Elementary Fluid Dynamics: The Bernoulli Equation

CVEN 311 Fluid Dynamics

HW #4

2.98, 2.104, 3.3, 3.10, 3.33, 3.39, 3.47, 3.50, 3.64, 3.77 (due 2/17 Fr. 5:00pm)

EX #1

2/23 (Thursday) 9:25 – 11:00 (10-min earlier & 10-min later)

Closed book; 1-page (letter size) sheet (front+back) allowed. Bring calculator!
Bernoulli Equation

- Assumptions needed for Bernoulli Equation
  - Inviscid (frictionless)
  - Steady
  - Constant density (incompressible)
  - Along a streamline

- Eliminate the constant in the Bernoulli equation? Apply at two points along a streamline.

- Bernoulli equation does not include
  - Mechanical energy to thermal energy
  - Heat transfer, shaft work

Hydraulic and Energy Grade Lines (neglecting losses for now)

The 2 cm diameter jet is 5 m lower than the surface of the reservoir. What is the flow rate (Q)?

What about the free jet?

Integrate $F=ma$ Normal to the Streamlines

- $-\frac{dp}{dn} = \rho \frac{V^2}{R_k} + \gamma \frac{dz}{dn}$
  - Multiply by $dn$

- $\int \frac{dp}{\rho} + \int \frac{V^2}{R_k} dn + \int \gamma dz = C$
  - Integrate

- $\frac{p}{\rho} + \int \frac{V^2}{R_k} dn + \gamma z = C$
  - If density is constant...

- $p + \rho \int \frac{V^2}{R_k} dn + \gamma z = C$
  - Normal to streamline

Bernoulli Equation

The Bernoulli Equation is a statement of the conservation of Mechanical Energy

\[ \frac{p}{\rho} + \frac{V^2}{2g} + z = C \]

- Pressure head

- Elevation head

- Velocity head

- Total head
Pressure Change Across Streamlines

\[ p + \rho \int_{B}^{n} \frac{V^2}{r} \, dn + \gamma z = C \]

If you cross streamlines that are straight and parallel, then \( p + \gamma z = C \) and the pressure is **hydrostatic**.

\[ p - \rho C^2 \int \frac{r}{2} \, dr + \gamma z = C \]
\[ V(r) = C r \]
\[ dn = -dr \]

As \( r \) decreases \( p \) **decreases**.

Summary

- By integrating \( F=ma \) along a streamline we found...
  - That energy can be converted between pressure, elevation, and velocity
  - That we can understand many simple flows by applying the Bernoulli equation
  - However, the Bernoulli equation cannot be applied to flows where viscosity is large or where mechanical energy is converted into thermal energy.

Jet Solution

The 2 cm diameter jet is 5 m lower than the surface of the reservoir. What is the flow rate \( Q \)?

\[ p_{1} + \frac{V_{1}^2}{2g} + z_1 = p_{2} + \frac{V_{2}^2}{2g} + z_2 \]
\[ z_2 = -5 \text{ m} \]

Pitot Tubes

- Used to measure air speed on airplanes
- Can connect a differential pressure transducer to directly measure \( V^2/2g \)
- Can be used to measure the flow of water in pipelines  **Point measurement!**
Example: Venturi

Find the flow \( Q \) given the pressure drop between point 1 and 2 and the diameters of the two sections. You may assume the head loss is negligible. Draw the EGL and the HGL.

\[
\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \\
\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} \\
\frac{p_1 - p_2}{\gamma} = \frac{V_2^2}{2g} \left[ 1 - \left( \frac{d_2}{d_1} \right)^4 \right] \\
V_2 = \sqrt{\frac{2g(p_1 - p_2)}{\gamma \left[ 1 - \left( \frac{d_2}{d_1} \right)^4 \right]}} \\
Q = CA \sqrt{\frac{2g(p_1 - p_2)}{\gamma \left[ 1 - \left( \frac{d_2}{d_1} \right)^4 \right]}}
\]

Example Venturi

\[
Q = VA \\
V_1 A_1 = V_2 A_2 \\
V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4} \\
V_1 d_1^2 = V_2 d_2^2 \\
V_1 = V_2 \frac{d_2^3}{d_1^3}
\]

Statics example

What is the air pressure in the cave air pocket?