Fill in your name and ID No. in the space above. There should be 9 pages including this one.

The exam is closed book, and one double-sided sheets of notes is permitted. **No collaboration with others!**

For multiple choice questions, choose the **single, BEST** answer.

For work-out problems, write down all general equations used and intermediate algebraic steps. Show all your work. Failure to do so will result in a lower score.

You have 90 minutes to complete the exam.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points earned</th>
<th>Total points possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple choice:</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Workout problem 1:</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Workout problem 2:</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Workout problem 3:</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>
Answers to the multiple choice questions. Circle the letter that corresponds to your answer for each of the multiple choice questions. Please note that only the answers you provide in this table will be graded.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I. True/False and multiple-choice questions worth 3 points each (30 pts total).

Circle the answer that is the most appropriate or closest numerically to your answer and then select that answer on the answer sheet (page 2).

1. Second-order Lagrange interpolation is more accurate than second-order Newton interpolation
   a. True
   b. False

2. Least-squares linear regression curves are not required to go through any of the data points on which the curves are based.
   a. True
   b. False

3. If \( a=[1;3] \) and \( b=[3\ 6] \), then the command \( a*b \) produces an error in Matlab
   a. True
   b. False
4. For the matrix multiplication $AB$ where $A$ and $B$ are multi-dimensional matrices
   
   a. The number of columns of $A$ must equal the number of rows of $B$.
   b. Changing the order of the multiplication results in a different answer (i.e. $AB \neq BA$).
   c. The inner matrix dimensions must agree.
   d. Only answers (a.) and (c.) are true.
   e. All three answers (a.), (b.) and (c.) are true.

5. A Matlab command that solves the matrix equation $Ax = b$ for $x$ is given by
   
   a. $x = b ./ A$
   b. $x = A' * b$
   c. $x = b * A^{-1}$
   d. $x = A^{-1} * b$
   e. Both answers (c.) and (d.)

6. If you use $LU$-decomposition to solve the matrix equation $Ax = b$ and $I$ is the identity matrix, which of the followings statements will be true?
   
   a. $LU = I$
   b. $A^{-1}A^{-1} = I$
   c. $LU = A$
   d. $x = Ub$
   e. None of the above

For problems 7 through 10, refer to the following curve-fitting problem. The consolidation $y$ in a soil column as a function of the load $x$ was measured as follows:

<table>
<thead>
<tr>
<th>$x$ [kN/m$^2$]</th>
<th>1</th>
<th>3</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ [mm]</td>
<td>3</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

$y = a_0 + a_1 x$

7. The value of the slope of the best-fit line using linear least-squares regression is
   
   a. 0.17
   b. 1.32
   c. 2.01
   d. 5.66
   e. 8.75
8. The value of the intercept of the best-fit line using linear least-squares regression is closest to
   a. -0.79
   b. 1.24
   c. 1.67
   d. 1.82
   e. 2.70

9. The coefficient of determination is the same value numerically as $r^2$.
   a. True
   b. False

10. A second-order interpolation for $y$ at $x = 6 \text{ kN/m}^2$ is closest to
    a. 8.9
    b. 9.3
    c. 9.5
    d. 9.8
    e. 10.3
II. Workout problems valued as indicated (70 pts total).
   Problem parts are valued as noted. State all important equations and assumptions.
   Show all your work.

11. Numerical differentiation (20 points). The concentration of algae in a lake during the fall is modeled as

   \[ C(t) = 2.5 \exp(-0.5t) \]  \hspace{1cm} (1)

   where \( C \) is the concentration in g/L and \( t \) is the time in months. Compute the net population decay rate \( C'(t) \) at \( t = 2 \) months using:

   a. The analytical solution (5 points).

   b. The first forward difference formula with a step size of 0.5 months (5 points).
c. The finite difference formula given by

\[ \frac{df}{dx}_{x_i} = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h} \]

with a step size of \( h = 0.5 \) months (5 points).

d. Which finite difference formula has a higher order accuracy? Please explain your answer. (5 points).
12. **Linear Regression (25 points)**. The velocity of the wind above the ground can be modeled by the boundary layer velocity profile

\[ u = ay^n \quad (2) \]

where \( u \) is the wind velocity, \( y \) is the height above the ground, and \( a \) and \( n \) are fitting parameters. The following table presents data measured in a cotton field near Snook:

<table>
<thead>
<tr>
<th>( y ) [m]</th>
<th>1</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u ) [m/s]</td>
<td>2</td>
<td>3</td>
<td>3.3</td>
</tr>
</tbody>
</table>

a. Find the parameters \( a \) and \( n \) for equation (2) using linear least-squares regression (15 points).

b. The wind profile may also be approximated by a cubic polynomial:

\[ u = ay^3 + by^2 + cy + d \]

Write down the equations you would solve to obtain the values of \( a, b, c, \) and \( d \) using matrix least squares (you do not have to calculate the result) (10 points).
13. **Matrix Algebra (25 points).** In solving a dynamics problem, a student obtains the following system of vibration equations

\[
\begin{align*}
7x_1 + 5x_2 + 4x_3 &= 35 \\
4x_1 + 5x_3 &= 28 \\
6x_1 + 7x_2 + 4x_3 &= 35
\end{align*}
\]

a. Write the system of equations in the form \( Ax = b \) (3 points).

b. Find the \( L \) and \( U \) matrices for the \( LU \)-decomposition of this system of equations using Gauss elimination with partial pivoting (7 points).
c. Solve the system of equations using

\[ A^{-1} = \begin{bmatrix}
5 & -8 & -25 \\
9 & 63 & 63 \\
-2 & -4 & 19 \\
9 & 63 & 63 \\
-4 & 19 & 20 \\
9 & 63 & 63
\end{bmatrix} \]

where \( A^{-1} \) is the inverse of the \( A \) matrix in part (a) (7 points).

d. What is the result of the matrix multiplication \( LU \) (5 points)