E\left(T_{rd-B}^p\right)\right\} + n\frac{\lambda}{n}(1-\alpha)\left\{a_B E\left(T_{rd-B}^d\right) + a_w E\left(T_{wt}^d\right) + a_v E\left(T_{rd}^d\right)\right\} + nF_v \tag{2}$$

Derivation of Computed Variables in Total Cost Function

For the FRT and DRC policies, the customers have the same ride time on the major transit. Shown in Figure 1, in the service area, Z is the point nearest to the city. Ride time is defined as the vehicle dwelling time plus vehicle running time between a terminal and point Z. For customers transferring at terminal $k \in \{1, 2, \ldots, n\}$, the ride time is

$$\left(k - \frac{1}{2}\right)\frac{W}{nV_B} + kS$$

Then one has the following results for customer ride time on the major transit:

$$E\left(T_{rd-B}^d\right) = E\left(T_{rd-B}^d\right) = \frac{1}{n}\sum_{k=1}^{n}\left[\left(k-\frac{1}{2}\right)\frac{W}{nV_B} + kS\right] = \frac{W}{2v_B} + \frac{n+1}{2}S \tag{3}$$

FRT Policy

The width of each zone is W/n. According to Quadrifoglio and Li (*3*), there are the following results for the FRT policy. In one zone, the expected walking time to the nearest bus stop $E(T_{wk})$ is

$$E\left(T_{wk}\right) = \frac{1}{4v_{wk}}\left(\frac{L}{N-1} + \frac{W}{n}\right) \tag{4}$$

The expected ride time of all customers is

$$\alpha E\left(T_{rd}^p\right) + (1-\alpha)E\left(T_{rd}^d\right) = \begin{cases} \frac{1}{2}\left[\frac{L}{v_b} + (N-1)s\right] & \text{for } a_w \le a_b \\ \left[\frac{\alpha}{3}\left(1 - \frac{1}{(N-1)^2}\right) + \frac{1}{2}\right]\left[\frac{L}{v_b} + (N-1)s\right] & \text{for } a_w > a_b \end{cases} \tag{5}$$

and the expected waiting time of all customers is

$$\alpha E\left(T_{wt}^p\right) + (1-\alpha)E\left(T_{wt}^d\right) = \begin{cases} \left[1 - \frac{1}{2(N-1)}\right]\left[\frac{L}{v_b} + (N-1)s\right] & \text{for } a_w \le a_b \\ \left[\frac{\alpha}{3}\left(\frac{1}{(N-1)^2} - 1\right) - \frac{1}{2(N-1)} + 1\right]\left[\frac{L}{v_b} + (N-1)s\right] & \text{for } a_w > a_b \end{cases} \tag{6}$$

DRC Policy

Let C represent the average cycle time of a DRC vehicle leaving and returning to a terminal. For the DRC policy, pickup customers will ride an average of $E(T_{rd}^d) = C/2$, since they can be dropped off uniformly anytime from time 0 to C of their cycle. They will need to wait $E(T_{wt}^p) = C/2 + C/2 = C$, since they will wait an average of $C/2$ from their show-up time to the end of the previous cycle and an additional average of $C/2$ waiting for the vehicle to reach them. Drop-off customers will need to wait an average of $E(T_{wt}^d) = C/2$, since they will show up and wait at the terminal uniformly from time 0 to C of the previous cycle. They will also ride an average of $E(T_{rd}^d) = C/2$, like the pickup customers.

Since the scheduling of customers is a vehicle-routing problem, it is difficult to derive C analytically. Approximating the commonly used insertion heuristic scheduling procedure with a nonbacktracking policy, Quadrifoglio and Li derived an analytical solution of C for the case of one zone (*3*). For each zone with demand λ/n and width W/n, C is the solution of the following equation:

$$aC^2 + bC + c = 0 \tag{7}$$

where

$$a = \frac{\lambda}{n}\left[\frac{\lambda}{n}\left(\frac{W}{3n} + sv_b\right) - v_b\right] \tag{8}$$

$$b = \frac{\lambda}{n}\left(\frac{W}{2n} + 2L + 2sv_b\right) - v_b \tag{9}$$

$$c = \frac{W}{6n} + sv_b \tag{10}$$

Two conditions should be satisfied: $C > 0$ and $b^2 - 4ac \geq 0$. Obviously $c > 0$; if $a > 0$, then $b > 0$, and both solutions of $C < 0$. When $a < 0$, only one solution of $C > 0$, and the cycle time C is

$$C = \frac{-b - \left(b^2 - 4ac\right)^{1/2}}{2a} \tag{11}$$

Since $a < 0$, the following condition should be satisfied:

$$n > \frac{1}{2}\left(\lambda s + \frac{\left[\left(\lambda s v_b\right)^2 + 4\lambda W v_b/3\right]^{1/2}}{v_b}\right) \tag{12}$$

However, a closed-form expression for C is not easy to derive. Let k represent the average number of customers for a cycle time. Assume $k/(k+1) = 1$, which is true when $k \to \infty$. According to Quadrifoglio and Li, a closed-form expression is obtained for the approximate cycle time, $\tilde{C}$, for each zone with demand λ/n and width W/n (*3*).

$$\tilde{C} = \frac{sv_b + \frac{W}{6n} + 2L}{v_b - \frac{\lambda}{n}\left(\frac{W}{3n} + sv_b\right)} \tag{13}$$

where n should satisfy Expression 12 to guarantee $\tilde{C} > 0$.

Optimal Number of Zones

FRT Policy

Substitute the computed variables in Equation 1 and obtain the FRT total cost $f(n)$ as

$$\begin{aligned} f(n) = {} & \frac{\lambda a_k}{4v_{wk}}\left(\frac{L}{N-1} + \frac{W}{n}\right) \\ & + \lambda\left[\frac{L}{v_b} + (N-1)s\right]\left[\frac{1}{2}a_b + \left(1 - \frac{1}{2(N-1)}\right)a_w + I\left(a_w - a_b\right)\right] \\ & + \lambda a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) + nF_b \end{aligned} \tag{14}$$

where

$$I = \frac{\alpha}{3}\left(\frac{1}{(N-1)^2} - 1\right) \qquad \text{if } a_w > a_b$$

$$I = 0 \qquad \text{if } a_w \leq a_b$$

Although n is a discrete variable, it is assume that it is a continuous variable to derive the optimal n. The derivative and the second derivative of function $f(n)$ for n are

$$\frac{df(n)}{dn} = -\frac{\lambda a_k W}{4v_{wk}n^2} + \frac{1}{2}\lambda a_B S + F_b \tag{15}$$

$$\frac{d^2 f(n)}{dn^2} = -\frac{\lambda a_k W}{2v_{wk}n^3} > 0 \tag{16}$$

Because

$$\frac{d^2 f(n)}{dn^2} > 0$$

the FRT total cost $f(n)$ is a convex function for $n > 0$. Thus, $f(n)$ has a global minimum when

$$\frac{df(n)}{dn} > 0$$

The optimal n value is

$$n = \left[\frac{\lambda a_k W}{2v_{wk}\left(\lambda a_B S + 2F_b\right)}\right]^{1/2} \tag{17}$$

If optimal n is not an integer, the optimal integer number of zones is, because of convexity, either $\lceil n \rceil$ or $\lfloor n \rfloor$, whichever has the minimum total cost.

DRC Policy

Substitute the computed variables in Equation 2 and obtain the analytical rigorous DRC total cost $r(n)$ and its derivative as

$$r(n) = \lambda\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right]C + \lambda a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) + nF_v \tag{18}$$

$$\begin{aligned} \frac{dr(n)}{dn} = {} & \frac{1}{2}\lambda a_B S + F_v + \lambda\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right] \\ & \times \frac{\lambda\left[\frac{\lambda}{n}\left(\frac{W}{n} + 2sv_b\right) - v_b\right]C^2 + \lambda\left[\frac{W}{n} + 2\left(L + sv_b\right)\right]C + \frac{W}{6}}{(2aC + b)n^2} \end{aligned} \tag{19}$$

where C, a, b are obtained from Equations 8 through 11.

In Equation 18, C is substituted with the approximation $\tilde{C}$ from Equation 13 and the approximate analytical DRC total cost $p(n)$ and its derivative are obtained as

$$\begin{aligned} p(n) = {} & \lambda\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right]\left[\frac{sv_b + \frac{W}{6n} + 2L}{v_b - \frac{\lambda}{n}\left(\frac{W}{3n} + sv_b\right)}\right] \\ & + \lambda a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) + nF_v \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{dp(n)}{dn} = {} & \frac{1}{2}\lambda a_B S + F_v - \lambda\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right] \\ & \times \frac{n^2 v_b\left(\frac{\lambda s^2 v_b + W}{6 + 2\lambda L s}\right) + \frac{2n\lambda W\left(sv_b + 2L\right)}{3} + \frac{\lambda W^2}{18}}{\left[n^2 v_b - \lambda\left(\frac{W}{3 + nsv_b}\right)\right]^2} \end{aligned} \tag{21}$$

Because of their convexity, when

$$\frac{dr(n)}{dn} = 0$$

or

$$\frac{dp(n)}{dn} = 0$$

the rigorous DRC total cost or the approximated DRC total cost has global minimum values. The corresponding optimal n has no closed-form expression, but it is possible to obtain a numerical solution and derive the optimal integer n as for the FRT policy.

SIMULATION DEVELOPMENT

A simulation model is developed to validate the DRC analytical modeling results. The simulation replicates the operations of the insertion heuristic algorithm described here, which is a widely used scheduling algorithm for demand responsive services.

Let $P_1, P_2, P_3, \ldots, P_m$ denote m customers. The insertion algorithm creates the customer sequence choosing the minimum additional distance at each insertion step in an $O(m^2)$ fashion, as follows:

1. Insert P_1: AP_1A is the only possible route.
2. Insert P_2: Possible routes include AP_2P_1A and AP_1P_2A; find the route R_2 with the minimum DRC running distance among the two possible routes. Suppose R_2 is route AP_1P_2A.
3. Insert P_3: Possible routes include $AP_3P_1P_2A$, $AP_1P_3P_2A$, and $AP_1P_2P_3A$; find the route R_3 with the minimum DRC running distance among the three possible routes.
4. . . .

(m) Insert P_m: Suppose the route R_{m-1} is generated by inserting P_{m-1}; Insert P_m to the route R_{m-1}; find the route R_m with the minimum DRC running distance among the m possible routes.

If one were to consider the insertion heuristic, the analytical derivation of the terms of the DRC total cost function described in the previous section would be difficult to perform because of the embedded vehicle-routing problem. Therefore, the analytical modeling of DRC assumes that vehicles follow a nonbacktracking policy (vehicles are not allowed to backtrack with respect to their primary forward direction to serve customers), which is a good approximation of the preceding insertion heuristic, especially for long and narrow service areas (*14*).

COMPUTATIONAL EXPERIMENT

Computational experiments are performed to test the analytical modeling. Three cases are analyzed. The parameter values are assumed to be as follows and as listed in Table 1:

1. A relatively large service area ($L = 2$ mi, $W = 6$ mi), with demand λ of 80 customers/h (density 6.67 customers/h/mi^2), a relatively high walking cost of \$40/h, a relatively high FRT bus cost of \$100/h;
2. A relatively large service area ($L = 2$ mi, $W = 6$ mi), with relatively high demand λ of 200 customers/h (density 16.67 customers/h/mi^2), a relatively low walking cost of \$20/h, a relatively high FRT bus cost of \$50/h; and
3. A relatively small service area ($L = 2$ mi, $W = 2$ mi), with relatively low demand λ of 10 customers/h (density 2.5 customers/h/mi^2), a relatively high walking cost of \$40/h, a relatively high FRT bus cost of \$100/h.

TABLE 1 Parameter Values

Case	Parameter	Value	Unit
Case 1	W	6	Miles
	λ	80	Customers/hour
	a_k	40	\$/customer/hour
	F_b	100	\$/vehicle/hour
Case 2	W	6	Miles
	λ	200	Customers/hour
	a_k	20	\$/customer/hour
	F_b	50	\$/vehicle/hour
Case 3	W	2	Miles
	λ	10	Customers/hour
	a_k	40	\$/customer/hour
	F_b	100	\$/vehicle/hour
Cases 1, 2, and 3	L	2	Miles
	α	0.6	
	a_w^h	10	\$/customer/hour
	a_w	20	\$/customer/hour
	a_v	10	\$/customer/hour
	a_b	10	\$/customer/hour
	a_B	10	\$/customer/hour
	F_v	100	\$/vehicle/hour
	v_{wk}	2	Miles/hour
	v_b	20	Miles/hour
	v_B	30	Miles/hour
	s	0.008333	Hour
	S	0.025	Hour
	d	0.25	Miles

Case 1

For the FRT policy $n = 4.7$ is obtained from Equation 17. When Equation 14 is used, the total cost (\$1,766/h) for $n = 5$ is less than the total cost (\$1,782.7/h) for $n = 4$. So the integer optimal number of zones for the FRT policy is five.

For the DRC policy, $n > 3.2$ with Expression 12. By using the rigorous formulas, with Equation 19 $n = 5.4$ is obtained when

$$\frac{dr(n)}{dn} = 0$$

With Equation 18, the total cost is \$1,280.9/h for $n = 5$ and is \$1,281.4/h for $n = 6$. So the optimal integer number of zones is five.

By using the approximation formulas, with Equation 21, $n = 5.3$ is obtained when

$$\frac{dp(n)}{dn} = 0$$

With Equation 20, the total cost is \$1,348.6/h for $n = 5$ and is \$1,362/h for $n = 6$. So the optimal integer number of zones is five, the same as that with rigorous formulas.

The simulations show that the minimum total cost for the DRC policy with the insertion heuristic algorithm is \$1,074.1/h. The optimal number of zones is five, which is the same as those from analytical rigorous and approximation formulas.

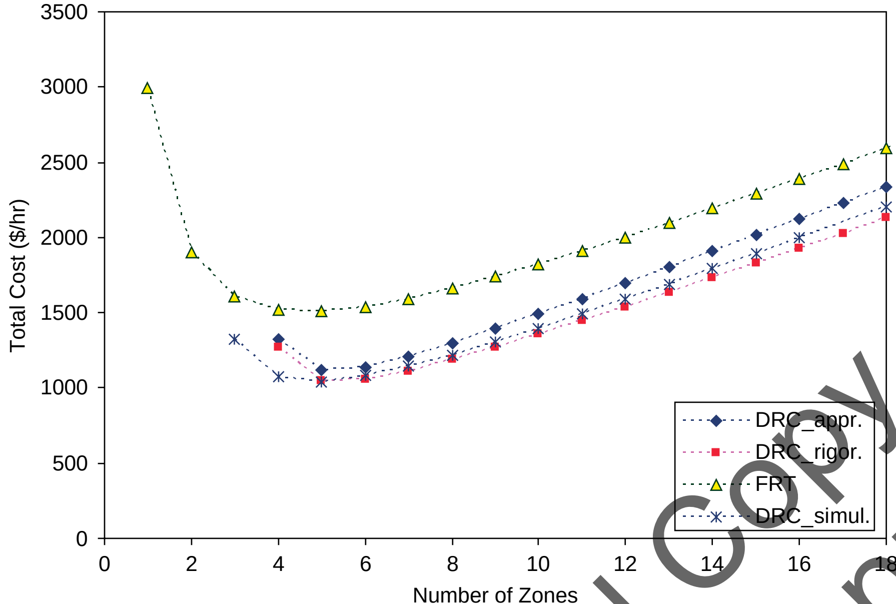

FIGURE 3 Total cost functions for Case 1 from three formulas (appr. = approximation, rigor. = rigorous, simul. = simulation).

The total costs for various numbers of zones are shown in Figure 3. Note the following observations:

- The minimum DRC total cost is less than that of the FRT policy, suggesting that the optimal configuration for this case would be a five-zone DRC feeder policy.
- For the DRC policy, the total costs obtained from the approximation formulas, the rigorous formulas, and the simulation are very close, validating the assumptions in the modeling approach.
- For the DRC policy, the total cost obtained from simulations is less than that from rigorous formulas when $n < 5$. This shows, as expected (*14*), that the nonbacktracking policy progressively worsens its effectiveness in approximating the insertion heuristic algorithm when the shape of the zone widens as a consequence of the reduction of the number of zones.

Case 2

Case 2 has a relatively high demand, low walking cost, and low FRT bus cost. Table 1 shows the input parameter values to the model. Figure 4 shows the values of total cost functions.

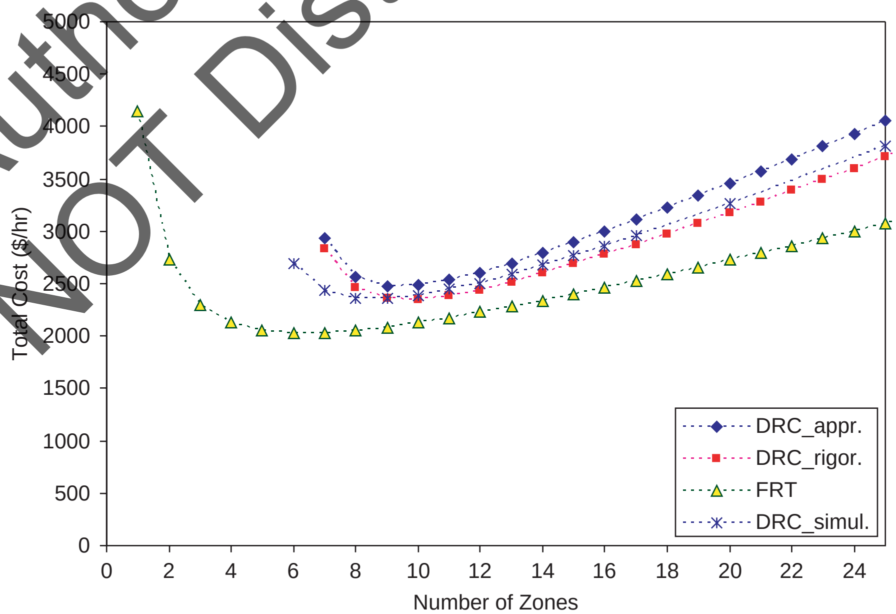

FIGURE 4 Total cost functions for Case 2 from three formulas.

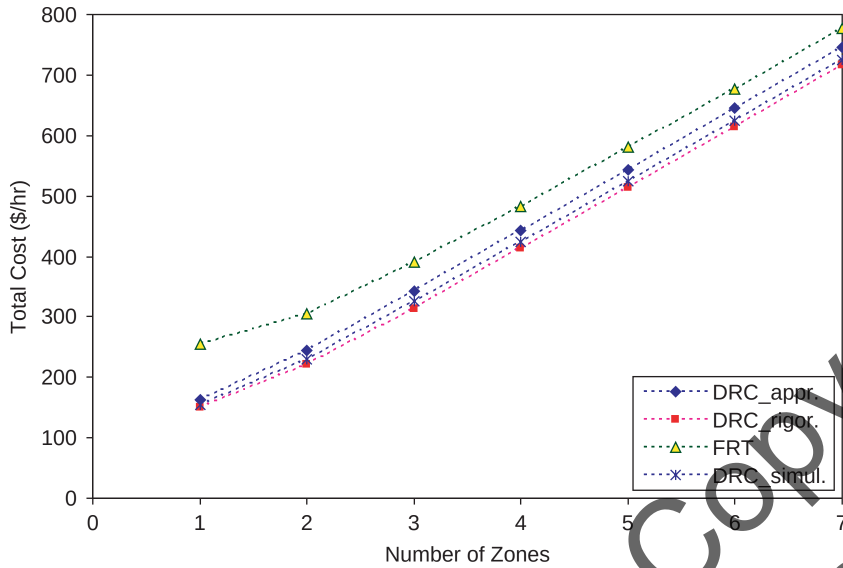

FIGURE 5 Total cost functions for Case 3 from three formulas.

For the FRT policy, the minimum total cost is $2,026/h for six zones. For the DRC policy, with the rigorous formulas, the minimum total cost is $2,346/h for 10 zones. With the approximation formulas, the minimum total cost is $2,480/h for nine zones. From simulations, the minimum total cost is $2,363/h for nine zones. As for Case 1, the optimal number of zones obtained from the approximation formula is very close to those from rigorous formulas and simulations. In this case, the minimum cost of the FRT is less than that of the DRC, suggesting that this configuration would require a six-zone FRT feeder service.

Case 3

Case 3 has a relatively small area. Table 1 shows the input parameter values. Figure 5 shows the values of total cost functions. For the FRT policy, the minimum cost is $255/h for one zone. For the DRC policy, the minimum costs are $164/h and $151/h, respectively, for one zone, with the approximation and rigorous formulas. Simulations also show that one zone is optimal with the minimum cost of $154/h.

As for the previous two cases, the approximated optimal number of zones and the minimal total cost are very close to those from the rigorous formulas and the simulations for the DRC policy. Both service policies suggest a single zone optimal design, but with lower cost for the DRC policy.

CONCLUSIONS

This paper addressed the problem faced by planners in designing feeder transit services and determined the optimal number of zones into which to divide a service area as well as the best operating policy (FRT versus DRC). An analytical model was developed that represents the total cost functions balancing customer service quality and vehicle operating cost. By analytical derivation, closed-form expressions were obtained for the FRT and approximation formulas for the DRC to determine the optimal number of zones. For the DRC, simulations were used to validate the results of the analytical formulas. All the case studies showed that the optimal number of zones and the total cost obtained from the approximation formulas are very close to those obtained from simulations.

The presented analytical formulation leads to a strictly convex optimization problem to minimize the cost by controlling the number of zones. This formulation provides evidence of the existence and uniqueness of the problem solution.

Limitations of results come from the simplified system configuration model. The formulation may be useful only for the service area close to a rectangle and the service area with a uniform land use pattern, which are, however, the majority of residential housing areas. A practical implementation of the partition of the whole service area in an optimal number of zones might be affected by possible street network constraints. However, in the planning and design phase of a new residential area, this can be considered before the road network is constructed.

In addition, the modeling assumes rectilinear movements of the vehicles among demand points that might not be realistic within some of the residential service areas with complex road network and land use patterns. Future research might include applications of the presented approach to real case studies with collected demand data and actual road networks.

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