Valuation of strategic options in public–private partnerships

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A B S T R A C T

This paper investigates the feasibility of and develops an economic valuation model for strategic options in Comprehensive Development Agreements (CDAs). A CDA is a form of public–private partnership whereby the right to price and collect revenues from toll roads is leased to a private entity for a long but finite period of time. In exchange, this provides local and state governments with a quick influx of cash and additional infrastructure. Uncertainty associated with such long-term leases is of substantial public concern. This paper examines five different strategic options, namely a buyout option, a conditional buyout option, a revenue-sharing option, and two types of minimum revenue guarantee options. The buyout option in particular could give the public sector additional control over the future use of leased facilities and address potential concerns regarding long-run uncertainty and possible unforeseen windfalls for the private sector. The paper’s contributions include the analysis, feasibility assessment and valuation of several strategic options, sensitivity analysis of the solutions, an economic consumer demand-based revenue model for purposes of cash flow simulation, and analysis of option price sensitivity to “moneyness”.

The main conclusion is that strategic options can provide useful risk reduction, but generally have significant value relative to the lease itself. By scaling down payoffs, options could be realistically included in CDAs and other PPPs. For some parameter values, option values to the developer and public authority are offsetting, allowing for costless risk reduction.

1. Introduction

The difficult state of public transportation budgets since the 1990s and particularly since the Great Recession of 2007–2009 has motivated the public sector to investigate “innovative financing” strategies to meet their budgetary shortfalls.1 Indeed, traditional revenue-raising mechanisms have struggled to keep up with the demand for public infrastructure construction and repair. For example, U.S. federal gas taxes that finance the Federal Highway Administration’s...
Trust Fund have remained at 18.4 cents per gallon. Meanwhile, costs have increased, vehicle efficiency has improved, and more revenues are dedicated to non-infrastructure needs (Puentes and Prince, 2003).

1.1. Innovative financing for transportation projects

To address this budgetary crisis, proponents of innovative financing have suggested as one of the delivery mechanisms transportation public–private partnerships, such as Comprehensive Development Agreements (CDAs). A CDA is a form of public–private partnership whereby the right to price and collect revenues from toll roads is leased to a private entity for a long but finite period of time, in exchange for providing local and state governments with an initial influx of cash and/or additional infrastructure. These arrangements include concessions and leases, can be used for new or existing projects, and generally involve design, construction, operation, and, eventually, transfer of the facilities. Typically, a higher degree of privatization of the project implies greater risk transference from the public sector to the participating private firm or consortium (see Fig. 1).

1.2. Contribution of the paper

This paper develops an analytical framework to value strategic options in CDAs, with an emphasis on new (Greenfield) transportation projects. The framework can be generalized to PPPs in different sectors. The strategic options considered are a buyout option, a conditional buyout option, and revenue-sharing options for the public agency, and minimum revenue guarantee options for the private sector (both annual and one-time cumulative). Although it is clear that all of these options have positive value, we cannot tell without further study their magnitudes or interaction effects. Our paper makes the following contributions:

- First, we analyze and provide valuations for different types of strategic options in PPPs, only one of which has received significant attention in the literature.
- Second, we compare the option solutions for different specifications and in particular the feasibility of “out-of-the-money” options (buyback price relative to original value; share of revenue-sharing; proportion of revenue guaranteed).
- Third, while much of the previous literature has directly modeled project cash flows, we open the “black box” of revenue generation and model the economic and transportation variables that determine cash flows.
- Fourth, this enables us to conduct sensitivity analysis on the option solutions to determine which parameters have the greatest impact on option valuation and therefore must be estimated the most accurately.

These options are, practically speaking, clauses written into CDA contractual agreements (or other PPPs). For example, the buyout option reduces the likelihood that CDAs will be seen ex post as a windfall for private sector developers at the expense of taxpayers and road users. Understanding how to price these options is important for contract negotiation. Even if they are included at no cost, the option price solution represents the value foregone by one of the parties. Moreover, including out-of-the-money options may be a useful method to reduce risk for both parties. Such options would have low cost, only paying off in exceptional circumstances, and would be easier to include in the contract. Lastly, for purposes of contract negotiations, such options may reduce the developer’s incentive to provide very low revenue estimates, and the refusal to allow for out-of-the-money options might signal less than honest bargaining.

A further benefit of strategic options is to counter the “prisoner’s dilemma” aspect of individual contract negotiations. It may be collectively beneficial for private firms to negotiate less aggressively in order to reduce windfall profit and therefore increase the public sector’s willingness to grant PPP contracts. However, an individual firm may be tempted to obtain a superior contract, even at the cost of making future PPP contracts less likely due to political backlash. Therefore, including strategic options can be beneficial for private operators collectively as they will reduce the likelihood of windfall profit without affecting firm compensation (options being priced in the contract).

The paper’s main findings suggest that baseline buyout, revenue-sharing, and minimum revenue guarantee options have significant value relative to the value of the concession. This is because the options are likely to be “in-the-money”, meaning there is a high probability they will be exercised and generate revenues. In contrast, the baseline conditional buyout option has only a small value, since it is most likely to be out-of-the-money throughout the life of the concession. The results suggest that while strategic options are useful to reduce revenue risk for both counterparties, keeping their cost low requires either scaling down their payoffs, designing them as initially deep out-of-the-money (so they provide worst-case insurance), or including options for both counterparties designed such that their values offset.

1.3. Previous literature

The literature on real options is large (for surveys, see e.g. Sick, 1995, or Trigeorgis, 1996). For real options in projects, analytical approaches are often not available, so numerical methods have been developed for valuation purposes (e.g.,

\footnote{An out-of-the-money option has zero intrinsic value and thus cannot be profitably exercised today, but remains valuable because it might be profitably exercised in the future (time value). An in-the-money option may be profitably exercised today.}
In this section, we situate our paper’s contribution to previous research. Garvin and Ford (2012) provide a recent survey of the literature on real options in transportation and construction.

Early contributions showed the promise of modeling real options in construction management to value flexibility (e.g., Ford et al., 2002). Much of the subsequent literature has examined minimum revenue guarantees to protect the developer against revenue risk. These are modeled as put options and may be exercised by the developer if revenues are below a predetermined threshold (e.g., Alonso-Conde et al., 2007; Cheah and Liu, 2006; Chiara, 2006; Chiara and Garvin, 2008; Chiara et al., 2007; Kim et al., 2011; Kokkaew and Chiara, 2013). Ashuri et al. (2012) model minimum revenue guarantee and traffic revenue cap options in BOT projects, using a binomial lattice and risk-neutral probabilities under the assumption of market completeness. Galera and Soliño (2010) analyze the value of a minimum traffic guarantee using data from an existing highway concession.

A few recent papers have explicitly recognized that traffic risk may not be hedgeable, so the assumption of market completeness may not hold. Brandao and Saraiva (2008) show how to estimate the value of a minimum revenue guarantee for traffic volume more accurately using market data. In particular, they address the issue of market incompleteness by calculating market prices of risk. Brandao et al. (2012) study the value of staged minimum demand guarantees provided by the Government to private developers for a 30-year concession in the Metro Line 4 of the São Paulo Metropolitan Subway System. Instead of assuming market completeness, they compute the risk premium based on a certainty-equivalent cash flow approach (Freitas and Brandao, 2010). Assuming that traffic volume evolves according to a geometric Brownian motion process, they find that these risk-reducing incentives increase the net value to the developer by 36% while only reducing the value to the public sector by 5%. In closely related work, Blank et al. (2009) model as real options a minimum traffic guarantee, a maximum traffic ceiling and an implicit option to abandon the project for an actual PPP in Brazil, the 4th Line of the Sao Paulo Metro. Their work shows how such options can be useful for risk-sharing purposes and further highlights the importance of estimating the market price of traffic risk.

A few papers have examined other types of options in transportation PPPs. For example, Cui et al. (2004) model a real option for the Government to buy a warranty on a highway construction project. Chan et al. (1992) study a collar option for a toll road PPP, leading to more cost-effective revenue risk management for the private developer. Ashuri et al. (2011) model the option to expand in a two-phase toll road construction project. Rose (1998) studies a special case of a buyout option with an application to Australian transportation public–private partnerships.

In the next section, we discuss advantages and inconveniences of PPPs and more specifically, CDAs, motivating the inclusion of strategic options in agreements. In Section 3, we define and discuss the strategic options analyzed in the paper, how we address market incompleteness, and what computational methods are used to price the options (i.e., simple Monte Carlo and Least Squares Monte Carlo). In Section 4, we describe the demand-side model used to obtain project cash flows as well as the financial model used to value the CDA, which is the underlying asset for the options. Baseline findings are presented in Section 5, together with a sensitivity analysis. We conclude in Section 6.

Some studies, however, use the classic Black–Scholes option pricing model even though it is not applicable when guarantees may be invoked prior to a set maturity date.
2. Benefits and challenges of CDAs and other PPPs

2.1. Why might the public and private sectors wish to partner?

For private firms, public–private partnerships are appealing as an asset class because they are broad-based, essential, real assets, because they provide stable long-run returns (assuming accurate project forecasts), and because there exist natural barriers to entry, even without the inclusion of a non-compete clause (defined below). Investing in infrastructure offers important diversification benefits (e.g., Gatti, 2012). Moreover, as road and transportation projects are less regulated than are utilities or telecommunications, the developer can benefit from quasi-monopoly power and economic rents.\(^3\)

For public authorities, entering into public–private partnerships is expected to provide a number of benefits. First, while the public sector is financially constrained, private firms have a greater capacity to access capital markets. The private sector offers economies of scale, financial innovation, a greater capacity for leverage, and monetization of projected revenue rather than only actual revenue. It can potentially leverage state funds toward transportation projects. This is also because a concession agreement is likely to be perceived by financial markets as more credible than a public authority, which could be swayed by political considerations into delaying or cancelling toll increases for instance. Indeed, the transaction should be beneficial because the private sector can borrow based on expected future toll increases, while the public sector’s bonding capacity is limited by considering only existing toll rates.\(^5\)

The public sector generally uses municipal bond market tax-exempt debt to finance transportation projects. The tax advantage of public financing implies that it has, other things being equal, lower borrowing costs (NCHRP, 2009). For example, Foote et al. (2008) estimate a 4.5% cost of debt for the Pennsylvania Turnpike Commission, while a private lease would borrow at 7.75%. However, it has been argued that tax-exempt debt involves a hidden subsidy whose cost to taxpayers should not be ignored, as it creates the illusion of a lower cost of borrowing. Moreover, the high rating of municipal bonds also implies that the public sector has a more limited range of debt instruments than does the private sector. Indeed, private sector funding covers a range of debt and equity instruments with varying risk-return profiles and typically longer time horizons, particularly for institutional investors (e.g., Gatti, 2012).

What happens in the event of default is another key difference between public and private borrowing to fund large-scale projects. The market perceives that public bonds, including revenue bonds, have the full backing of Government agencies. This implicit guarantee lowers the cost of debt for public financing (NCHRP, 2009). On the other hand, private-sector sponsors of PPPs such as corporations, institutional investors, or investment banks wish to limit their exposure in case of project default. To reduce contamination risk, sponsors use project finance and create special purpose vehicles (or companies) to hold project assets and cash flows, with most of the funding composed of syndicated bank loans (Gatti, 2012; Yescombe, 2014). Project finance debt is non-recourse, so the sponsor’s assets are protected, but this implies a higher cost of borrowing.

2.2. Examples from the USA and other countries

In the following we focus on CDAs although the analysis and model are generalizable to various PPPs, by changing the model parameters that generate cash flow projections. Though often considered successful, CDAs are frequently controversial.\(^5\) Consider for example SR-91, an urban CDA in in California. In 1995, the State of California awarded a private firm with a Build-Operate-Transfer (BOT) contract to improve the congested Route 91 (SR-91) by adding to the highway a 10-mile high occupancy/toll (HOT) lane. The HOT lane would relieve congestion by allowing motorists to pay a toll to access it, while drivers in high occupancy vehicles would enter at no cost. The firm paid $128 million for a 35-year lease, after which it would return to the Government the tollway in good condition along with all rights to collect tolls.

Eight years later, the Government paid the developer $207.5 million to buy back operating rights to SR-91. Why the lease increased in value when there were eight fewer years’ worth of cash flows to collect remains an interesting question and further motivates investigating buyout options. The evidence suggests that the buyback price was higher than the initial concession price because, first, the HOT lane addition to SR-91 was successful. Indeed, the number of vehicle trips increased from 5.69 million in 1996 to 9.27 million in 1998, and despite several toll rate increases stabilized around 8.25 million. Second, the State of California wanted to further relieve congestion through improvements to adjacent freeways, but was unable to do so because the SR-91 lease included a strict non-compete clause prohibiting road improvements that would reduce the developer’s revenue stream.

Moreover, public agencies may be concerned about a CDA developer having conflicting incentives, e.g., an efficient toll collection vs. a smooth traffic flow, and also about relinquishing too many rights to developers, particularly regarding toll increases or improvements to competing roads, as in SR-91. Furthermore, local and state governments do not wish to appear

\(^6\) We thank a reviewer for helping to clarify this and other related issues.

\(^5\) We are grateful to a reviewer for emphasizing this literature.

\(^4\) In Texas, plans for a Trans-Texas Corridor to be built through CDAs were developed by Governor Perry, but the state legislature strongly endorsed in June 2007 a two-year moratorium on CDAs. In July 2009, the legislature voted against the potential use of CDAs in transportation, and instead authorized $2 billion in bonds for road building and other contracts. However, CDAs that had already been approved were not bought back. For example, SH-130 in Texas is expected to continue as planned. While the first segments were opened in 2006, two additional segments are being built by Cintra-Zachry as part of a $1.3 billion agreement whereby the developer can collect tolls for a period of 50 years, with a revenue-sharing provision with the State of Texas.
to be helping private firms make windfall profits. For example, consider a 50-year Greenfield\textsuperscript{7} CDA initially valued at $400 million based on available forecasts. Once the tollway opens, it becomes clear that demand is higher than expected, and moreover that regional economic growth is greater than expected.\textsuperscript{8} After five years, the revised value of remaining cash flows is $800 million. The public may be unhappy, putting pressure on political leaders who seek reelection.

The evidence suggests that CDAs, and particularly Greenfields, suffer from optimism bias and are likely to be unprofitable (Bain and Wilkins, 2002).\textsuperscript{3} It is less clear, however, what explains optimism bias, which can occur on the cost or benefit side. Rather than a systematic failure to forecast, it may be the result of asymmetric information and incentive incompatibilities (management and consulting fees). Optimism increases the likelihood of project approval. On the benefit side, traffic volume forecasts are typically overstated. Indeed, privately financed toll road concessions are commonly awarded to bidding teams submitting the highest traffic (and hence revenue) projections (Bain, 2009). On the cost side, strategic misrepresentation occurs in particular because of capital fixed costs (not O&M). Flyvbjerg (2008) suggests mark-up factors for Greenfield projects and reference-class forecasting. In the context of asymmetric information, Danninger (2005), Flyvbjerg (2008), and the European Investment Bank (2011) also note from international PPPs that over-forecasts are a standard response by bidders to incentive structures between the principal (public sector) and the agent (concessionaire) which are inserted to address public approval.

Moreover, private developers may worry not only about revenue uncertainty but also the possibility that the public sector will try to change the terms of the agreement (i.e., expropriation risk). To be sure, it is difficult in practice to determine how profitable private toll operators are. This is because they typically operate several entities, some being profitable, while others are not. The financial structure may be opaque, allowing one entity to export profits to another through fees or management services.\textsuperscript{10}

The prices and durations of concessions have also attracted much attention—for example, the 99-year lease of the Chicago Skyway for $1.83 billion and the 75-year lease of the Indiana Toll Road for $3.8 billion (see e.g. Arshad, 2010).\textsuperscript{11} In both cases, the winning bid was much larger than the second-highest one, possibly suggesting a winner’s curse. Moreover, the prices paid were about five times the price of French concessions that had superficially similar characteristics. Indeed, the concession price as a multiple of earnings was about 60 for the U.S. concessions, but only about 12 for the French concessions. The high prices paid for U.S. concessions, however, can be explained by their more favorable terms, including clauses allowing for substantial toll rate increases. Other contract provisions (general restrictions, buyback provisions, etc.) also help explain the very different prices paid for U.S. concessions, however, can be explained by their more favorable terms, including clauses allowing for substantial toll rate increases. Other contract provisions (general restrictions, buyback provisions, etc.) also help explain the very different valuations (Bel and Foote, 2007). The French concessions may have found a better balance between operator profits and user benefits. While U.S. concessions were designed to maximize taxpayer welfare, European concessions maximized toll-payer welfare.

### 3. Valuation of strategic options: definitions and computational methods

This section motivates strategic options for both public and private parties to reduce risk and increase the mutual appeal of CDAs. Then, we describe the five options modeled in this paper, three from the public agency’s perspective and two from the developer’s (concessionaire).

#### 3.1. Can options facilitate Comprehensive Development Agreements?

For both the public and private parties, the main concern is uncertainty associated with difficult-to-forecast revenue streams over a very long period of time, e.g. 50 years or more. Options can reduce uncertainty for both parties. The existing literature has focused mainly on options for private developers, such as minimum revenue guarantees (Cheah and Liu, 2006; Chiara, 2006; Chiara and Garvin, 2008; Chiara et al., 2007).

Sources of risk may be categorized as being due to: (i) revenue generation, (ii) operation and maintenance, (iii) government actions and force majeure, (iv) termination or counterparty payment uncertainty, and (v) disputes. In this paper, we assume that expropriation risk and counterparty default risk are negligible, although this may not be true for some emerging economies (e.g., Schwartz and Trolle, 2010). Post-implementation, the main source of uncertainty is business risk associated with uncertain demand for the toll road, depending on regional and national macroeconomic conditions as well as whether a

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\textsuperscript{7} CDAs are classified as Greenfield or Brownfield projects. A Greenfield project consists of new construction, often in a newly developed area. In addition to construction risk, such projects face substantial revenue uncertainty associated with difficult-to-forecast demand for the new project, particularly if the geographical location is still in economic development. Indeed, in some cases the urban development fails to materialize and there is almost no demand for the project. In contrast, a Brownfield project is a mature one, typically already built and located in a well-established city or region. There is much less risk, and less demand uncertainty, associated with the upgrade of a Brownfield project compared with new construction (Greenfield).

\textsuperscript{8} There exists a clear relationship between project maturity and the risk premium expected by investors. The risk premium is very high for Greenfield projects, but falls sharply as projects become more mature, i.e., for Brownfield projects. For example, research done by the Macquarie Infrastructure Group (undated) on the profitability of the Indiana toll road finds that the risk premium is at least 7% for new construction projects such as South Bay or Westlink M7, but only about 3% for established projects such as Lusoponte or M4 or M5 Motorways.

\textsuperscript{9} For example, Bain and Wilkins (2002) find that only four out of 32 toll road cases studied reached their forecasted performance. On average, actual volume was only 70% of the volume forecast (Bain and Plantagie, 2007).

\textsuperscript{10} We are indebted to a reviewer for bringing this issue to our attention.

\textsuperscript{11} Note that although the Indiana and Chicago Skyway agreements are for 75 and 99 years, respectively, legislation limits CDAs in Texas to have durations of no more than 50 years. In contrast, durations in Spain are limited to 20 years for Brownfields and 40 years for Greenfields.
non-compete clause has been included. Clear delineations of risk-sharing between public and private parties is often difficult and contracts may lack in transparency (Peters and Perotta, 2006).12

3.1.1. Buyout option

The Texas legislature in 2007–2008, for example, favored a clause generally known as “termination for convenience,” whereby the public agency would buy back the CDA at its own discretion. In the past, buyout prices have been negotiated ex post, e.g. the SR-91 toll highway in California. We therefore consider a contractual clause giving the public authorities the right to buy back the CDA for a predetermined exercise price, either to collect revenues itself or to lease out to a new private firm at a better price. How much would this option cost?

Financial options derive value from an underlying asset price. The equivalent of asset price here is the updated net present value (NPV) of remaining cash flows in the CDA. Every period, more information is learned about the profitability of the concession. It is likely that the greatest amount of uncertainty is resolved in the first few periods. The updated NPV must decrease, other things being equal, because there are fewer years’ worth of cash flows remaining. However, if traffic volume greatly exceeds expectations, as in the case of SR-91, the updated NPV may be greater than the exercise price. Thus, it is modeled as an American call option on a dividend-yielding asset, for which early exercise, i.e. buying back the CDA, might be optimal.13

Moreover, there is a second effect: if the project is successful, operator financial risk is reduced, bringing down the cost of capital and increasing project value independently of the higher project cash flows. For an unsuccessful project, risk and cost of capital both increase with time, further lowering project value.14

A necessary condition for the buyout option to be exercised at a given period is for the revised, period-t NPV to be greater than the exercise price. A sufficient condition is for the period of exercise to be the optimal stopping date. The problem is therefore one of solving for the optimal stopping date, first, and then valuing the option based on its expected payoff. The option payoff at time \( t \), \( \Pi_t = \max(V_t - X, 0) \) equals the difference between \( V_t \), the updated value of the concession at \( t \), i.e. the NPV of all remaining cash flows, and the exercise price \( X = \phi \cdot V_t \), which equals a constant \( \phi > 0 \)

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The choice of a lower public sector discount rate is typically motivated by its financing alternatives to CDAs, e.g., municipal bond rates. Note, however, that public sector agencies frequently are limited in their bonding capacity due to previously incurred debt (e.g., Navigant, 2012). This constraint could further motivate delegation of responsibilities to the private sector through a PPP.16 Finally, assuming a lower public discount rate would only improve the mutually advantageous nature of the transaction, as the buyout price paid by the public agency could be simultaneously lower than the public’s valuation and higher than the private sector’s.

3.1.2. Conditional buyout option

A special case of interest is a buyout option that may be exercised only if “windfall profit” is earned. Specifically, this buyout option may be exercised by the Government at date \( t \) if the Internal Rate of Return over the period \( [0, \ldots, t] \) exceeds a predetermined threshold, such as 12% (i.e., twice the cost of capital). Since the trigger is unlikely to be reached, the exercise price is set to zero, i.e. \( \phi = 0 \). If the threshold is reached, the option is exercised. Indeed, given a zero exercise price and a declining value of the CDA over time (fewer years of revenues left), there is no reason to delay. Lastly, it is assumed that there is a five-year vesting period: the option may not be exercised before year 6 at the earliest. Surprisingly, our findings show

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12 Political risk is particularly difficult to evaluate and “price” in a financial model. This could be, for example, uncertainty associated with new political leaders or parties who did not support the previous administration’s endorsement of a CDA or a change in legislative provisions.

13 In the numerical solution, however we assume there is only one exercise date per period (i.e., each month), so technically it is Bermudan-style (see e.g., Chiara, 2006).

14 We thank a reviewer for emphasizing this point.

15 This is because, even though the public discount rate is likely lower than the private discount rate (e.g., Groot, 2003), it makes more economic sense to assume that the public agency will exercise the option if it could, in principle, buy back the concession and profitably sell it to a new developer at a new price reflecting current market conditions. For this to be true, the public must use the market valuation of the CDA. In a previous version of the paper, it was assumed that the public agency used a lower discount rate than the private sector, following Groot (2003). However, this assumption unnecessarily complicated the analysis, so it has been removed.

16 In their Final Report, Navigant (2012) document the debt capacity of the NY-NJ Port Authority. They write (p. 2): “Public Private Partnerships (“PPPs”) may represent an opportunity for the Port Authority to execute certain of its significant capital projects but need to be evaluated in context with other available alternatives.” They further explain that although the Port Authority’s debt in Consolidated Bonds has increased $5.7 billion in 2000 to $15.5 billion at the end of 2011, it is expected that up to $26 billion (total) could be issued by 2020 (Navigant, 2012, p. 12–13). We are grateful to a reviewer for bringing to our attention this study and its relevance for debt capacity in the model.
that this option has a low value despite the zero exercise price. This is because there are very few simulation paths in which the option is exercised, and when it is, there are typically few years left to the life of the lease.\textsuperscript{17} The present value of a few distant years of cash flows is very low.

### 3.1.3. Revenue-sharing option

This type of clause held by the public agency seems to be preferred to buyout options by private firms involved in CDAs. It provides the public agency with a share of revenues if the latter are much higher than what was anticipated at the time the contract was signed. The option is defined such that if net revenues exceed a predetermined amount, then a share $\alpha$ of “excess profits” is paid by the developer to the Government, while the developer retains a share $(1-\alpha)$.

This clause is modeled as $T$ distinct European call options, each with price $C_E$ for $i = \{1, \ldots, T\}$.\textsuperscript{18} There is one revenue-sharing option for each year in the life of the contract. E.g., the year 1 option can only be exercised at the end of year 1 based on that year’s financial statements. Unused options are lost. At the end of each period $t$ (e.g., fiscal year), net cash flows to equity-holders $CF_t$ are compared with a predetermined value $K_i$ (exercise price) calculated as time-0 expected cash flows to equity-holders. Each option has a payoff of: $P^{CE}_t = \max(0, CF_t - K_i)$, where $0 < \alpha < 1$ is the share of “excess profit” returned by the developer to the Government, $CF_t$ are the realized cash flows for year $i = \{1, 2, \ldots, 50\}$, and $K_i = E_0[CF_i]$ are the time-0 forecasted cash flows for each option. In the baseline case, $\alpha = 0.5$. Naturally, increasing $\alpha$ makes the options more valuable. The total cost to the public agency of purchasing all $T$ options is the sum of the $T$ option values.\textsuperscript{19}

### 3.1.4. Minimum revenue guarantee options

The developer may be granted minimum revenue guarantee options, providing a revenue floor and reducing revenue risk. This type of option has been previously studied (see e.g. Chiara, 2006; Chiara et al., 2007).\textsuperscript{20} Options to the developer and to the Government may be calibrated to have offsetting prices, providing risk reduction without affecting the CDA value. We consider two specifications: (i) a series of European-style options paying off annual revenue shortfalls, and (ii) an American-style put option paying off a one-time cumulative revenue shortfall in exchange for returning the CDA to the Government. To our knowledge, only the first has received attention in the literature. Moreover, the American-style option involves an optimal exercise decision that can interact with the buyout option exercise date.

First, consider a series of $T$ annual revenue guarantee options modeled as European-style put options. Each option, for year $i = \{1, 2, \ldots, 50\}$, may be exercised once, and only one option may be exercised each year. Each put option $i$ pays off the difference between a fraction of $K_i$, the time-0 expectation of year-$i$ cash flows to equity-holders, and the actual year-$i$ cash flows to equity-holders: $P^{CE}_t = \max(0, \theta K_i - CF_t)$, with $K_i = E_0[CF_i]$ and the baseline revenue guarantee set at $\theta = 0.50$ (i.e., a 50% guarantee). Other parameter values are also explored. Thus, unless $\theta = 1$, it is not sufficient for cash flows to equity-holders to be below expectations for the option to be exercised. Rather, they must be below a fraction $0 < \theta < 1$ of expectations.

Second, we consider a cumulative-value revenue guarantee and abandonment option. The developer who exercises this option receives a payoff based on the cumulative revenue shortfall, but in return forfeits the remaining years of the concession. The option pays off, at a single date $t$, $P^{RGC}_t = \max(0, K_t - S_t)$, the first term, $K_t = E_0[\sum_{i=0}^{T} CF_i] - \sum_{i=0}^{T} CF_i$ is a revenue shortfall. It equals a fraction of the time-0 expectation of cumulative cash flows to equity-holders over the period $0, \ldots, t$, minus the actual cumulative cash flows to equity-holders over the same period. The second term, $S_t = \sum_{i=t+1}^{T} \frac{CF_i}{(1+r)^i}$, is the present value at time-$t$ of the remaining years of cash flows. It is assumed in the baseline model that $\theta = 0.50$. This is an American-style put option with a time-varying exercise price. To value this American option, the optimal exercise date must first be determined. Intuitively, if the project is unsuccessful, it is profitable to delay exercise because cumulative shortfall grows over time. However, the payoff loses value in present-value terms. Note further that $S_t$ is likely to be small when $K_t$ is large, because in each simulated time path, cash-flows are correlated.

### 3.2. Option valuation under incomplete markets

Textbook real option valuation assumes market completeness. In this setting, however, this assumption may not be reasonable because traffic risk is not a traded asset—it cannot be hedged. As in Brandao and Saraiva (2008), we replace instead the true drift $\mu$ of the stochastic process describing asset value $V_t$ with the drift adjusted for the market price of risk, $(\mu - \sigma \lambda)$, where $\sigma$ is volatility (here, traffic volume volatility) and $\lambda$ is the market price of risk (here, traffic volume risk). The values of these parameters are obtained from calculations presented below.

\textsuperscript{17} Other buyout option specifications are certainly possible, for different IRR thresholds or $\phi$, but the purpose is not to be exhaustive, but rather to show a few important option types. Moreover, as we explain, it is possible to calibrate options to deliver cost-effective risk-reduction based on specific objectives of the public or private counterparty. We are grateful to a reviewer for suggesting that we investigate this type of option in the CDA.

\textsuperscript{18} We use the subscript $i$ instead of $t$ to make it clear that the option payoff is for a specific European option $i$ at its only possible exercise date, unlike the buyout option, which is unique but has a different payoff value at every date.

\textsuperscript{19} To avoid a possible bias, the $T$ options associated with a given simulation are priced using the same time series path. This means the profitability of the $T$ options on a given path is positively correlated.

\textsuperscript{20} The previous literature has also considered loan guarantees as real options in PPPs that may be available to the private developer (Takashima et al., 2010).
To see how this approach works, consider a continuous-time framework where the asset value $V$ follows geometric Brownian motion under the physical measure $P$, $dV = \mu V dt + \sigma V dW$, for $W$ a Brownian motion under $P$. Now define $dV^* = dW + \lambda dt$ as a Brownian motion under the risk-neutral measure $Q$. Hence, we can write: $dV = (\mu - \lambda) dt + \sigma V dW^*$ under $Q$. Then, we obtain $N$ simulated paths under the risk-neutral measure $Q$. In each path, we determine the option payoff and discount it at the risk-free rate. These payoffs are valid even under market incompleteness.

Traffic volume uncertainty is the most important source of risk in this model (see e.g., Brandao and Saraiva, 2008). Therefore, the market price of traffic risk is also the market price of project risk. To obtain the risk premium, $\lambda_k \sigma_k$, given a market price of traffic risk $\lambda_k$ and traffic volatility $\sigma_k$, we use the CAPM and write: $\lambda_k \sigma_k = \beta_p \left( r_p - r_f \right) \times \frac{\sigma_p}{\sigma_k}$, where $\beta_p$ is the project beta, $\sigma_k$ is the volatility of traffic revenues, and $\sigma_p$ is the volatility of the asset (CDA) value. The project beta is known from the literature, while the second and third are estimated from the simulations. The specific values used are $\beta_p = 0.7, E[r_m] - r_f = 0.07, r_f = 0.02, \frac{\sigma_p}{\sigma_k} = 0.644$, and therefore the risk premium is $\lambda_k \sigma_k = 0.03156$.

3.3. Computational methods for strategic option valuation

3.3.1. European-style strategic options

To value the annual revenue-sharing options and annual minimum revenue guarantees, we follow the procedure outlined in Section 3.2. We use the time series paths under the risk-neutral measure $Q$, such that $dV = (\mu - \lambda) dt + \sigma V dW^*$ (see Section 3.2). The option values are computed from 500,000 simulated paths of 50 periods each, assuming a project lifetime of 50 years. Each option value is computed as the Monte Carlo average payoff of $N$ paths under $Q$, including all payoffs of zero for paths where the option is not exercised (Boyle, 1977; Boyle et al., 1997). The payoffs are discounted $i$ periods at the risk-free rate $r_f$.

3.3.2. Least Squares Monte Carlo approach for American-style options

The American-style buyout, conditional buyout, and cumulative shortfall minimum revenue guarantee options are priced by Least Squares Monte Carlo (LSMC, see Longstaff and Schwartz, 2001). Monte Carlo evaluation is not applicable here because these options can be exercised at any time before expiry, so their solution involves an optimal stopping problem. To value these options, first we must determine the optimal date of exercise. The challenge for the decision-maker is that it is not sufficient for exercise to be profitable. It must also be more profitable than the expected value from continuing to hold the strategic option. The purpose of LSMC is to evaluate, at every period, the “continuation value” of holding the option longer, and to compare it with the value from immediately exercising the option. If, at a given period, the continuation value exceeds the payoff from exercising the option, then it is optimal to delay exercise by one period. The continuation value is calculated using Ordinary Least Squares across all simulated paths, so it is a linear regression using $N = 500,000$ observations.

The approach has three steps. First, $N$ paths are simulated to obtain time series of project asset values. Time series paths under the risk-neutral measure $Q$ are used, where the drift is adjusted as described above, and payoffs are discounted at the risk-free rate $r_f$. Second, we determine at every period whether it is optimal to exercise the option, i.e. whether the payoff is positive:

$$P_i = \max(V_i - X, 0) \quad \text{for the buyout option}$$

$$P_i = \max(R_i - S_i, 0) \quad \text{for the minimum revenue guarantee option}$$

Third, we find the continuation value using Ordinary Least Squares regressions over a polynomial in $V_i$ (i.e., a linear combination of a set of basis functions). The OLS problem at every period may be written as follows:

$$\min_{\beta} \left( P_i - P^L(V_i) \cdot \beta \right)' \times (P_i - P^L(V_i) \cdot \beta)$$

where $P^L(.)$ is a polynomial operator of order $L$. We estimate the continuation value as follows:

$$CV(V_i, X) = P^L(V_i) \times \hat{\beta}$$

where $\hat{\beta}$ is the estimated coefficient of the OLS regression. The decision rule is: if $CV^p(V_i, X) > P_i(V_i, X)$ then we delay exercising the option because the expected value from exercising it in the future is greater. If, however, $CV^p(V_i, X) \leq P_i(V_i, X)$, then we exercise the option. Once we have determined the optimal exercise date $t_n$ in every path $n = \{1, 2, \ldots, N\}$, we discount the payoff in path $n$ by $r_f$ periods using the risk-free rate $r_f$. Finally, we compute as the solution the Monte Carlo sample average of the option values, and report Monte Carlo standard errors.

21 We thank a reviewer for clarifying this point and suggesting how to explain it concisely.

22 We use powers of the state variables (up to the fourth power), as Longstaff and Schwartz (2001, p. 142) find that the solutions are “remarkably robust to the choice of basis functions” and thus typically not affected by the choice of polynomials (e.g., Hermite, Legendre, Chebychev, etc.).
4. Revenue and CDA valuation model

This section describes a consumer demand-based model developed for purposes of obtaining simulated time series paths of project cash flows to equity-holders and asset values. Rather than specifying the stochastic process directly, we model the economic variables that determine each period’s cash flows as generated by the concession, as well as the remaining concession value at each period. This approach enables us to conduct sensitivity analysis on the strategic options. The following Fig. 2 shows the steps involved to obtain, first, project cash flows, then asset values and option values.

The baseline model considered is a BOT Greenfield CDA with a contract period of 50 years. The developer is responsible for the project construction, and subsequently begins operation and toll collection for a period of 50 years. The CDA itself is a six-lane toll highway. Since there is not much historical data to estimate CDA model parameters, we calibrate them based on a careful review of the literature on transportation CDAs. Then, we verify the robustness of solutions using sensitivity analysis. For convenience, all model parameters are summarized and briefly described in Table 1.

4.1. Valuation of the CDA: project cash flows, cost of equity, debt and taxes

To value the concession, we need to determine project cash flows and the weighted average cost of capital (WACC). Project cash flows are obtained as the solution of the optimum toll price \( P \) times quantity \( (\text{traffic volume}) \ Q(P) \), based on the demand model (see Section 4.2). To find the value of the CDA, we compute the discounted sum of all free cash flows to equity-holders. These are cash flows available to return to the equity-holders after paying for debt service, taxes and costs of operation and maintenance. The discount rate that is used is the cost of levered equity, i.e., the expected return to equity-holders given the project’s level of undiversifiable risk, adjusted by the tax advantage of debt (leverage).

The developer’s profit is generally taxable, although CDAs may include certain tax exemptions. Based on the literature for U.S. CDAs, it is assumed that the developer pays an average corporate tax rate of 17.5% on all cash flows net of debt service payments and operating and maintenance costs. However, private developer entities may use transfer pricing, corporate management fee payments between entities, and tax management (see Peters, 2007). This means a seemingly unprofitable venture may in fact be an excellent investment for the sponsors and investment banks involved, once all fees are included. Although our analysis does not account for the value of such fees to sponsors in determining project profitability, the option price solutions remain correct under the assumption that the underlying asset value is correctly measured. Exploring the magnitude of management and consulting fees in relation to project value, and their impact on strategic options, appears to be an important topic for further research.

Regarding tax management, we examine in the section on sensitivity analysis the impact on the option price solutions of doubling the tax rate to 35% or halving it to 8.75%, reflecting different levels of tax management. We assume that the developer chooses an optimal capital structure to maximize the tax shield from interest payments, and maintains this target debt-to-equity ratio throughout the life of the concession by rolling over debt if necessary. This might involve paying back a syndicated bank loan after a few years using newly issued project bonds, and then paying bondholders coupons periodically until the end of the CDA, at which point the principal is paid off as a bullet payment.

It is assumed in the baseline model that 25% of the expected total cost of the CDA for the developer (construction cost plus lease) consists of equity, while the remainder consists of senior debt, for a baseline \( D/E \) ratio of 3. Once the agreement is signed, the amount of equity in the project is fixed. Thus, construction cost overruns are added to the senior debt. In the simulations, therefore, the \( D/E \) ratio is slightly greater than 3, but never substantially so. For example, suppose the project’s cost is $1.8 billion, including $600 million for construction and $1.2 billion for a 50-year BOT agreement. Sponsors provide $450 million in equity, expecting returns of about 11.6%, while senior debtholders contribute $1.35 billion expecting returns of 5%. No junior or mezzanine debt is used. Note that re-gearing debt to maintain a fixed debt-to-equity ratio, instead of gradually paying down principal, could be a profitable capital structure strategy for the developer.

Operation and maintenance costs are a significant expense, as the agreement typically includes a performance contract that can result in non-compliance points and liquidated damages if the developer does not meet the stipulated requirements. O&M costs can represent 25–30% of the value of operating cash flows, e.g., $8–9 million for the Chicago Skyway during 2002–2004 (Arshad, 2010, p. 112). For SH-130 Texas Segment 5–6, they are about $78 million annually (Holzmann, 2012). The CDA in this paper is a smaller project, so plausible values are a $1 million fixed cost and a variable cost that is a function of traffic volume, averaging $2.37 million. These values are consistent with per-mile O&M costs available from the works cited.


24 E.g., whether to use straight-line or Modified Accelerated Cost Recovery System (MACRS) asset depreciation, whether assets are recorded at book or market value, and whether tax havens are used.

25 In a previous version of the paper, we assumed that debt was paid off in equal installments during the life of the project, so the Book Debt-to-Equity ratio would decrease monotonically.

26 The 5% return to debtholders reflects a 3% credit risk spread. A more involved credit risk analysis would be beyond the scope of the paper.

27 We thank a reviewer for bringing this issue to our attention.

28 See e.g. point 7.5 on “asset condition” in the SH-130 segments 5–6 facility technical requirements document [Texas Department of Transportation, 2007].
Table 1
Description of the model variables and parameters. All truncated normal distributions are for the support \([-\infty, \infty]\).

<table>
<thead>
<tr>
<th>Variable or parameter</th>
<th>Description</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Duration of project construction, (\log M) distributed truncated (N(\mu_M, \sigma_M^2))</td>
<td>(E[M] = 3)</td>
</tr>
<tr>
<td>C</td>
<td>Annual cost of project construction, (\log C) distributed truncated (N(\mu_C, \sigma_C^2))</td>
<td>(E[C] = $200 million)</td>
</tr>
<tr>
<td>(r_e)</td>
<td>Cost of levered equity (expected return to equityholders)</td>
<td>Baseline 11.6%</td>
</tr>
<tr>
<td>(r_f)</td>
<td>Risk-free rate approximated by the U.S. 3-month treasury bill yield</td>
<td>2%</td>
</tr>
<tr>
<td>(r_m - r_f)</td>
<td>Equity market risk premium</td>
<td>7%</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Financial beta for the project (based on sector beta)</td>
<td>0.7</td>
</tr>
<tr>
<td>PHPTV</td>
<td>Peak-hour proportion of traffic volume, (0 &lt; x &lt; 1)</td>
<td>0.11</td>
</tr>
<tr>
<td>PFIV</td>
<td>Proportion of trucks (for each axle class) as a fraction of total vehicles, (0 &lt; x &lt; 1)</td>
<td>0.5 (total)</td>
</tr>
<tr>
<td>TTA</td>
<td>Proportion of toll avoidance, (0 &lt; x &lt; 1)</td>
<td>0.01</td>
</tr>
<tr>
<td>PHTV</td>
<td>Peak-hour traffic volume, in vehicles per hour per lane</td>
<td>1800</td>
</tr>
<tr>
<td>AEPIIV</td>
<td>Axle-equivalent peak-hour traffic volume, computed using the axle-equivalent of each axle class of trucks</td>
<td>2025</td>
</tr>
<tr>
<td>Lanes</td>
<td>Number of lanes in the CDA toll highway</td>
<td>6</td>
</tr>
<tr>
<td>Days</td>
<td>Effective days</td>
<td>330</td>
</tr>
<tr>
<td>(A_0)</td>
<td>Baseline annual traffic volume when tolls are set to 0, such that (A) equals</td>
<td>36.085 million</td>
</tr>
<tr>
<td>(Q)</td>
<td>Quantity demanded, i.e., traffic volume for a given price (toll rate), such that (Q = A(\mu + \psi)^{-\gamma})</td>
<td>Varies</td>
</tr>
<tr>
<td>(P)</td>
<td>Toll rate (if axle-equivalent) or vector of toll rates (if multiple vehicle classes)</td>
<td>Varies</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Parameter determining traffic volume upper bound as (P \rightarrow 0)</td>
<td>0.5</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Elasticity of demand parameter</td>
<td>0.3 (initial)</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>Maximum traffic volume ensuring average traffic speed does not fall below (V_0)</td>
<td>80 km/h</td>
</tr>
<tr>
<td>(V_0)</td>
<td>Average traffic speed below which the developer is financially penalized, based on the correspondence</td>
<td>80 km/h</td>
</tr>
<tr>
<td>(\bar{V}_T)</td>
<td>Free-flow average traffic speed</td>
<td>110 km/h</td>
</tr>
<tr>
<td>(\bar{B})</td>
<td>Curvature parameter such that (B = \bar{V}_T/(\bar{V}_T - V_0))</td>
<td>2.687</td>
</tr>
<tr>
<td>(g_0)</td>
<td>Rate of traffic volume growth such that (g_0 = g_0(\alpha t + \sum_{i=1}^n \psi_i))</td>
<td>Varies</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Intensity of the deterministic downward trend in traffic growth</td>
<td>0.20</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Random shock to the growth rate, distributed truncated (N(0, \sigma_\rho^2))</td>
<td>(E[\rho] = 0)</td>
</tr>
<tr>
<td>(A_t)</td>
<td>Time-varying traffic volume base measure, (A_t = A_0 \exp(\alpha t))</td>
<td>Varies</td>
</tr>
<tr>
<td>(\chi)</td>
<td>Effect on the elasticity parameter (\chi) of improvements to competing roads, where (\chi) is distributed truncated (N(0, \sigma_\chi^2)) and may occur each period with a probability distributed Bernoulli(p)</td>
<td>(E[\chi] = 0)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Average corporate tax on net revenues</td>
<td>0.175</td>
</tr>
<tr>
<td>(E(D + E))</td>
<td>Proportion of financing from equity, the remainder through senior debt</td>
<td>0.25</td>
</tr>
<tr>
<td>(\pi)</td>
<td>Proportion of excess revenue-sharing returned to the public sector</td>
<td>0.5</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Buyout price multiplier (1 + original concession value)</td>
<td>1.0</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Proportion of minimum revenue guarantee to developer</td>
<td>0.75</td>
</tr>
<tr>
<td>IRR(_\text{trigger})</td>
<td>IRR threshold for the conditional buyout option</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Fig. 2. Schematic representation of the model: how CDA value and option values are computed from the variables.
To obtain the levered cost of equity which is used to discount the free cash flows to equity-holders (project sponsors), first we compute the unlevered cost of capital using the Capital Asset Pricing Model:

$$E[r_D] = r_f + \beta_p (E[r_m] - r_f)$$

where $E[r_D]$ is the expected return on the project, $r_f = 0.02$ is the risk-free rate, $\beta_p = 0.7$ is the project beta based on the literature (assuming an average sector beta), and $E[r_m] - r_f = 0.07$ is the market risk premium. For the risk-free rate, we use the 3-month U.S. Treasury bill yield. The CAPM provides an unlevered cost of capital of $E[r_D] = 0.069$.\(^3\) Then, to determine the expected return to equity-holders, $E[r_E]$, we assume that the project is financed with 75% senior debt and 25% equity, that the interest rate paid on the senior debt is $r_d = 0.05$ (i.e., a 3% spread above the risk-free rate), and that the average tax rate paid is $\tau = 0.175$. Note that the tax shield of interest payments on debt implies that the Weighted Average Cost of Capital (WACC) is 0.06, lower than the unlevered cost of capital which is 0.069. The levered cost of equity is calculated from the tax-adjusted WACC formula:

$$\text{WACC} = \frac{D}{D+E}(1-\tau)r_d + \frac{E}{D+E}E[r_E]$$

which implies that that the levered cost of equity, used to discount cash flows to equity-holders, is:

$$E[r_E] = E[r_D] + (1-\tau)\frac{D}{E}(E[r_D] - r_d) = 0.116$$

This expected return is below the Internal Rate of Return (IRR) for successful projects. For example, the IRR for SR-91 in California was about 17%, while the fair market value rate of return for SH-130 in Texas was 18%.

### 4.2. Demand model: basic framework

In this section, we describe the main components of the demand model, which is used to compute optimal toll rates, quantity (traffic volume), and cash flows to equity-holders. Additional information about the model and its parameters is presented in Appendix A. Assume that quantity demanded, i.e., traffic volume, equals $Q = A(P + \psi)^{-\gamma}$ where $A$ is determined as described in Appendix A.3, $P$ is the unit toll rate or vector of rates, $\gamma$ is a parameter governing price elasticity of demand, and $\psi$ is a parameter ensuring that traffic volume reaches a finite value as price goes to zero. The no-toll corner solution is then $Q = A\psi^{-\gamma}$. An upper bound reflecting maximum capacity is imposed to ensure that traffic volume does not grow unboundedly. This maximum capacity is defined as the traffic volume beyond which safety becomes an issue. The developer’s problem is to choose an optimal toll rate $P^\ast$ such that revenue is maximized, $P \in \text{arg max} P \cdot Q$ subject to the following constraints. First, $P$ and $Q$ are non-negative. Second, there is a maximum toll rate increase, which is a known constant, e.g., 2%.\(^3\) The simulations suggest that it is nearly always optimal for the developer to fully use the allowable toll rate increase, i.e. the constraint always binds. This result is consistent with a fairly inelastic demand for using the toll road. Therefore, the constraint is meaningful to the contract agreement. This result suggests that limits on allowable rate increases are relevant, and that due to the difficulty of supervision, developers may obtain economic rents in their absence. A notable limitation of this model is that it does not consider rational expectations. Modeling rational expectations would make the model more accurate and internally consistent, but at the cost of greater complexity.

Third, there is a level of service constraint. Traffic volume must not exceed the volume $Q_0$ at which average speed reaches the minimum speed $V_0$ as defined in the CDA to ensure a steady movement of traffic—for example, $V_0 = 80$ km/h. If average speed falls below $V_0$, the developer is penalized a large share of the period’s cash flows, e.g., at least 25% and as much as 100%. To avoid adding a variable, we transform this minimum speed constraint into a maximum quantity constraint using a congestion correspondence based on Li’s (2008) modification of the Newell-Franklin speed-flow model:

$$Q = Q_0 \left(\frac{V}{V_0}\right) \left(1 - \frac{1}{B} \ln \left(\frac{V_f - V}{V_f - V_0}\right)\right)^{-1}$$

where $Q$ and $V$ are the actual traffic quantity (volume) and average speed, respectively, $V_f$ is the free-flow traffic speed, e.g., $V_f = 110$ km/h, $V_0$ is the contractually determined minimum average speed below which the developer is financially penalized, $Q_0$ is the traffic volume above which speed falls below $V_0$, and lastly $B = V_0/(V_f - V_0)$. This correspondence allows us to turn the minimum average speed constraint into a maximum traffic volume constraint in the developer’s revenue maximizing problem.\(^3\)

Lastly, note that although a constant risk-free interest is used, sensitivity analyses are presented later.\(^3\)

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\(^{29}\) The 6.9% figure is low compared to the literature, but this due to the currently low risk-free rate and equity market risk premium.

\(^{30}\) In a previous version of this paper, the constraint was the maximum of {constant, inflation, real GDP growth rate} but as a reviewer pointed out, modeling stochastic processes for inflation and real GDP complicates the analysis without contributing much in terms of new insights. Indeed, the results are largely similar if we assume constant inflation and real GDP growth rate (e.g., 2% per annum each).

\(^{31}\) Additional results (available upon request) suggest that, for the model parameters considered, incurring penalties by letting $Q > Q_0$ is never optimal. Note that in Greenfield projects, however, the constraint is generally non-binding, because even with healthy traffic growth, volume remains far below $Q_0$. Nonetheless, we include the constraint in the model, for example in the event of an exceptional growth boom.

\(^{32}\) In previous versions of this paper, stochastic interest rates were used based on the CKLS model (Chan et al., 1992). However, this added little to the analysis.
5. Findings

5.1. Results of the simulation model

This section presents the results, summarized in Table 2, that are obtained for the option values as well as an interpretation of the findings. The value of the CDA itself, given the model parameters, is $514.7 million. Under baseline specifications, the buyout, revenue-sharing, and minimum revenue guarantee options are valuable, but the conditional buyout option is not.

5.1.1. Buyout option

The buyout option for the baseline CDA has a price of $57.83 million, with a Monte Carlo standard error of $0.623 million. The option price represents 11.2% of the value of the CDA, which is $514.7 million. The distribution of optimal stopping times, for paths where exercise was optimal, is roughly lognormal with most of the probability mass between 5 and 20 years, implying that it is generally optimal to buy back the lease well before the midpoint of the contract duration. These results are noteworthy because they seem consistent with the case of the SR-91 road in California, which was bought back after only eight years.

The conditional buyout option has very little value. For a zero exercise price and an IRR threshold of 12%, its value is only $3.7 million, or 0.7% of the CDA value. This is because the IRR threshold is seldom reached, and if so, the project is already mature and there are few years left of cash flows.

5.1.2. Revenue-sharing option

The price of each individual annual revenue-sharing options (with a share $a = 0.5$ going to the public agency) increases nearly monotonically in option maturity, even though payoffs are discounted for more years. This is because the first few years of operation of a Greenfield project generate little or no profit. On average, each option costs about $0.7 million. To acquire revenue-sharing options for all $T = 50$ years, the public agency should pay the developer $33.82 million, which represents 6.6% of the value of the CDA.

5.1.3. Minimum revenue guarantee options

We consider two MRG options: a single, American-style cumulative guarantee requiring the developer to return the lease, and a series of annual European-style put options. Both have substantial value. For a fraction of guaranteed revenue of $\theta = 0.5$, the American-style MRG option is worth $81.76$ million, or 15.9% of the CDA value. The sum of all 50 European-style MRG options, also with $\theta = 0.5$, is worth $124.29$ million, or 24.1% of the CDA value.

5.2. Option moneyness

An obstacle to the inclusion of strategic options in CDAs or PPPs is their cost, which we have shown to be potentially high relative to CDA value. However, as discussed earlier, setting the options to be deeply out-of-the-money provides a way to include them at a lower cost and still provide some risk reduction. Such out-of-the-money options would only be exercised under exceptional circumstances, such as a toll road vastly over- or underperforming its expectations. Table 3 presents option price solutions for all options considered and for different levels of “moneyness”. Solutions are presented for different values of: the exercise price multiplier $\phi$ (for the buyout option); the IRR trigger and $\phi$ (conditional buyout option); the

<table>
<thead>
<tr>
<th>Option</th>
<th>Type</th>
<th>Price (in $ millions)</th>
<th>Standard error (in $ millions)</th>
<th>Price as a proportion of CDA value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Options for the government</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buyout option, 5-year vesting period, $\phi = 1$</td>
<td>American call</td>
<td>57.83</td>
<td>0.623</td>
<td>11.2</td>
</tr>
<tr>
<td>Conditional buyout option, 5-year vesting period, IRR trigger 12%, $\phi = 0$</td>
<td>American call</td>
<td>3.693</td>
<td>0.188</td>
<td>0.72</td>
</tr>
<tr>
<td>Series of annual revenue-sharing options, with share $a = 0.50$, (sum of all $T$ options)</td>
<td>European call</td>
<td>33.82</td>
<td>&lt;1.0</td>
<td>6.6</td>
</tr>
<tr>
<td>Options for the private sector developer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single, cumulative minimum revenue guarantee, fraction guaranteed $\theta = 0.50$</td>
<td>American put</td>
<td>81.76</td>
<td>1.02</td>
<td>15.9</td>
</tr>
<tr>
<td>Series of annual minimum revenue guarantee options (sum of all $T$ options), fraction guaranteed $\theta = 0.50$</td>
<td>European put</td>
<td>124.29</td>
<td>&lt;1.0</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Note: Results are based on the author’s calculations from 550,000 simulations generated from the model presented in the paper, discarding the first 50,000 as a burn-in stage.
fraction of excess revenue shared $\alpha$ (revenue-sharing option); and for the fraction of revenue guaranteed $\theta$ (minimum revenue guarantees).

The buyout option has a baseline value of $57.83$ M, given $\phi = 1$ (i.e., the exercise price equals the original CDA value). The option value nearly doubles to $108.32$ M if $\phi = 0.5$ (exercise price equal to half of the original CDA value). More interesting, however, is to consider out-of-the-money options. The option value decreases to $24.58$ M if $\phi = 1.5$ and to $6.65$ M for $\phi = 2.0$. Thus, out-of-the-money buyout options can be valued at only 1–5% of the CDA price. The conditional buyout option value, assuming $\phi = 0$, is very sensitive to the IRR threshold selected. From the baseline $3.693$ M (for a 12% IRR), the value falls to essentially zero for a 18% IRR, but increases to $7.258$ M for a 9% IRR and $65.7$ M for a 6% IRR. Thus, the conditional buyout appears to be a less reliable instrument due to its high sensitivity to the chosen IRR threshold.

Revenue-sharing options are valued at a baseline $33.82$ M for $\alpha = 0.50$. Their value increases to $50.72$ M if $\alpha = 0.75$, while it falls to $6.76$ M and $16.91$ M for $\alpha = 0.10$ and $\alpha = 0.25$ respectively. Therefore, once again it is possible to set out-of-the-money revenue-sharing options valued at only 1–5% of the CDA value.

For the developer, increasing the fraction of revenue guaranteed from $\theta = 0.50$ to 0.75 raises the value of the cumulative and annual MRG options from $81.76$ M to $129.66$ M and from $124.29$ M to $191.54$ M, respectively. Lowering $\theta$ to 0.25, the options fall in value to $45.02$ M and $71.79$ M. Lastly, considering a guarantee fraction of only $\theta = 0.10$, options are still valued at $30.04$ M and $47.39$ M. These results suggest that for Greenfield projects, even deep out-of-the-money revenue guarantees have considerable value, which is not true for deep out-of-the-money buyout options.

5.3. Model uncertainty and sensitivity analysis

The option price solutions described in the previous section are for specific project parameters and are therefore subject to model uncertainty. To evaluate model robustness, we conduct a sensitivity analysis of the solutions by varying key model parameters by ±50% of the original value. We also include a robustness check for the inclusion of a non-compete clause. The results of the sensitivity analysis are summarized in Table 4. The same random variable draws are used throughout the sensitivity analysis. A total of 50,000 runs are used for the sensitivity analysis. We examine the effect of changing model parameter values on the pricing solutions for the buyout option only. Note that the option price solution for the basic model was $57.83$ million given a CDA worth $514.7$ million.

5.3.1. Inclusion of a no-compete clause

The developer asks for a no-compete clause because it restricts the development of alternative roads and thus reduces the elasticity of the demand, increasing revenues. To model the no-compete clause, we assume that no random shocks to the demand elasticity parameter $\gamma$ may occur. As a result, the buyout option price increases to $62.61$ million. Implicitly, there is substantial value attached to the possibility of making marginal improvements to adjacent roads. The higher value reflects the role in the demand model of allowing for marginal improvements to adjacent freeways. If the public agency is not allowed to make such road improvements, demand is more inelastic than it would be otherwise and revenues increase substantially over time as toll rates increase.

Note: Results are based on the author’s calculations from 550,000 simulations generated from the model presented in the paper, discarding the first 50,000 as a burn-in stage.
5.3.2. Interest rate level

The buyout option price solution is sensitive to the risk-free rate used in the analysis. Lowering the rate from 0.02 to 0.01 increases the option value to $134.02 M, while doubling the interest rate lowers the option value to $43.93 M.

5.3.3. Demand model uncertainty

If the standard deviation \( r_Q \) of the natural log of initial traffic volume is doubled from 0.5 to 1, the option price increases to $72.75 million. If, however, \( r_Q \) is halved to 0.025, the option price falls to $55.64 million. The changes are of the correct sign but are smaller than expected, given how important the resolution of early period uncertainty is for traffic volume.

For some parameters such as vehicles per hour per lane, option price does not change monotonically. From a baseline of 1200 vehicles, option value decreases if the number of vehicles is half or double the baseline. The buyout option price is very sensitive to assumptions about traffic growth volatility \( m \). If \( m \) is doubled from 0.01 to 0.02, the option price increases substantially to $90.22 million while if \( m \) is halved, the option price falls to $32.48 million. This finding suggests that out-of-the-money options may be very useful to the public sector to limit windfall profits to the developer at a reasonable cost. Such an option is unlikely to be exercised because the probability of windfall profits is low. Lastly, we examine volatility in the (log) of construction duration. Surprisingly, the option value is not very sensitive to this parameter.

5.3.4. Average corporate tax rate

In the baseline model, the average corporate tax rate is 17.5%, to reflect a certain level of tax management. Doubling this tax rate lowers the buyout option price to $70.64 million, while halving the tax rate increases the option value to $44.52 million.

5.3.5. Duration of the concession

It is no surprise that option value is increasing in the duration of the concession. Doubling the concession duration from 50 to 100 years implies an increase in option value from $57.83 to $84.56 M, while reducing it from 50 to 25 years leads to a decrease in value to $34.32 M.

6. Conclusion and extensions

The need for innovative financing in public transportation budgeting has encouraged the use of transportation CDAs such as toll highway Build-Operate-Transfer arrangements and other leases and concessions. Although there are important benefits both for the private sector and for public authorities, there is also significant concern because of uncertainty regarding the long duration of such agreements, the difficulty of forecasting revenue streams, and the possibility of public backlash against toll increases or private developer windfall profit.

This paper asks whether strategic options, included as clauses or special provisions in CDAs, can be of mutual benefit and therefore encourage the development of CDAs to address transportation financing difficulties. Although the framework is amenable to the analysis of a fairly wide range of problems, we focus on typical Greenfield highway CDAs, which appear to be one of the thorniest policy issues in this area. A consumer demand-based framework is developed to analyze the sources of risk in CDAs, particularly in how they affect project cash flows, and examines the potential usefulness of strategic...
options to reduce uncertainty, focusing on the public sector’s options. Model parameters are calibrated using estimates from historical data and results from the established literature.

Several types of strategic options are considered, namely buyout, conditional buyout, revenue-sharing (annual), and minimum revenue guarantee (annual and one-time). They are priced using simple Monte Carlo (for European-style options) and Least Squares Monte Carlo (for American-style options). By definition, strategic or real options have non-negative value. However, it is difficult to know the magnitude of their value without modeling them. Our findings suggest that the baseline buyout and revenue-sharing options would be costly to include in a CDA, as they could be worth between one-sixth and one-fourth of the CDA concession price. However, the options become more affordable to the public sector if the exercise price (buyout price) is increased or if the degree of revenue-sharing is lowered, respectively. Therefore, out-of-the-money strategic options—which are unlikely to be exercised—can be included at relatively low cost to protect against the likelihood of a developer earning windfall profit. Such an option might also help with the initial negotiations by giving the developer the incentive to aim for consistent target returns instead of speculating on a low probability of extremely high profit. In contrast, more generous options (in terms of moneyness) are probably too costly to include in CDAs or other PPPs. More generally, for both the public agency and the developer, scaling option payoffs or changing the exercise price so the option is out-of-the-money will lower its initial value and make it easier to include in a CDA.

The paper therefore emphasizes that modified options with scaled-down payoff structures could be promising and more affordable alternatives to the baseline options. Finally, it is reasonable to conclude that, of the different options, revenue-sharing may be preferable because, while it reduces risk for the public sector, it does not increase uncertainty for the developer as much as a buyout option might. The sensitivity analysis of key model parameters conducted in the paper shows that the option price solutions are robust to most, but not all, parameters. In particular, the accuracy of the solutions depends on the accuracy of the traffic volume and volume growth parameters, which are estimated using available historical data on similar, existing CDAs.

Note that complications arise when several strategic options are included in the same CDA (see e.g. Trigeorgis, 1993). Although a thorough investigation of these effects is beyond the scope of the paper, preliminary results suggest that interaction effects lower the value of options. The magnitude of the decrease in option value varies according to the type of options included, but it can be significant. For example, the value of the developer’s annual minimum revenue guarantee options decreases by about 80% if the Government holds a buyout option. Moreover, the interaction of strategic options can allow for offsetting costs, thereby reducing risk for both counterparties without altering the original value of the CDA. For example, it is possible to find parameter values \((\alpha', \theta')\) for the proportions of revenue-sharing and revenue guarantees, respectively, that lead to equal and offsetting option values. Effectively, the developer pays for put options by selling the Government call options.

Several other extensions may be considered. The demand model could be revised to allow for rational expectations, to improve accuracy and internal consistency. Moreover, one might generalize the traffic volume model to incorporate, for example, High-Occupancy Vehicle (HOV) lanes and “shadow” toll pricing.

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Appendix A. Demand model: additional details

This appendix contains additional information about the demand model and its parameters. In the following, note that all truncated Normal random variables are limited to the support \([-3\sigma, 3\sigma]\) to avoid extreme outliers in the simulations.

A.1. Maturity of the project

The analysis concerns a Greenfield CDA could easily be adapted to a Brownfield, by changing model parameters appropriately. The main difference between Greenfield and Brownfield projects is project maturity and project location maturity, which translate into more (Greenfield) or less (Brownfield) traffic and revenue uncertainty.\(^{34}\)

\(^{34}\) Results presented in a previous version of the paper have been omitted here for brevity, but are available upon request.\(^{35}\)

\(^{35}\) In a Brownfield project, there is a history of demand data available, so forecasts are generally reliable. Moreover, the project is mature and the city or region is economically developed, so traffic growth will generally be small but more predictable. On the other hand, with a Greenfield project forecasts of initial traffic and subsequent growth are unreliable because the project is not yet built, there is no demand to observe, and it is difficult to use data from established Greenfield projects elsewhere in the country because conditions can be very different. This paper focuses on the case of a Greenfield project because uncertainty is much higher, but the framework allows for analysis of options in a Brownfield project, if the model parameters are appropriately adjusted.
A.1.1. Construction risk

Risk in construction costs and duration are substantial in Greenfield projects. Despite performance measures, cost overruns are not uncommon (Bain and Plantagie, 2007), and new projects can be delayed by political or environmental concerns, as in the case of the I-80 in Pennsylvania and New Jersey. The New Jersey Turnpike and Garden State Parkway privatization plan is referred to as monetization in New Jersey. To account for the role of political and environmental risk leading to higher costs and/or delayed construction, we assume that both cost and duration of construction are stochastic. Clearly, the actual cost of construction affects the developer’s profits, while the duration of construction delays the opening of the project, thereby reducing the NPV of the cash flows from the lease.

Assume that the log of annual construction cost is distributed truncated Normal with a mean of $200 million. The log of construction duration also follows a truncated normal distribution with mean three years. The total cost of construction is therefore a random variable itself and equals the product of $M$ years times the respective stochastic annual costs $C_m$. On average, the project would cost $600 million, and this is the value included in the concession price (adjusting to current value dollars). Following the literature, we assume that total cost is roughly monotonic in the duration of construction, such that the random variables for duration and for annual cost are independent. Specifically, $\ln M$ is distributed as truncated Normal $(\mu_m, \sigma_m^2)$ and $\ln C$ is distributed as truncated Normal $(\mu_c, \sigma_c^2)$ such that the total project cost equals the sum of $M$ different annual costs.

For purposes of model tractability, we ignore the costs associated with a project being terminated before construction begins (e.g., costs of planning and development) and we also assume that once construction begins, it can only be delayed, not cancelled.

A.2. Traffic model parameters

The following presents a description of the model parameters, the values used based on the research literature, and their interpretation. The model primitives are parameters for the peak-hour proportion of traffic, proportion of trucks and axle equivalent (for toll rate and road damage purposes), proportion of toll violations, and peak-hour traffic volume in vehicles per hour per lane (vphpl).

The peak-hour proportion of traffic volume is a fraction between 0 and 1 that describes how much of a day’s traffic volume occurs during peak hour(s). This value is typically 0.07–0.11. The proportion of trucks as a fraction of total vehicles is important to accurately estimate toll revenues as well as road damage. Instead of assuming a flat toll rate for all trucks, we model different axle-classified truck categories, since truck toll rates are usually axle-based as well. This allows us to define a single toll rate $P$ which becomes the developer’s principal decision variable. Trucks are then charged an axle-based multiplier of $P$, in effect paying a toll equal to that of several cars.

For toll road segments near state or national borders, enforcement of toll payments can be more difficult, and a fraction of users will escape toll payment enforcement. A proportion of toll avoidance is specified based on geographical location of the toll road segment. Lastly, peak-hour traffic volume in terms of vphpl is specified, where, for example, 1800 would correspond to relatively dense free-flow traffic, while 1200 would be light, and 600 would be very sparse.

To compute the annual traffic volume corner solution $A_0$ (corresponding to a toll rate or rate vector $P = 0$), we first obtain the effective peak-hour volume adjusted for truck axle-based equivalent volume and toll rate violations, then divide by the peak-hour proportion to obtain daily volume per lane. Next we multiply daily volume per lane by the number of lanes in the toll road and the number of effective days in a year. Effective days represent a way to annualize daily traffic when it is clear that traffic varies according to the day of the week, for example work commuting as opposed to weekend trips. The parameter $A_r$ represents a baseline scenario measure of annual traffic volume and is used in the demand model to derive actual quantity or volume of traffic.

A.3. Traffic volume, growth rates and ramp-up period

Assume the rate of traffic growth contains both stochastic and deterministic components. The shape of the traffic growth curve is calibrated using estimates and data from actual projects. Previous evidence suggests that, generally speaking, traffic volume in Greenfield projects should be first convex increasing in time, then reaches an inflection point, and finally becomes concave increasing in time. Although traffic volume must be asymptotically bounded above, this bound is not expected to be reached in a Greenfield project. For example, U.S. interstate highways can be considered early Greenfields and these never reached upper bounds. Thus, given time we should not expect new Greenfields to reach them either. For purposes of valuing the lease, the expectations (means) of stochastic components are used.

For a Greenfield project, traffic growth rates for all periods $t$ are modeled as:

$$g_t = g_0 \exp \left(-\alpha t + \sum_{t=1}^{t} \nu_t \right)$$

where the first-period growth rate $g_0$ is distributed as Uniform over the support 0.15–0.50, and so has an expectation of 0.325. In subsequent periods, the growth rate $g_t$ is expected to decrease gradually at the exponential rate $\exp(-\alpha t)$, not including random shocks, $\nu_t$ representing local or regional economic/demographic growth. In the baseline model,
\( \omega = 0.20 \), and \( v \) is distributed as truncated \( N(0, \sigma^2) \) over the support \([-3\sigma, 3\sigma]\). These values were selected to eliminate the possibility of unrealistic outliers. The parameter values are calibrated to replicate growth and volume trend data from existing Greenfield project data. Traffic growth should be correlated with regional population or economic growth, but since the latter variable is not used elsewhere, it is excluded from the set of simulated variables and it is assumed that traffic growth implicitly reflects localized population growth. Fig. 3 shows a large number of traffic growth paths over time.

Traffic volume in each period is determined as follows. First, the developer increases toll rates optimally according to the formula defined earlier but subject to a maximum of 4% each year. Second, traffic growth \( g_t \) is computed using the above equation together with a draw of the random variable \( v \). Traffic growth \( g_t \) affects the parameter \( A \) in the demand model, as traffic growth moves causes the demand curve to move, as well as the intercept at \( P = 0 \). Effectively:

\[
A_t = A_0 \exp(g_t) = A_0 \exp\left\{ g_0 \exp\left( -\omega t + \sum_{i=1}^{t} v_i \right) \right\}
\]

The demand equation for \( Q \) is revised using the new value of \( A \). Third, a new traffic volume \( Q_t \) is computed from the demand equation using the new, period-\( t \) values \( P_t, A_t \), and \( g_t \). The latter parameter is further discussed in the sub-section on non-compete clauses.

### A.4. Non-compete clause

A key provision in a CDA is whether a non-compete clause is included. Such a clause can have important and expensive consequences, as shown by SR-91 in California. In the baseline model, we exclude the non-compete clause because public authorities have been, particularly since SR-91, reticent to allow them. However, the effect on option values of allowing a non-compete clause is examined in the section on sensitivity analysis. For a given corridor, monopoly power may be created in the absence of easement for alternative roads. In this case, regulation would be necessary to protect road users, as it is well known from economic theory that monopoly prices are associated with sub-optimally low quantities (i.e., road use).\(^{36}\)

The elasticity parameter \( \gamma \) is assumed to be relatively high at the beginning of the Greenfield project, implying demand is elastic. Over time, however, the parameter value decreases according to a deterministic time trend, as drivers become accustomed to using the toll road. In the absence of a non-compete clause, there is a positive probability every period that improvements will be made to alternative, i.e., competing roads. These road improvements affect the demand for the toll highway by increasing the elasticity parameter \( \gamma \), such that demand becomes more elastic because drivers have more choices and as a result are more flexible. The effect of competing road improvements is captured through a Bernoulli process. Each period, a single Bernoulli\((p)\) draw is made. A Bernoulli success is interpreted as improvements made to competing roads such that \( \gamma \) increases by an amount that is normally distributed.

\(^{36}\) We thank a reviewer for emphasizing that monopoly power may be created in such a case.
An important issue that is, however, beyond the scope of this paper is to evaluate externalities associated with diversion of traffic to alternative roads following toll increases. If diversion is substantial, congestion on toll-free, alternative roads may increase the cost associated with time lost in congestion as well as the number of accidents.

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