SLOPE STABILITY ANALYSIS
WITH
TOTAL STRESS ANALYSIS
AND
EFFECTIVE STRESS ANALYSIS

(Extracted from Fundamentals of Geotechnical Engineering,
Braja Das, Brooks/Cole Publishers)
Solution  We are given $\phi = 15^\circ$ and $c = 29$ kN/m$^2$. If $FS_e = 3$, then $FS_c$ and $FS_\phi$ should both be equal to 3. We have

$$FS_c = \frac{c}{c_d}$$

or

$$c_d = \frac{c}{FS_e} = \frac{c}{FS_c} = \frac{29}{3} = 9.67 \text{ kN/m}^2$$

Similarly,

$$FS_\phi = \frac{\tan \phi}{\tan \phi_d}$$

$$\tan \phi_d = \frac{\tan \phi}{FS_\phi} = \frac{\tan \phi}{FS_c} = \frac{\tan 15}{3}$$

or

$$\phi_d = \tan^{-1} \left[ \frac{\tan 15}{3} \right] = 5.1^\circ$$

Substituting the preceding values of $c_d$ and $\phi_d$ into Eq. (10.40) gives

$$H = \frac{4c_d}{\gamma} \left[ \frac{\sin \beta \cos \phi_d}{1 - \cos(\beta - \phi_d)} \right] = \frac{4 \times 9.67}{16.5} \left[ \frac{\sin 45 \cos 5.1}{1 - \cos(45 - 5.1)} \right] \approx 7.1 \text{ m}$$

10.5 Analysis of Finite Slope with Circularly Cylindrical Failure Surface—General

In general, slope failure occurs in one of the following modes (Figure 10.6):

1. When the failure occurs in such a way that the surface of sliding intersects the slope at or above its toe, it is called a slope failure (Figure 10.6a). The failure circle is referred to as a toe circle if it passes through the toe of the slope and as a slope circle if it passes above the toe of the slope. Under certain circumstances, it is possible to have a shallow slope failure, as shown in Figure 10.6b.

2. When the failure occurs in such a way that the surface of sliding passes at some distance below the toe of the slope, it is called a base failure (Figure 10.6c). The failure circle in the case of base failure is called a midpoint circle.

Various procedures of stability analysis may, in general, be divided into two major classes:
1. **Mass procedure.** In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is hardly the case in most natural slopes.

2. **Method of slices.** In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each of the slices is calculated separately. This is a versatile technique in which the nonhomogeneity of the soils and pore water pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface.
10.6 Mass Procedure of Stability Analysis

(b) Shallow slope failure

(c) Base failure

FIGURE 10.6 (Continued)

The fundamentals of the analysis of slope stability by mass procedure and method of slices are presented in the following sections.

10.6 Mass Procedure of Stability Analysis (Circularly Cylindrical Failure Surface)

Slopes in Homogeneous Clay Soil with $\phi = 0$
(Undrained Condition)

Figure 10.7 shows a slope in a homogeneous soil. The undrained shear strength of the soil is assumed to be constant with depth and may be given by $\tau = c_u$. To make
the stability analysis, we choose a trial potential curve of sliding $AED$, which is an arc of a circle that has a radius $r$. The center of the circle is located at $O$. Considering the unit length perpendicular to the section of the slope, we can give the total weight of the soil above the curve $AED$ as $W = W_1 + W_2$, where

\[ W_1 = (\text{area of } FCDEF)(\gamma) \]

and

\[ W_2 = (\text{area of } ABFEA)(\gamma) \]

Note that $\gamma$ = saturated unit weight of the soil.

Failure of the slope may occur by the sliding of the soil mass. The moment of the driving force about $O$ to cause slope instability is

\[ M_d = W_1 l_1 - W_2 l_2 \]  

(10.36)

where $l_1$ and $l_2$ are the moment arms.

The resistance to sliding is derived from the cohesion that acts along the potential surface of sliding. If $c_d$ is the cohesion that needs to be developed, the moment of the resisting forces about $O$ is

\[ M_r = c_d(\DeltaED)(1)(r) = c_d r^2 \theta \]  

(10.38)

For equilibrium, $M_r = M_d$; thus,

\[ c_d r^2 \theta = W_1 l_1 - W_2 l_2 \]
\[ c_d = \frac{W_1 h_1 - W_2 h_2}{r^2 \theta} \]  

(10.44)

The factor of safety against sliding may now be found:

\[ FS_s = \frac{\gamma h}{c_d} = \frac{c_u}{c_d} \]  

(10.45)

Note that the potential curve of sliding, \( AED \), was chosen arbitrarily. The critical surface is the one for which the ratio of \( c_u \) to \( c_d \) is a minimum. In other words, \( c_d \) is maximum. To find the critical surface for sliding, a number of trials are made for different trial circles. The minimum value of the factor of safety thus obtained is the factor of safety against sliding for the slope, and the corresponding circle is the critical circle.

Stability problems of this type were solved analytically by Fellenius (1927) and Taylor (1937). For the case of critical circles, the developed cohesion can be expressed by the relationship

\[ c_d = \gamma H m \]

or

\[ H_o = \frac{c_u}{\gamma m} \]  

(10.46)

Note that the term \( m \) on the right-hand side of the preceding equation is nondimensional and is referred to as the stability number. The critical height (that is, \( FS_s = 1 \)) of the slope can be evaluated by substituting \( H = H_o \) and \( c_u = c_u \) (full mobilization of the undrained shear strength) into Eq. (10.46). Thus,

\[ H_o = \frac{c_u}{\gamma m} \]  

(10.47)

Values of the stability number \( m \) for various slope angles \( \beta \) are given in Figure 10.8. Terzaghi and Peck (1967) used the term \( \gamma H l / c_d \), the reciprocal of \( m \), and called it the stability factor. Figure 10.8 should be used carefully. Note that it is valid for slopes of saturated clay and is applicable to only undrained conditions (\( \phi = 0 \)).

In reference to Figure 10.8, consider these issues:

1. For slope angle \( \beta \) greater than 53\(^\circ\), the critical circle is always a toe circle. The location of the center of the critical toe circle may be found with the aid of Figure 10.9.
For $\beta > 53^\circ$:
All circles are toe circles.

For $\beta < 53^\circ$:
Toe circle
Midpoint circle
Slope circle

FIGURE 10.8 (a) Definition of parameters for midpoint circle-type failure; (b) plot of stability number against slope angle (Redrawn from Terzaghi and Peck, 1967)

2. For $\beta < 53^\circ$, the critical circle may be a toe, slope, or midpoint circle depending on the location of the firm base under the slope. This is called the 
depth function, which is defined as

$$D = \frac{\text{vertical distance from the top of the slope to the firm base}}{\text{height of the slope}}$$  (10.48)
3. When the critical circle is a midpoint circle (that is, the failure surface is tangent to the firm base), its position can be determined with the aid of Figure 10.10.

4. The maximum possible value of the stability number for failure at the midpoint circle is 0.181.

Fellenius (1927) also investigated the case of critical toe circles for slopes with $\beta < 53^\circ$. The location of these can be determined using Figure 10.11 and Table

FIGURE 10.9 Location of the center of critical circles for $\beta > 53^\circ$
FIGURE 10.10 Location of midpoint circle

10.1. Note that these critical toe circles are not necessarily the most critical circles that exist.

EXAMPLE 10.4

A cut slope in saturated clay (Figure 10.12) makes an angle of 56° with the horizontal.

a. Determine the maximum depth up to which the cut could be made. Assume that the critical surface for sliding is circularly cylindrical. What will be the nature of the critical circle (that is, toe, slope, or midpoint)?
FIGURE 10.11 Location of the center of critical toe circles for \( \beta < 53^\circ \)

b. Referring to part a, determine the distance of the point of intersection of the critical failure circle from the top edge of the slope.

c. How deep should the cut be made if a factor of safety of 2 against sliding is required?

Solution

a. Since the slope angle \( \beta = 56^\circ > 53^\circ \), the critical circle is a toe circle. From Figure 10.8, for \( \beta = 56^\circ \), \( m = 0.185 \). Using Eq. (10.47), we have

\[
H_a = \frac{c_a}{\gamma m} = \frac{24}{(15.7)(0.185)} = 8.26 \text{ m} \approx 8.25 \text{ m}
\]

Table 10.1 Location of the center of critical toe circles (\( \beta < 53^\circ \))

<table>
<thead>
<tr>
<th>( n' )</th>
<th>( \beta ) (deg)</th>
<th>( \alpha_1 ) (deg)</th>
<th>( \alpha_2 ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>45</td>
<td>28</td>
<td>37</td>
</tr>
<tr>
<td>1.5</td>
<td>33.68</td>
<td>26</td>
<td>35</td>
</tr>
<tr>
<td>2.0</td>
<td>26.57</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>3.0</td>
<td>18.43</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>5.0</td>
<td>11.32</td>
<td>25</td>
<td>37</td>
</tr>
</tbody>
</table>

Note: For notations of \( n' \), \( \beta \), \( \alpha_1 \), and \( \alpha_2 \), see Figure 10.11.
b. Refer to Figure 10.13. For the critical circle, we have

\[
BC = EF = AP - AE = H_o (\cot \alpha - \cot 56^\circ)
\]

From Figure 10.9, for \( \beta = 56^\circ \), the magnitude of \( \alpha \) is 33°, so

\[
BC = 8.25(\cot 33 - \cot 56) = 7.14\text{ m} \approx 7.15\text{ m}
\]

c. Developed cohesion is

\[
\frac{c_u}{F_S} = \frac{24}{2} = 12\text{ kN/m}^2
\]
From Figure 10.8, for $\beta = 56^\circ$, $m = 0.185$. Thus, we have

$$H = \frac{c_d}{\gamma m} = \frac{12}{(15.7)(0.185)} = 4.13 \text{ m}$$

**EXAMPLE 10.5**

A cut slope was excavated in a saturated clay. The slope made an angle of $40^\circ$ with the horizontal. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assume an undrained condition and $\gamma_{sat} = 17.29$ kN/m$^3$.

a. Determine the undrained cohesion of the clay (use Figure 10.8).

b. What was the nature of the critical circle?

c. With reference to the toe of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

**Solution**

a. Referring to Figure 10.8, we find

$$D = \frac{9.15}{6.1} = 1.5$$

$$\gamma_{sat} = 17.29 \text{ kN/m}^3$$

$$H_o = \frac{c_d}{\gamma m}$$

From Figure 10.8, for $\beta = 40^\circ$ and $D = 1.5$, $m = 0.175$, so

$$c_d = (H_o)(\gamma)(m) = (6.1)(17.29)(0.175) = 18.5 \text{ kN/m}^2$$

b. Midpoint circle
c. From Figure 10.10, for $D = 1.5$ and $\beta = 40^\circ$, $n = 0.9$, so

$${\text{distance}} = (n)(H_o) = (0.9)(6.1) = 5.49 \text{ m}$$

**Slopes in Homogeneous Soil with $\phi > 0$**

A slope in a homogeneous soil is shown in Figure 10.14a. The shear strength of the soil is given by

$$\tau = c + \sigma' \tan \phi$$

The pore water pressure is assumed to be 0. $AC$ is a trial circular arc that passes through the toe of the slope, and $O$ is the center of the circle. Considering unit length
\[ \tau_f = c + \sigma' \tan \phi \]

FIGURE 10.14 Analysis of slopes in homogeneous soils with \( \phi > 0 \)
perpendicular to the section of the slope, we find

weight of the soil wedge $ABC = W = (\text{area of } ABC)(\gamma)$

For equilibrium, the following other forces are acting on the wedge:

1. $C_d$—the resultant of the cohesive force that is equal to the unit cohesion developed times the length of the cord $AC$. The magnitude of $C_d$ is given by (Figure 10.14b).

   $$C_d = c_d(AC)$$  \hspace{1cm} (10.49)

$C_d$ acts in a direction parallel to the cord $AC$ (Figure 10.14b) and at a distance $a$ from the center of the circle $O$ such that

$$C_d(a) = c_d(AC)r$$

or

$$a = \frac{c_d(AC)r}{C_d} = \frac{AC}{AC}r$$  \hspace{1cm} (10.50)

2. $F$—the resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of $F$ will pass through the point of intersection of the line of action of $W$ and $C_d$.

Now, if we assume the full friction is mobilized ($\phi_d = \phi$ or $FS_d = 1$), then the line of action of $F$ will make an angle $\phi$ with a normal to the arc, and thus it will be a tangent to a circle with its center at $O$ and having a radius of $r \sin \phi$. This circle is called the friction circle. Actually, the radius of the friction circle is a little larger than $r \sin \phi$.

Since the directions of $W$, $C_d$, and $F$ are known and the magnitude of $W$ is known, we can plot a force polygon, as shown in Figure 10.14c. The magnitude of $C_d$ can be determined from the force polygon. So the unit cohesion developed can be found:

$$c_d = \frac{C_d}{AC}$$

Determining the magnitude of $c_d$ described previously is based on a trial surface of sliding. Several trials must be made to obtain the most critical sliding surface along which the developed cohesion is a maximum. So it is possible to express the maximum cohesion developed along the critical surface as

$$c_d = \gamma H[f(\alpha, \beta, \theta, \phi)]$$  \hspace{1cm} (10.51)

For critical equilibrium—that is, $FS_c = FS_d = FS_x = 1$—we can substitute $H = H_c$ and $c_d = c$ into Eq. (10.51):

$$c = \gamma H_c[f(\alpha, \beta, \theta, \phi)]$$
FIGURE 10.15 Taylor's stability number for \( \phi > 0 \)

or

where \( m \) = stability number. The values of \( m \) for various values of \( \phi \) and \( \beta \) (Taylor, 1937) are given in Figure 10.15. Example 10.6 illustrates the use of this chart.

Calculations have shown that, for \( \phi \) greater than about 3°, the critical circles are all toe circles. Using Taylor's method of slope stability (as shown in Example 10.6), Singh (1970) provided graphs of equal factors of safety, \( FS_n \), for various slopes, and these are given in Figure 10.16. In these charts, the pore water pressure was assumed to be 0.

**EXAMPLE 10.6**

A slope with \( \beta = 45^\circ \) is to be constructed with a soil that has \( \phi = 20^\circ \) and \( c = 2.5 \) kN/m³. The unit weight of the compacted soil will be 18.9 kN/m³.

- **a.** Find the critical height of the slope.
- **b.** If the height of the slope is 10 m, determine the factor of safety with respect to strength.
Solution

a. We have

\[ m = \frac{c}{\gamma H_\sigma} \]

From Figure 10.15, for \( \beta = 45^\circ \) and \( \phi = 20^\circ \), \( m = 0.06 \). So

\[ H_\sigma = \frac{c}{\gamma m} = \frac{24}{(18.9)(0.06)} = 21.1 \text{ m} \]
b. If we assume that full friction is mobilized, then, referring to Figure 10.15 (for $\beta = 45^\circ$ and $\phi_d = \phi = 20^\circ$), we have

$$m = 0.06 = \frac{c_d}{\gamma H}$$

or

$$c_d = (0.06)(18.9)(10) = 11.34 \text{ kN/m}^3$$
Thus,

\[ FS_\phi = \frac{\tan \phi}{\tan \phi_\theta} = \frac{\tan 20}{\tan 20} = 1 \]

and

\[ FS_c = \frac{c}{c_\theta} = \frac{24}{11.34} = 2.12 \]

Since \( FS_c \neq FS_\phi \), this is not the factor of safety with respect to strength.

Now we can make another trial. Let the developed angle of friction, \( \phi_\theta \), be equal to 15°. For \( \beta = 45° \) and the friction angle equal to 15°, we find from Figure 10.15

\[ m = 0.085 = \frac{c_\theta}{\gamma H} \]

or

\[ c_\theta = (0.085)(18.9)(10) = 16.07 \text{ kN/m}^2 \]

For this trial,

\[ FS_\phi = \frac{\tan \phi}{\tan \phi_\theta} = \frac{\tan 20}{\tan 15} = 1.36 \]

and

\[ FS_c = \frac{c}{c_\theta} = \frac{24}{16.07} = 1.49 \]

Similar calculations of \( FS_\phi \) and \( FS_c \) for various assumed values of \( \phi_\theta \) are given in the table.

<table>
<thead>
<tr>
<th>( \phi_\theta )</th>
<th>( \tan \phi_\theta )</th>
<th>( FS_\phi )</th>
<th>( m )</th>
<th>( c_\theta ) (kN/m²)</th>
<th>( FS_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.364</td>
<td>1.0</td>
<td>0.06</td>
<td>11.34</td>
<td>2.12</td>
</tr>
<tr>
<td>15</td>
<td>0.268</td>
<td>1.36</td>
<td>0.085</td>
<td>16.07</td>
<td>1.49</td>
</tr>
<tr>
<td>10</td>
<td>0.176</td>
<td>2.07</td>
<td>0.11</td>
<td>20.79</td>
<td>1.15</td>
</tr>
<tr>
<td>5</td>
<td>0.0875</td>
<td>4.16</td>
<td>0.136</td>
<td>25.70</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The values of \( FS_\phi \) are plotted against their corresponding values of \( FS_c \) in Figure 10.17, from which we find

\[ FS_c = FS_\phi = FS = 1.45 \]
10.7 Method of Slices

Stability analysis using the method of slices can be explained by referring to Figure 10.18a, in which $AC$ is an arc of a circle representing the trial failure surface. The soil above the trial failure surface is divided into several vertical slices. The thickness of each slice need not be the same. Considering unit length perpendicular to the cross-section shown, the forces that act on a typical slice (nth slice) are shown in Figure 10.18b. $W_n$ is the effective weight of the slice. The forces $N_n$ and $T_n$ are the normal and tangential components of the reaction $R$, respectively. $P_n$ and $P_{n+1}$ are the normal forces that act on the sides of the slice. Similarly, the shearing forces that act on the sides of the slice are $T_n$ and $T_{n+1}$. For simplicity, the pore water pressure is assumed to be 0. The forces $P_n$, $P_{n+1}$, $T_n$, and $T_{n+1}$ are difficult to determine. However, we can make an approximate assumption that the resultants of $P_n$ and $T_n$ are equal in magnitude to the resultants of $P_{n+1}$ and $T_{n+1}$ and also that their lines of action coincide.