Ch. 7 Stress Transformation

Given the stresses (normal & shear) on any orthogonal plane...

2D

Find the stresses on any other plane.

Recall: We proved this is possible (in 2D)

The equations that resulted are repeated in $(\tau_x, \tau_y, \tau_z)$, $(\sigma_x, \sigma_y, \sigma_z)$

The graphical form of these equations $T_{nt} (\sigma_n, \theta)$
To Construct Mohr's Circle: (assumes one of the given 3 planes is a pr. plane) PLANE STRESS.

1) On the element define x-y axes normal to two of the planes on which stresses are known.

2) Identify and write down the values of \( \sigma_x = \), \( \sigma_y = \), and \( \tau_{xy} = \) with signs.

3) On a \( T-\sigma \) axis, plot two points \((\sigma_x, T_{xy})\) and \((\sigma_y, T_{xy})\). Label these points.

(Note: Author plots \( T \) downward.)

4) Connect these two points with a diameter, compute

\[
C = \frac{\sigma_x + \sigma_y}{2}, \quad R = \left[\left(\sigma_x - C\right)^2 + T_{xy}^2\right]^{1/2}
\]

5) Draw Mohr's Circle.
How to Use Mohr's Circle

- To find principal stresses
- To find max shear stress (on any plane)
- To find stresses on any given plane

Principal stresses: The normal stress on any plane that has no shear stress is a “principal stress.”

(The plane is a “principal plane”)

To find $\sigma_1, \sigma_2, \sigma_3 \Rightarrow$ identify points where $\tau = 0$

$$\sigma_1 = C + R$$
$$\sigma_2 = C - R$$
$$\sigma_3 = 0 \text{ from stress block}$$

To find max shear stress

$$\tau_{\text{max}} = R \text{ (max shear stress in x-y plane)}$$

$$\tau_{\text{max}} = \left(\frac{\sigma_1 - \sigma_3}{2}\right)^{\frac{1}{2}} = \frac{\sigma_1 - 0}{2} = \frac{\sigma_1}{2} \text{ max shear stress on any plane}$$
To find the stress on any given plane:

1. Draw a normal to the desired plane $n$.
2. Choose a transverse direction $t$, such that $(m,t)$ is a rotated $(x,y)$ system.
3. Identify the angle from $x$ to $n$ ($180^\circ$ in this case).
4. On the circle locate the point that is twice this angle from $(0_x,0_y)$ in some direction as on element.
5. Label that point $(\tau_n, -\tau_t)$ and determine the values of $\tau_n = \tau_t = \ldots$
Example

Given the state of stress shown:

1) Draw MC
2) Find $\sigma_1, \sigma_2, \sigma_3, T_{max}$ in plane $xy$ or any plane.
3) Find the stresses on the plane $A-A$

Soln

\[
\begin{align*}
\sigma_x &= +100 \text{ psi} \\
\sigma_y &= -200 \text{ psi} \\
T_{xy} &= +50 \text{ psi}
\end{align*}
\]

\[
C = \frac{100-200}{2} = -50 \text{ psi}
\]

\[
R = \sqrt{150^2 + 50^2} = 158.1 \text{ psi}
\]

\[
\begin{align*}
\sigma_1 &= C + R = -50 + 158.1 = 108.1 \text{ psi} \\
\sigma_2 &= C - R = -50 - 158.1 = -208.1 \text{ psi} \\
\sigma_3 &= 0
\end{align*}
\]

\[T_{\text{max}} = 158.1 \text{ psi (in } xy \text{ plane)}\]

\[T_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} = 158.1 \text{ psi}\]
Find stresses on plane A-A.

\[ \tan \phi = \frac{50}{150} = \frac{1}{3} \]

\[ \phi = 18.43^\circ \]

\[ 80 + 18.43 = 98.43 \]

\[ \beta = 180 - 98.43 = 81.56^\circ \]

\[ \sigma_n = C - R \cos \beta = -50 - 158.1 \cos 81.56^\circ = -73.2 \text{ psi} \]

\[ -T_{\text{nt}} = -R \sin \beta = -158.1 \sin 81.56^\circ = -156.4 \text{ psi} \]

\[ T_{\text{nt}} = +156.4 \text{ psi} \]